TEXAS MATH SOLUTION

# Algebra II

## Teacher's Implementation Guide

### Skills Program Edition

SY 2022-2023

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Cover Design by Anne Milliron



ISBN: 978-1-63862-078-5
Teacher's Implementation Guide

Printed in the United States of America
1 2 3 4 5 6 7 8 9 CC 21 20 19 18 17

# Our Manifesto

**WE BELIEVE** that quality math education is important for all students, to help them develop into creative problem solvers, critical thinkers, life-long learners, and more capable adults.

**WE BELIEVE** that math education is about more than memorizing equations or performing on tests—it's about delivering the deep conceptual learning that supports ongoing growth and future developments.

**WE BELIEVE** all students learn math best when teachers believe in them, expect them to participate, and encourage them to own their learning.

**WE BELIEVE** all teachers teach math best when they really know the content, have the desire and right mindset, and get the resources and support they need to build cultures of collaborative learning.

**WE BELIEVE** our learning solutions and services can help accomplish all of this, and that by working together with educators and communities we serve, we guide the way to better math learning.

LONG + LIVE + MATH

At Carnegie Learning, we choose the path that has been proven most effective by research and classroom experience. We call that path the Carnegie Learning Way. Follow this code to take a look inside.

# ACKNOWLEDGMENTS

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- Lumina Datamatics, Ltd.
- Mathematical Expressions, LLC

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www.pixabay.com

## Special Thanks

- Alison Huettner for project management and editorial review.
- Jacyln Snyder for her contributions to the Teacher's Implementation Guide facilitation notes.
- Harry Lynch for his contributions and review of the Statistics and Probability strand.
- Madison Kalo for her design contributions.
- The members of Carnegie Learning Cognitive Scientist Team—Brendon Towle, John Connelly, Bob Hausmann, Chas Murray, and Martina Pavelko—for their insight in learning science and collaboration on MATHia® Software.
- John Jorgenson, Chief Marketing Officer, for all his insight and messaging.
- Carnegie Learning Education Services Team for content review and providing customer feedback.
- The entire Carnegie Learning staff for their hard work and dedication to transforming math education.
- The families of the authoring team for their continued support.

# ACKNOWLEDGMENTS


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## Special Thanks

- The entire Carnegie Learning Production Team, with extreme gratitude for Sara Kozelnik, Julie Leath, Lindsay Ryan, Angela Cerbone, Lenore MacLeod, Hannah Mumm, Olivia Rangel, Chloe Thompson and Emily Tope, for their patience, attention to detail, and around-the-clock hours that made the production of this textbook possible.
- Thank you to all the Texas educators and education professionals who supported the review process and provided feedback for this resource.

"Mathematics is so much more than rules and algorithms. It is learning to reason, to make connections, and to make sense of the world. We believe in Learning by Doing™—students need to actively engage with the content if they are to benefit from it. Your classroom environment will determine what type of discourse, questioning, and sharing will take place. Students deserve a safe place to talk, to make mistakes, and to build deep understanding of mathematics. My hope is that these instructional materials help you shift the mathematical authority in your class to your students. Be mindful to facilitate conversations that enhance trust and reduce fear."

**Sandy Bartle Finocchi, Chief Mathematics Officer**

"Your students come to you, not as clean slates, but as messy boards full of knowledge that they have gained in previous math classes and also in the world. The lessons in this book are designed to build off what students already know. I encourage you to build confidence in your students by asking them questions to uncover what they already know, connecting their prior experiences with new ideas, providing them time to make connections and to persevere through problems, and giving only the support necessary to keep them on the right path."

**Amy Jones Lewis, Senior Director of Instructional Design**

"At Carnegie Learning, we have created an organization whose mission and culture is defined by student success. Our passion is creating products that make sense of the world of mathematics and ignite a passion in students. Our hope is that students will enjoy our resources as much as we enjoyed creating them."

