



**TEXAS MATH
SOLUTION**

Algebra II

Student Textbook

Skills Program Edition

SY 2022-2023

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with Josh Fisher, Janet Sinopoli, and Victoria Fisher**



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Our Manifesto

WE BELIEVE that quality math education is important for all students, to help them develop into creative problem solvers, critical thinkers, life-long learners, and more capable adults.

WE BELIEVE that math education is about more than memorizing equations or performing on tests—it's about delivering the deep conceptual learning that supports ongoing growth and future development.

WE BELIEVE all students learn math best when teachers believe in them, expect them to participate, and encourage them to own their learning.

WE BELIEVE teachers are fundamental to student success and need powerful, flexible resources and support to build dynamic cultures of collaborative learning.

WE BELIEVE our learning solutions and services can help accomplish this, and that by working together with educators and communities we serve, we guide the way to better math learning.

LONG + LIVE + MATH

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High School Math Solution Authors

- Sandy Bartle Finocchi, Chief Mathematics Officer
- Amy Jones Lewis, Senior Director of Instructional Design
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“ Mathematics is so much more than memorizing rules. It is learning to reason, to make connections, and to make sense of the world. We believe in Learning by Doing™—you need to actively engage with the content if you are to benefit from it. The lessons were designed to take you from your intuitive understanding of the world and build on your prior experiences to then learn new concepts. My hope is that these instructional materials help you build a deep understanding of math. ”

Sandy Bartle Finocchi, Chief Mathematics Officer

“ You have been learning math for a very long time—both in school and in your interactions in the world. You know a lot of math! In this course, there’s nothing brand new. It all builds on what you already know. So, as you approach each activity, use all of your knowledge to solve problems, to ask questions, to fix mistakes, and to think creatively. ”

Amy Jones Lewis, Director of Instructional Design

“ At Carnegie Learning, we have created an organization whose mission and culture is defined by your success. Our passion is creating products that make sense of the world of mathematics and ignite a passion in you. Our hope is that you will enjoy our resources as much as we enjoyed creating them. ”

Barry Malkin, CEO, Carnegie Learning

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Glossary

LESSON STRUCTURE

Each lesson has the same structure. Key features are noted.

3

Planting the Seeds

Exploring Cubic Functions

Warm Up
Use the Distributive Property to rewrite each expression.

1. $a(2a - 1)(5 + a)$
2. $(9 - x)(x + 3)$
3. $b^2(10 - b) + b^2$
4. $(w - 2)(w + 3)(w + 1)$

Learning Goals ①

- Represent cubic functions using words, tables, equations, and graphs.
- Interpret the key characteristics of the graphs of cubic functions.
- Analyze cubic functions in terms of their mathematical context and problem context.
- Connect the characteristics and behaviors of a cubic function to its factors.
- Compare cubic functions with linear and quadratic functions.
- Build cubic functions from linear and quadratic functions.

Key Terms

- cubic function
- relative maximum
- relative minimum

② You have calculated the volume of various geometric figures. How can you use what you know about volume to build an algebraic function?

LESSON 3: Planting the Seeds • 1

1. Learning Goals

Learning goals are stated for each lesson to help you take ownership of the learning objectives.

2. Connection

Each lesson begins with a statement connecting what you have learned with a question to ponder.

Return to this question at the end of this lesson to gauge your understanding.

3. Getting Started

Each lesson begins with Getting Started. When working on Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.

3

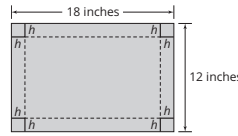
GETTING STARTED

Our Business Is Growing

The Plant-A-Seed Planter Company produces planter boxes. To make the boxes, a square is cut from each corner of a rectangular copper sheet. The sides are bent to form a rectangular prism without a top. Cutting different sized squares from the corners results in differently sized planter boxes. Plant-A-Seed takes sales orders from customers who request a sized planter box.

It may help to create a model of the planter by cutting squares out of the corners of a sheet of paper and folding.

Each rectangular copper sheet is 12 inches by 18 inches. In the diagram, the solid lines indicate where the square corners are cut, and the dotted lines represent where the sides are bent for each planter box.



1. Complete the table given each planter box is made from a 12 inch by 18 inch copper sheet. Include an expression for each planter box's height, width, length, and volume for a square corner side of length h .