**Barry Malkin, CEO, Carnegie Learning**

At Carnegie Learning, we choose the path proven most effective by research and classroom experience. We call that path the **Carnegie Learning Way.**

# Our Instructional Approach

Carnegie Learning's instructional approach is based upon the collective knowledge of our researchers, instructional designers, cognitive learning scientists, and master practitioners. It is based on a scientific understanding of how people learn and a real-world understanding of how to apply the science to the classroom. At its core, our instructional approach is based on three simple yet critical components:

### ENGAGE

**Activate student thinking by tapping into prior knowledge and real-world experiences.** Provide an introduction that generates curiosity and plants the seeds for deeper learning.

### DEVELOP

**Build a deep understanding of mathematics through a variety of activities.** Students encounter real-world problems, sorting activities, worked examples, and peer analysis—in an environment where collaboration, conversations, and questioning are routine practices.

### DEMONSTRATE

**Reflect on and evaluate what was learned.** Ongoing formative assessment underlies the entire learning experience, driving real-time adjustments, next steps, insights, and measurements.

# Our Research

Carnegie Learning has been deeply immersed in research ever since it was founded by cognitive and computer scientists from Carnegie Mellon University. Our research extends far beyond our own walls, playing an active role in the constantly evolving field of cognitive and learning science. Our internal researchers collaborate with a variety of independent research organizations, tirelessly working to understand more about how people learn, and how learning is best facilitated. We supplement this information with feedback and data from our own products, teachers, and students, to continuously evaluate and elevate our instructional approach and its delivery.

# Our Support

We're all in. In addition to our instructional resources, implementing Carnegie Learning in your classroom means you get access to an entire ecosystem of ongoing classroom support, including:

- **Professional Learning:** Our team of Master Math Practitioners is always there for you, from implementation to math academies to a variety of other options to help you hone your teaching practice.
- **Texas Support Center:** We've customized a Support Center just for you and your students. The Texas Support Center provides articles and videos to help you implement the Texas Math Solution, from the basics to get you started to more targeted support to guide you as you scaffold instruction for all learners in your classroom. Visit **www.CarnegieLearning.com/texas-help** to explore online and to access content that you can also share with your students and their caregivers.
- **MyCL:** This is the central hub that gives you access to all of the products and resources that you and your students will need. Visit MyCL at **www.CarnegieLearning.com/login**.
- **LONG + LIVE + MATH:** When you join this community of like-minded math educators, suddenly you're not alone. You're part of a collective, with access to special content, events, meetups, book clubs, and more. Because it's a community, it's constantly evolving! Visit **www.longlivemath.com** to get started.

Scan this code to visit the Texas Support Center and look for references throughout the Front Matter to learn more about the robust resources you will find in the Support Center.

# Our Blend of Learning

The Texas Math Solution delivers instructional resources that make learning math attainable for all students. Learning Together and Learning Individually resources work in parallel to engage students with various learning experiences they need to understand the mathematics at each grade level.

For **Learning Together**, the student textbook is a consumable resource that empowers students to become creators of their mathematical knowledge. This resource is designed to support teachers in facilitating active learning so that students feel confident in sharing ideas, listening to each other, and learning together.

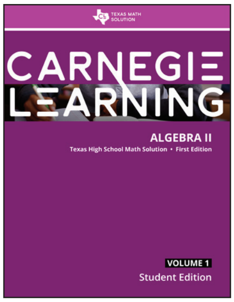
Over the course of a year, based on the recommended pacing, teachers will spend approximately 60% of their instructional time teaching whole-class activities as students learn together.

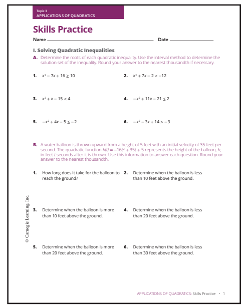
For **Learning Individually**, the Skills Practice provides students the opportunity to engage with problems that target each lesson's skills, concepts, and applications. This resource is designed to target discrete skills for development and mastery, therefore, scaffolding and extension opportunities are provided in the problem sets.

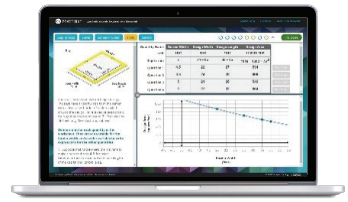
An additional Learning Individually resource is MATHia®, an intelligent software that provides just-in-time support and tracks student progress against fine-grained skills to deliver the right content they need to become proficient with the mathematics.

Over the course of the year, based on the recommended pacing, teachers will spend approximately 40% of their instructional time monitoring students as they work and learn individually.

| Learning Together | Learning Individually | |
|---|---|---|



## TEXTBOOK

I am a record of student thinking, reasoning, and problem solving. My lessons allow students to build new knowledge based upon prior knowledge and experiences, apply math to real-world situations, and learn together in a collaborative classroom.

My purpose is to create mathematical thinkers who are active learners that participate in class.

## SKILLS PRACTICE

I am targeted practice of each lesson's skills, mathematical concepts, and applications for each topic in the student textbook.

My purpose is to provide additional problem sets for teachers to assign as needed for additional practice or remediation.

## MATHia

I am designed to empower students to learn individually at their own pace with sophisticated AI technology that personalizes their learning experiences, while giving teachers real-time insights to monitor student progress.

My purpose is to coach students alongside teachers as students learn, practice, do, and look forward.

Visit the Texas Support Center for additional information on the Learning Individually resources.

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## Topic 2: Radical Functions



# Module 5: Inverting Functions

## Topic 1: Exponentials and Logarithmic Functions



## Topic 2: Exponential and Logarithmic Equations

## End of Course Topic

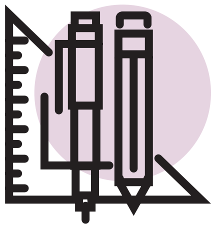
In a word, every single piece of Carnegie Learning's Texas Math Solution is intentional. Our instructional designers work alongside our master math practitioners, cognitive scientists, and researchers to intentionally design, draft, debate, test, and revise every piece, incorporating the latest in learning science.

# Intentional Mathematics Design

Carnegie Learning's Texas Math Solution is thoroughly and thoughtfully designed to ensure students build the foundation they'll need to experience ongoing growth in mathematics.

### Mathematical Coherence

The arc of mathematics develops coherently, building understanding by linking together within and across grades, so students can learn concepts more deeply and apply what they've learned to more complex problems going forward.



### Mathematical Process Standards

Carnegie Learning is organized around the Mathematical Process Standards to encourage experimentation, creativity, and false starts, which is critical if we expect students to tackle difficult problems in the real world, and persevere when they struggle.

### Multiple Representations

Carnegie Learning recognizes the importance of connecting multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

### Transfer

Carnegie Learning focuses on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.

# Texas Math Solution Overview

The instructional materials in the Carnegie Learning Texas Math Solution cover functions, figures, and data sets, from their fundamental concepts to the connections between them. We think about these interrelated ideas in a holistic way to integrate students' understanding with their developing habits of mind.

Our classroom activities emphasize active learning and making sense of the mathematics, and we ask deep questions that require students to thoroughly understand the material.

### WHAT ARE THE CARNEGIE LEARNING TEXAS MATH SOLUTION GUIDING PRINCIPLES?

The Texas Math Solution has been strongly influenced by scientific research into the learning process and student motivations for academic success. Its guiding principles are active learning, discourse through collaboration, and personalized learning.

## Active Learning

The research makes it clear that students need to actively engage with content in order to benefit from it. Studies show that as instruction moves up the scale from entirely passive to fully interactive, learning becomes more robust. All of the activities we provide for the classroom encourage students to be thoughtful about their work, to consider hypotheses and conclusions from different perspectives, and to build a deep understanding of mathematics. The format of the student text, as a consumable workbook, supports active instruction.

## Discourse through Collaborative Learning

Effective collaboration encourages students to articulate their thinking, resulting in self-explanation. Reviewing other students' approaches and receiving feedback on their own provides further metacognitive feedback. Collaborative problem-solving encourages an interactive instructional model, and we have looked to research to provide practical guidance for making collaboration work. The collaborative activities within our lessons are designed to promote active dialogue centered on structured activities.

## Personalized Learning

One of the ways to build intrinsic motivation is to relate activities to students' existing interests. Research has proven that problems that capture student interests are more likely to be taken seriously. In the textbook, problems often begin with the students' intuitive understanding of the world and build to an abstract concept, rather than the other way around.