Square Corner Side Length (inches)	Height (inches)	Width (inches)	Length (inches)	Volume (cubic inches)
0				
1				
2				
3				
4				
5				
6				
7				
h				

2 • TOPIC 1: Composing and Decomposing Functions

idth

Ask

yourself:

What patterns do you notice in the table?

box



LESSON 3: Planting the Seeds • 3

4

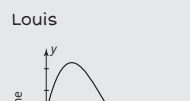
ACTIVITY 3.1 Building a Cubic Function from a Situation

Let's consider the graph of the *cubic function* you created.



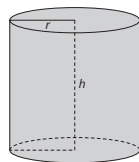
1. Louis, Ahmed, and Heidi each used graphing technology to analyze the volume function, $V(h)$, and to sketch the graph. They disagree about the shape of the graph.

A **cubic function** is a function that can be written in the general form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$.



ACTIVITY 3.2 Building a Cubic Function from a Quadratic and Linear Function

The Plant-A-Seed Company also makes cylindrical planters for city sidewalks and store fronts. The cylindrical planters come in a variety of sizes, but all have a height that is twice the radius.



1. Why do you think Plant-A-Seed might want to manufacture different sizes of a product, but maintain a constant ratio of height to radius?

Remember:
A constant ratio makes the cylindrical planters similar.

2. Consider differently sized cylindrical planters.

a. Complete the table.

Radius	Height (inches)	Base Area (square inches)	Volume (cubic inches)
0			
1			
2			
3			
4			
			2000
x			

b. Describe how you determined the volume when you are given the radius.

Volume of a cylinder: $V = (\text{base area})(\text{height})$
Area of a circle: $A = \pi r^2$

4. Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about answer-getting. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, worked examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

5. Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

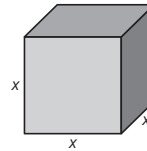
Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

NOTES

5 TALK the TALK

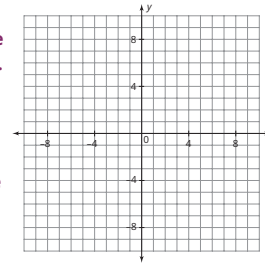
Cubism

Consider a cube, which has equal length, width, and height, x .



1. Recall that one way to determine the volume of a cube is to multiply the area of the base by its height.

a. Sketch a graph of the function that represents the area of the base of the cube.



b. Sketch a graph of the function that represents the height of the cube.

c. Sketch a graph of the function that represents the volume of the cube.

2. Which general shape does this cubic function match? Explain your reasoning.

ASSIGNMENT

Assignment

LESSON 3: Planting the Seeds

6

Write

- Provide an example of each key term.
- relative minimum
 - relative maximum
 - cubic function

7

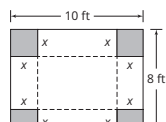
Remember

A cubic function is a polynomial function of degree 3 that can be written in the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$. The graph has 2 general shapes.

8

Practice

- Cynthia is an engineer at a manufacturing plant. Her boss asks her to use rectangular metal sheets to build storage bins with the greatest possible volume. Each rectangular sheet is 8 feet by 10 feet. Cynthia's sketch shows the squares to be removed from the corners of each sheet. The



Use a tablet computer or use the QR code if possible to view the practice questions.



- indicate where the metal storage bins without top
- Write a function $V(x)$ to represent the volume of the bins. Explain your reasoning.
 - Graph the function $V(x)$. Determine the domain of the function. Explain your reasoning.
 - Determine the maximum volume of a bin. Explain your reasoning.
 - Determine any relative maximums or relative minimums of $V(x)$. Then, determine the intervals over which the function is increasing and decreasing.
 - Determine the x - and y -intercepts of the graph of $V(x)$. What do they represent in this problem situation?
 - Nikki's boss asks her to make several bins with volumes of exactly 40 cubic feet. Determine the bin dimensions that will work.

9

Stretch

- Nikki is an engineer at a manufacturing plant. Her boss asks her to use rectangular metal sheets to build storage bins with the greatest possible volume. Each rectangular sheet is 8 feet by 10 feet. Nikki's sketch shows the squares to be removed from the corners of each sheet. The dashed lines indicate where the metal storage bins without top
- Write a function $V(x)$ to represent the volume of the bins. Explain your reasoning.
 - Graph the function $V(x)$. Determine the domain of the function. Explain your reasoning.

- Determine the maximum volume of a bin. What are the dimensions of a bin with the maximum volume?
- Determine any relative maximums or relative minimums of $V(x)$. Then, determine the intervals over which the function is increasing and decreasing.
- Determine the x - and y -intercepts of the graph of $V(x)$. What do they represent in this problem situation?
- Nikki's boss asks her to make several bins with volumes of exactly 40 cubic feet. Determine the bin dimensions that will work.

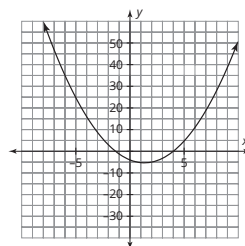
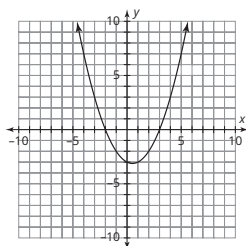
10

Review

- Dilate each function by the given factor to create a new function of higher degree. Sketch the graph and then identify the zeros of the new function.

a. $f(x) = \left(\frac{1}{2}x + 1\right)(x - 3)$
Sketch $(x + 1) \cdot f(x)$.

b. $g(x) = (3x + 4)\left(\frac{1}{4}x + 2\right)$
Sketch $(x - 1) \cdot f(x)$.



- The figures shown represent a visual pattern of tiles.
 - Create a table to display the number of squares used in each of the first 6 figures.
 - Create a graph of the data points in your table on the coordinate plane shown. Draw a smooth curve to connect the points.
 - Describe the pattern as linear, exponential, quadratic, or none of these. Explain your reasoning.
- Solve the equation $x^2 - 6x + 35 = 10$.

6. Write

Reflect on your work and clarify your thinking.

7. Remember

Take note of the key concepts from the lesson.

8. Practice

Use the concepts learned in the lesson to solve problems.

9. Stretch

Ready for a challenge?

10. Review

Remember what you've learned by practicing concepts from previous lessons and topics.

PROBLEM TYPES YOU WILL SEE

Worked Example

When you see a Worked Example:

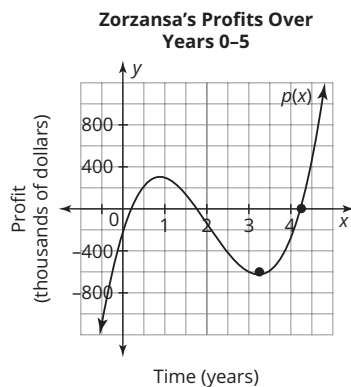
- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

Ask Yourself:

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

Worked Example

You can determine the average rate of change of Zorzansa's profit for the time interval (3.25, 4.25).



Substitute the input and output values into the average rate of change formula.

Evaluate the expression.

$$\begin{aligned} \frac{f(b) - f(a)}{b - a} &= \frac{f(4.25) - f(3.25)}{4.25 - 3.25} \\ &= \frac{0 - (-600)}{1} \\ &= \frac{600}{1} = 600 \end{aligned}$$

The average rate of change for the time interval (3.25, 4.25) is approximately \$600,000 per year.

Who's Correct?

When you see a

Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or not correct.

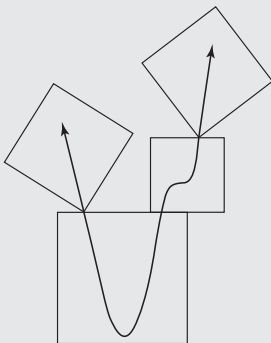
Ask Yourself:

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

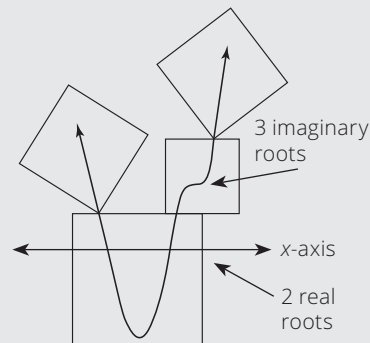
4. Novena created this graph of a fourth degree polynomial. Armondo said that she is incorrect, that it is a fifth degree polynomial. Who is correct? For the student who is incorrect, explain the error in their thinking.



Novena



Armondo



Thumbs Up

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.

Ask Yourself:

- Why is this method correct?
- Have I used this method before?

Augie



The cubic function $f(x) = (x - 3)(x - 1)(x + 4)$ has the three zeros given. I can verify this by solving the equations $x - 3 = 0$, $x - 1 = 0$, and $x + 4 = 0$.