**Functions • Figures • Data Sets**

Transformation

Equivalence & Congruence

Proportionality & Similarity

### HOW IS THE CONTENT DEVELOPED IN A MATHEMATICALLY COHERENT WAY?

Throughout the high school math courses of the Texas Math Solution, students examine and investigate functions, figures, and data sets. Within each category, we strive to extend and connect students' experience in middle school around the critical mathematical ideas of transformation, equivalence and congruence, and proportionality and similarity.

## Transformation

Transforming functions and figures builds from an understanding of the fundamental behaviors of translations, rotations, reflections, and dilations. These behaviors apply in the same ways to different function types in algebra and to geometric figures on the plane. Understanding the structure of transformations leads to connections across multiple domains in multiple courses.

## Equivalence & Congruence

Equivalence is approached in two ways. First, understanding equivalence using multiple relationships of the same function or data set reveals different properties or key characteristics. Second, understanding equivalence in terms of expressions allows students to compose and decompose equations, make sense of solutions, and solve problems. Congruence is treated similarly: understanding congruence using rigid motions highlights key characteristics that are true for both figures, which leads to establishing triangle congruence criteria, an important underpinning for formal proof. The concept of equivalence is extended to the analysis of data, where students learn the critical skill of representing data in equivalent but differently useful ways, enabling them to make analyses and decisions.

## Proportionality & Similarity

Developing proportional reasoning is a life-long journey that begins in middle school: from ratios and proportions to understanding how linear functions relate to sequences with common differences and how exponential functions relate to sequences with common ratios. Exploring dilations and the relationships that hold true in similar figures develops spatial reasoning. Analyzing similarity in right triangles extends to right triangle trigonometry, connecting the algebra and geometry domains.

### HOW IS THE MATHEMATICS CONTENT DELIVERED TO PROMOTE PRODUCTIVE MATHEMATICAL PROCESSES?

Students deserve math learning that develops them into creative problem solvers, critical thinkers, life-long learners, and more capable adults, while teachers deserve instructional resources that will support them in bringing learning to life. There are three organizing principles that guide these resources.

## Seeing Connections

Activities make use of models—e.g., real-world situations, graphs, diagrams, and worked examples—to help students see and make connections between different topics. In each lesson, learning is linked to prior knowledge and experiences so that students build their new understanding on the firm foundation of what they already know. We help students move from concrete representations and an intuitive understanding of the world to more abstract representations and procedures. Activities thus focus on real-world situations to demonstrate the usefulness of mathematics.

## Exploring Structure

Questions are phrased in a way that promotes analysis, develops higher-order-thinking skills, and encourages the seeking of mathematical relationships. Students inspect a given function, figure, or data set, and in each case, they are asked to discern a pattern or structure. We want students to become fluent in seeing how the structure of each representation—verbal, graphic, numerical, and algebraic—reveals properties of the function it defines. We want students to become fluent at composing and decomposing expressions, equations, and data sets. We want them to see how the structure of transformations applies to all function types and rigid motions. As students gain proficiency in manipulating structure, they become capable of comparing, contrasting, composing, decomposing, transforming, solving, representing, clarifying, and defining the characteristics of functions, figures, and data sets.

## Reflecting and Communicating

A student-centered approach focuses on students thinking about and discussing mathematics as active participants in their own learning. Through articulating their thinking in conversations with a partner, in a group, or as a class, students integrate each piece of new knowledge into their existing cognitive structure. They use new insights to build new connections. Through collaborative activities and the examination of peer work—both within their groups and from examples provided in the lessons—students give and receive feedback, which leads to verifying, clarifying, and/or improving the strategy.

# Algebra II Content at a Glance

This Year at a Glance highlights the sequence of topics and the number of blended instructional days (1 day is a 45-minute instructional session) allocated in Algebra II in the Texas Math Solution. The suggested pacing information includes time for assessments, providing you with an instructional map that covers 180 days of the school year. As you set out at the beginning of the year, we encourage you to still modify this plan as necessary to meet the range of needs for your students.