Thumbs Down

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

Ask Yourself:

- Where is the error?
- Why is it an error?
- How can I correct it?

Emily



A cubic function must have three zeros. I know this from the Fundamental Theorem of Algebra. However, the number of real and imaginary zeros can vary. The function may have 0, 1, 2, or 3 imaginary zeros.

MATHEMATICAL PROCESS STANDARDS

Texas Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The “I can” expectations listed below align with the TEKS Mathematical Process Standards and encourage students to develop their mathematical learning and understanding.

► **Apply mathematics to problems arising in everyday life, society, and the workplace.**

I can:

- use the mathematics that I learn to solve real-world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

► **Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.**

I can:

- explain what a problem “means” in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

► **Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.**

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

► **Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.**

I can:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

► **Create and use representations to organize, record, and communicate mathematical ideas.**

I can:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

► **Analyze mathematical relationships to connect and communicate mathematical ideas.**

I can:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

► **Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.**

I can:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.

ACADEMIC GLOSSARY

There are important terms you will encounter throughout this book. It is important that you have an understanding of these words as you get started on your journey through the mathematical concepts. Knowing what is meant by these terms and using these terms will help you think, reason, and communicate your ideas.

Visit the Students & Caregivers Portal on the Texas Support Center at www.CarnegieLearning.com/texas-help to access the Mathematics Glossary for this course anytime, anywhere.



ANALYZE

Definition

To study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

Related Phrases

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

EXPLAIN YOUR REASONING

Definition

To give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

Related Phrases

- Show your work
- Explain your calculation
- Justify
- Why or why not?

Related Phrases

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

REPRESENT

Definition

To display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

Related Phrases

- Predict
- Approximate
- Expect
- About how much?

ESTIMATE

Definition

To make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

Related Phrases

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

DESCRIBE

Definition

To represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

Thought Bubbles

Look for these icons as you journey through the textbook. Sometimes they will remind you about things you already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are here to help and guide your learning.



Remember:



Think

about:



Ask

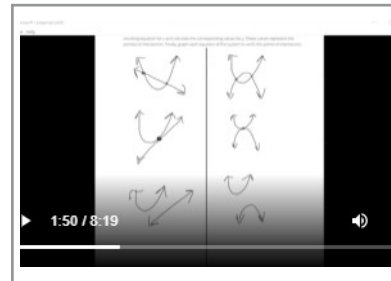
yourself:

Side notes are included to provide helpful insights as you work.

Resources for Students and Caregivers

Student Lesson Overview Videos

Each lesson has a corresponding lesson overview video(s) for you to use and reference as you are learning. The videos provide an overview of key concepts, strategies, and/or worked examples from the lessons.



Topic Summary

A Topic Summary is provided at the end of each topic. The Topic Summary lists all key terms of the topic and provides a summary of each lesson. Each lesson summary defines key terms and reviews key concepts, strategies, and/or worked examples.

Extending Linear Relationships Summary

KEY TERMS

- Gaussian elimination
- solution of a system of linear inequalities
- linear programming
- matrix (matrices)
- dimensions
- square matrix
- matrix element
- matrix multiplication
- identity matrix
- multiplicative inverse of a matrix
- matrix equation
- coefficient matrix
- variable matrix
- constant matrix
- absolute value
- reflection
- line of reflection
- argument of a function
- linear absolute value equation
- linear absolute value inequality
- equivalent compound inequality

LESSON 1 Gauss in Das Haus

Recall that a system of two linear equations can be solved algebraically by using either the substitution method or the linear combinations method. Systems of two linear equations can have one solution, no solution, or an infinite number of solutions.

A system of two equations involving one linear equation and one quadratic equation can be solved using methods similar to those for solving a system of two linear equations. These systems have one solution, two solutions, or no solutions. The solution(s) can be verified by graphing equations on the same coordinate grid and then calculating the point(s) of intersection.

For example, you can solve the following system of two equations in two variables algebraically then verify the solution graphically.

$$\begin{cases} y = x + 1 \\ y = x^2 - 3x + 4 \end{cases}$$

TOPIC 1: Sum

$$\begin{aligned} x^2 - 3x + 4 &= x + 1 \\ x^2 - 4x + 3 &= 0 \\ (x - 3)(x - 1) &= 0 \end{aligned}$$

$x = 3$ or $x = 1$

Substitute $x = 3$ into the linear equation.
 $y = 3 + 1 = 4$

Substitute $x = 1$ into the linear equation.
 $y = 1 + 1 = 2$

The solutions to the system are (3, 4) and (1, 2).

To solve a system of three linear equations using substitution, the first step is to solve for one variable in one of the equations. Then substitute this expression for that variable in the other two equations. The two new equations will then have only two unknown variables and can be solved using either substitution or linear combinations.

The goal of **Gaussian elimination** is to use linear combinations to isolate one variable for each equation. When using this method, you can:

- swap the positions of two equations.
- multiply an equation by a nonzero constant.
- add one equation to the multiple of another.

For example, you can solve the system $\begin{cases} x + 5y - 6z = 24 \\ -x - 4y + 5z = -21 \\ 5x - 4y + 2z = 21 \end{cases}$ using Gaussian elimination.

Add the first and second equation and replace the second equation.

$$\begin{aligned} x + 5y - 6z &= 24 \\ -x - 4y + 5z &= -21 \\ \hline y - z &= 3 \end{aligned} \rightarrow \begin{cases} x + 5y - 6z = 24 \\ y - z = 3 \\ 5x - 4y + 2z = 21 \end{cases}$$

Multiply the first equation by -5 and add it to the third equation. Replace the third equation.

$$\begin{aligned} -5x - 25y + 30z &= -120 \\ 5x - 4y + 2z &= 21 \\ \hline -29y + 32z &= -99 \end{aligned} \rightarrow \begin{cases} x + 5y - 6z = 24 \\ y - z = 3 \\ -29y + 32z = -99 \end{cases}$$

Multiply the second equation by 29 and add it to the third equation. Replace the third equation.

$$\begin{aligned} 29y - 29z &= 87 \\ -29y + 32z &= -99 \\ \hline 3z &= -12 \end{aligned} \rightarrow \begin{cases} x + 5y - 6z = 24 \\ y - z = 3 \\ 3z = -12 \end{cases}$$

2 • TOPIC 1: Extending Linear Relationships

Mathematics Glossary

A course-specific mathematics glossary is available to utilize and reference while you are learning. Use the glossary to locate definitions and examples of math key terms.

Glossary

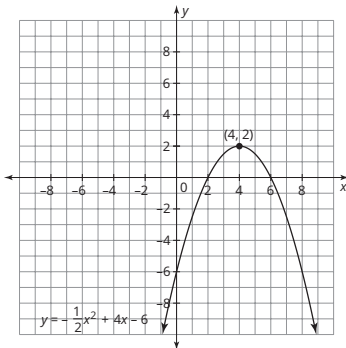
A

absolute maximum

A function has an absolute maximum if there is a point that has a y -coordinate that is greater than the y -coordinates of every other point on the graph.

Example

The ordered pair $(4, 2)$ is the absolute maximum of the graph of the function $f(x) = -\frac{1}{2}x^2 + 4x - 6$.

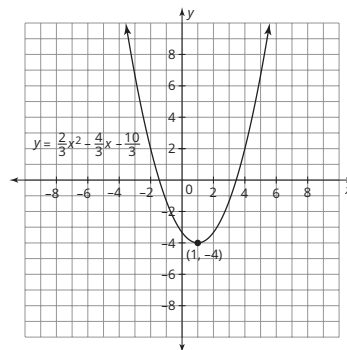


absolute minimum

A function has an absolute minimum if there is a point that has a y -coordinate that is less than the y -coordinates of every other point on the graph.

Example


The ordered pair $(1, -4)$ is the absolute minimum of the graph of the function $y = \frac{2}{3}x^2 - \frac{4}{3}x - \frac{10}{3}$.



Module Family and Caregiver Guides

Each module guide will provide a different highlight of the academic glossary, description and examples of TEKS Mathematical Process Standards, and an overview of a different component of our instructional approach known as The Carnegie Learning Way. Also included is a module overview of content, specific key terms, visual representations, and strategies you are learning in each topic of the module.

The purpose of the Family and Caregiver Guides is to bridge student learning in the classroom to student learning at home. Our goal is to empower you and your family to understand the concepts and skills learned in the classroom so that you can review, discuss, and solidify the understanding of these key concepts together. Videos will also be available on the Students & Caregivers Portal on the Texas Support Center to provide added support.

MODULE 1 FAMILY AND CAREGIVER GUIDE 

Read and share with your student.