## Texas Algebra II: Year at a Glance

*1 Day Pacing = 45-minute Session

| Module | Topic | Pacing | TEKS |
|---|---|---|---|
| Process Standards are embedded in every module: 2A.1A, 2A.1B, 2A.1C, 2A.1D, 2A.1E, 2A.1F, 2A.1G | | | |
| **1 Exploring Patterns in Linear and Quadratic Relationships** | 1: Extending Linear Relationships | 17 | 2A.2A, 2A.3A, 2A.3B, 2A.3C, 2A.3D, 2A.3E, 2A.3F, 2A.3G, 2A.6C, 2A.6D, 2A.6E, 2A.6F, 2A.7I |
| | 2: Exploring and Analyzing Patterns | 19 | 2A.3A, 2A.3B, 2A.4A, 2A.4D, 2A.4F, 2A.5B, 2A.7A, 2A.7B, 2A.8A, 2A.8C |
| | 3: Applications of Quadratics | 15 | 2A.2B, 2A.2C, 2A.3A, 2A.3C, 2A.3D, 2A.4B, 2A.4E, 2A.4H, 2A.7I, 2A.8A, 2A.8B, 2A.8C |
| | | **51** | |
| **2 Analyzing Structure** | 1: Composing and Decomposing Functions | 11 | 2A.2A, 2A.7B, 2A.7I, 2A.8A |
| | 2: Characteristics of Polynomial Functions | 11 | 2A.2A, 2A.6A, 2A.7I |
| | | **22** | |
| **3 Developing Structural Similarities** | 1: Relating Factors and Zeroes | 10 | 2A.2A, 2A.7B, 2A.7C, 2A.7D, 2A.7E |
| | 2: Polynomial Models | 8 | 2A.4E, 2A.7B, 2A.8A, 2A.8B, 2A.8C |
| | | **18** | |
| **4 Extending Beyond Polynomails** | 1: Rational Functions | 16 | 2A.2A, 2A,6G, 2A.6H, 2A.6I, 2A.6J, 2A.6K, 2A.6L, 2A.7C, 2A.7E, 2A.7F, 2A.7I |
| | 2: Radical Functions | 17 | 2A.2A, 2A.2B, 2A.2C, 2A.2D, 2A.4C, 2A.4F, 2A.4G, 2A.6A, 2A.6B, 2A.7G, 2A.7H, 2A.7I |
| | | **33** | |
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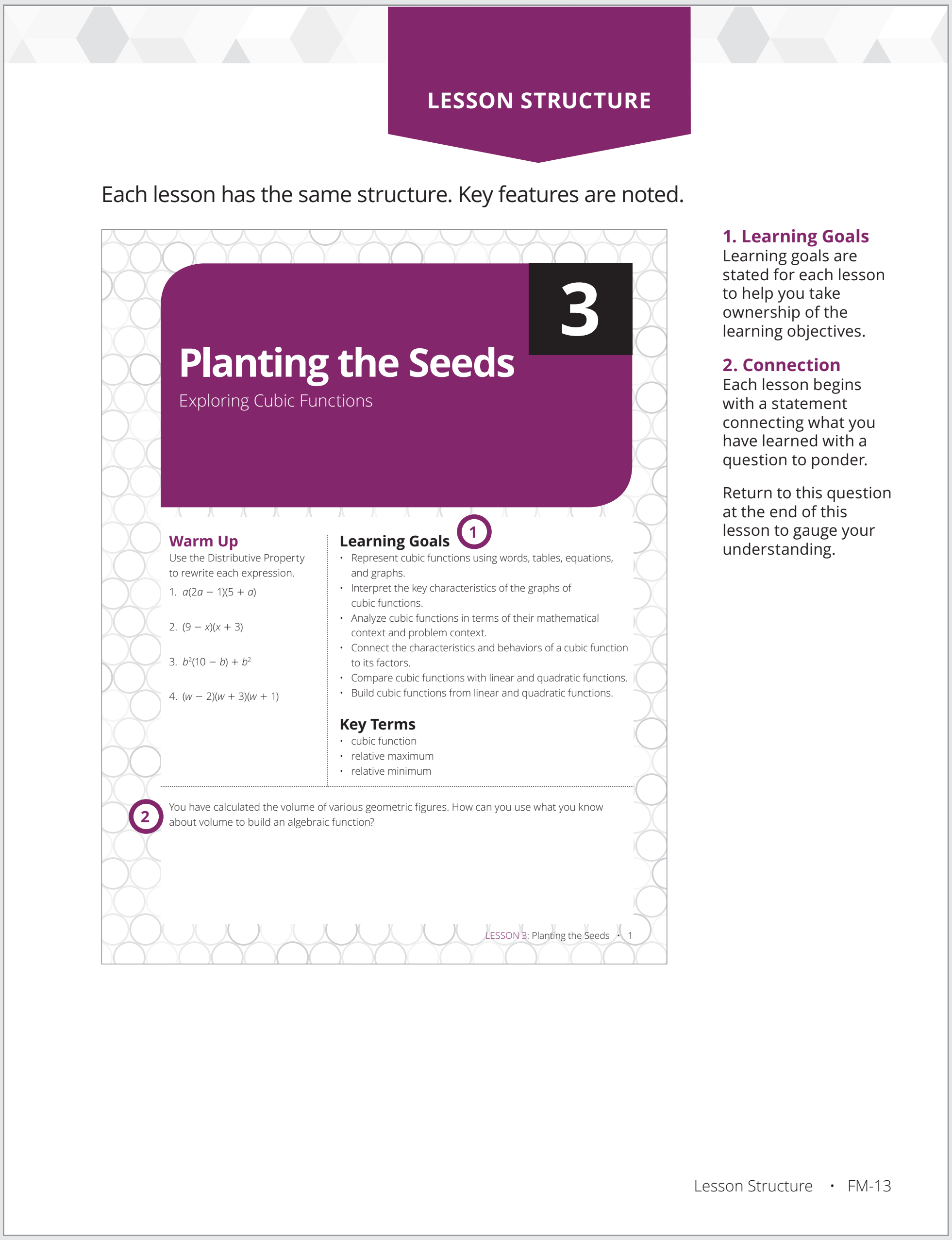
| | 3: Applications of Exponential Functions | 10 | 2A.2A, 2A.4C, 2A.5A, 2A.5B, 2A.6A, 2A.6C, AQR.2H, P.2F, P.5A, P.5E |
| | | **45** | |
| **End of Course Formative Assessment** | Performance Tasks | 11 | 2A.2A, 2A.3A, 2A.3C, 2A.3D, 2A.4F, 2A.4H, 2A.5A, 2A.5B, 2A.5D, 2A.6H, 2A.6I, 2A.6J |
| | | **11** | |
| | **Total Days:** | **180** | |