How to support your student as they learn about

Exploring Patterns in Linear and Quadratic Relationships

Mathematics is a connected set of ideas, and your student knows a lot. Encourage them to use the mathematics they already know when encountering new concepts in this topic.

Module Introduction

In this module your student will progress from simple functions to the more complex polynomial, rational, radical, and logarithmic functions. There are 3 topics in this module: *Extending Linear Relationships, Exploring and Analyzing Patterns, and Applications of Quadratics.* Your student will use what they already know about the absolute value of numbers and their understanding of the structure of equations in this module.

Academic Glossary


Each module will focus on an important term. Knowing and using these terms will help your student think, reason, and communicate their math ideas.

Term	Analyze
Definition	<ul style="list-style-type: none"> To study or look closely for patterns. To break a concept down into smaller parts to gain a better understanding of it.
Questions to Ask Your Student	<ul style="list-style-type: none"> Do you see any patterns? Have you seen something like this before? What happens if the shape, model, or numbers change?
Related Phrases	<ul style="list-style-type: none"> Examine Evaluate Determine Observe Consider Investigate What do you notice?


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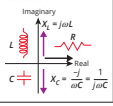
Analyze the figures shown. Can you create an expression to represent the number of blocks where n represents the figure number?




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MODULE 1 FAMILY AND CAREGIVER GUIDE 

Module Overview

TOPIC 1	TOPIC 2	TOPIC 3
Extending Linear Relationships	Exploring and Analyzing Patterns	Applications of Quadratics
17 Days	15 Days	15 Days
Your student will advance their ability to write and solve systems of equations, analyze solutions and use new methods such as Gaussian elimination and matrices to solve systems of linear equations in three variables.	Your student will analyze and describe different patterns and use algebraic expressions to represent the patterns. Then they will expand their knowledge of complex numbers and use complex numbers to solve quadratic equations.	Your student will model real-world scenarios using quadratic functions, inequalities, and systems with a linear function or expression. They will also develop the fundamentals for inverses of functions and use inverses to further solve problems.
Did you know that? The first signs of mobile usage was in 19th/2nd century BCE in Chinese math books.	Did you know that? Imaginary numbers are often used in electricity, specifically alternating current (AC) electronics. AC electricity changes between positive and negative. Using imaginary and real numbers helps those working with AC electricity do calculations and avoid electrocution.	What is the world? A simple form of inverse functions that we use everyday but maybe don't think about is the temperature outside! Normally we use Fahrenheit but Celsius is actually just the inverse of it!
		$C = \frac{5}{9}(F - 32)$ $F = \frac{9}{5}C + 32$

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MODULE 1 FAMILY AND CAREGIVER GUIDE 

Math Process Standards

Each module will focus on a process (or a pair of processes) that will help your student become a mathematical thinker. The "I can" statements listed below help your student to develop their mathematical learning and understanding.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I can:




- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.

Look for examples of these processes in the Topic Summaries.

The Carnegie Learning Way


Our Instructional Approach

Carnegie Learning's instructional approach is based on how people learn and real-world understandings. It is based on three key components:

ENGAGE	DEVELOP	DEMONSTRATE
		
Purpose: Provide an introduction that creates curiosity and uses what students already know and have experienced.	Purpose: Build a deep understanding of mathematics through different activities.	Purpose: Reflect on and evaluate what was learned.
Questions to Ask: How does this problem look like something you did in class?	Questions to Ask: Do you know another way to solve this problem? Does your answer make sense?	Questions to Ask: Is there anything you do not understand?

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ONLINE RESOURCES FOR FAMILIES AND CAREGIVERS
<https://www.carnegielearning.com/texas-help/students-caregivers/>

MODULE 1 FAMILY AND CAREGIVER GUIDE 

Topic 1: Extending Linear Relationships

Key Terms

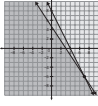
- Gaussian elimination
- solution of a system of linear inequalities
- linear programming
- matrix (matrices)
- dimensions
- square matrix
- matrix element
- matrix multiplication
- identity matrix
- multiplicative inverse of a matrix
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- linear absolute value equation
- linear absolute value inequality
- equivalent compound inequality

The solution of a system of inequalities is the intersection of the solutions to each inequality.


Every point that both solutions share satisfies all inequalities in the system.


A matrix (plural matrices) is an array of numbers composed of rows and columns. A matrix is usually designated by a capital letter.

A line of reflection is the line upon which an object is flipped or mirrored across.



$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 3 & 1 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$





Follow the link to access the Student Glossary:
<https://www.carnegielearning.com/texas-help/students-caregivers/>

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Topic Family Guides

Each topic contains a Family Guide that provides an overview of the math of the topic and answers the questions, “Where have we been?” and “Where are we going?” Additional components of the Family Guide are, as follows: new notation or strategy taught in the topic, definitions of a few key terms, connection of math to the real world, related standardized test question sample, or talking points for caregivers to support your learning.

We recognize that learning outside of the classroom is crucial to student success at school. While we don’t expect families and caregivers to be math teachers, the Family Guides are designed to assist families and caregivers as they talk to you about what you are learning. Our hope is that both you and your family will read and benefit from these guides.

Carnegie Learning Family Guide
Algebra II

Module 1: Extending Linear Relationships

TOPIC 1: EXTENDING LINEAR RELATIONSHIPS

Students begin this topic by reviewing what they know about systems of linear equations. They apply this knowledge to solve systems involving a linear and a quadratic equation and systems of three linear equations in three variables. Students also use systems of linear inequalities and linear programming to model optimal solutions to real-world situations. They use matrices to solve systems of linear equations in three variables.

Next, they calculate the absolute value of given values before considering the linear absolute value function. Students first graph the function $f(x) = x$, and then graph $f(x) = |x|$ and $f(x) = |-x|$, discussing how each graph changed. Students explore transformations of the function before moving on to solve and graph linear absolute value equations and inequalities based on real-world situations.

Where have we been?

Students enter this topic with a wide range of experiences with linear functions. Students have set up and solved systems of equations since late middle school and early high school. They have investigated properties of real numbers, including the multiplicative identity and multiplicative inverse. In this topic, students will extend these properties to a new object—a matrix.

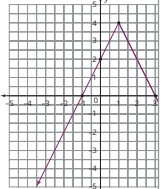
Where are we going?

Although derived from linear relationships, linear absolute value functions are more complex than the linear functions students have dealt with previously. They share characteristics with linear functions familiar to students, but they also serve as a bridge to the nonlinear functions the student studies during the remainder of this course: quadratic functions, polynomial functions, radical and rational functions.

Linear Absolute Value Function


The coordinate plane shows the graph of the linear absolute value function $f(x) = -2|x - 1| + 4$.

The graph increases to a vertex and then decreases and is symmetric across a vertical line through the vertex.



Systems

Your body is an amazing collection of different systems. Your cardiovascular system pumps blood throughout your body, your skeletal system provides shape and support, and your nervous system controls communication between your senses and your brain. Your skin, including your hair and fingernails, is a system all by itself—the integumentary system—and it protects all of your body's other systems. You also have a digestive system, endocrine system, excretory system, immune system, muscular system, reproductive system, and respiratory system.



Talking Points

Absolute value is an important topic to know about for college admissions tests.

Here is an example of a sample question:

What are the values of n and p so that $-n|2p - 6| > 0$?

For the product to be greater than 0, the factors must be either both greater than 0 or both less than 0.

Since one of the factors is an absolute value, the factors cannot be both less than 0, so they are both greater than 0.

This means that n must be less than 0, and p cannot be equal to 3.

The solution is all values such that $n < 0$ and $p \neq 3$.

Key Terms

linear programming
Linear programming is a branch of mathematics that determines the maximum and minimum value of linear expressions on a region produced by a system of linear inequalities.

matrix
A matrix (plural matrices) is an array of numbers composed of rows and columns.

absolute value
The absolute value of a number is its distance from zero on the number line.

line of reflection
A line of reflection is the line that the graph is reflected across.

linear absolute value equation
An equation in the form $|x + a| = c$ is a linear absolute value equation.

2 • TOPIC 1: Extending Linear Relationships



Students and Caregivers Portal

Research has proven time and again that family engagement greatly improves a student's likelihood of success in school.

The Students & Caregivers Portal on the Texas Support Center provides:



- Getting to Know Carnegie Learning video content to provide an introduction to the instructional materials and research.
- Articles and quick tip videos offering strategies for how families and caregivers can support student learning.
- Access to instructional resources to support students and caregivers.

To access new content and resources, visit the Students and Caregivers Portal on the Texas Support Center at <https://www.CarnegieLearning.com/texas-help/students-caregivers/>