Each lesson of the Texas Math Solution has the same structure. This consistency allows both you and your students to track your progress through each lesson. Key features of each lesson are noted.

# Lesson Structure


LESSON STRUCTURE

Each lesson has the same structure. Key features are noted.

3

Planting the Seeds

Exploring Cubic Functions

Warm Up

Use the Distributive Property to rewrite each expression.

1. $a(2a - 1)(5 + a)$
2. $(9 - x)(x + 3)$
3. $b^2(10 - b) + b^2$
4. $(w - 2)(w + 3)(w + 1)$

Learning Goals (1)

- Represent cubic functions using words, tables, equations, and graphs.
- Interpret the key characteristics of the graphs of cubic functions.
- Analyze cubic functions in terms of their mathematical context and problem context.
- Connect the characteristics and behaviors of a cubic function to its factors.
- Compare cubic functions with linear and quadratic functions.
- Build cubic functions from linear and quadratic functions.

Key Terms

- cubic function
- relative maximum
- relative minimum

(2) You have calculated the volume of various geometric figures. How can you use what you know about volume to build an algebraic function?

LESSON 3: Planting the Seeds • 1

**1. Learning Goals**
Learning goals are stated for each lesson to help you take ownership of the learning objectives.

**2. Connection**
Each lesson begins with a statement connecting what you have learned with a question to ponder.

Return to this question at the end of this lesson to gauge your understanding.

Lesson Structure • FM-13

## ENGAGE

### Establishing Mathematical Goals to Focus Learning

Create a classroom climate of collaboration and establish the learning process as a partnership between you and students.

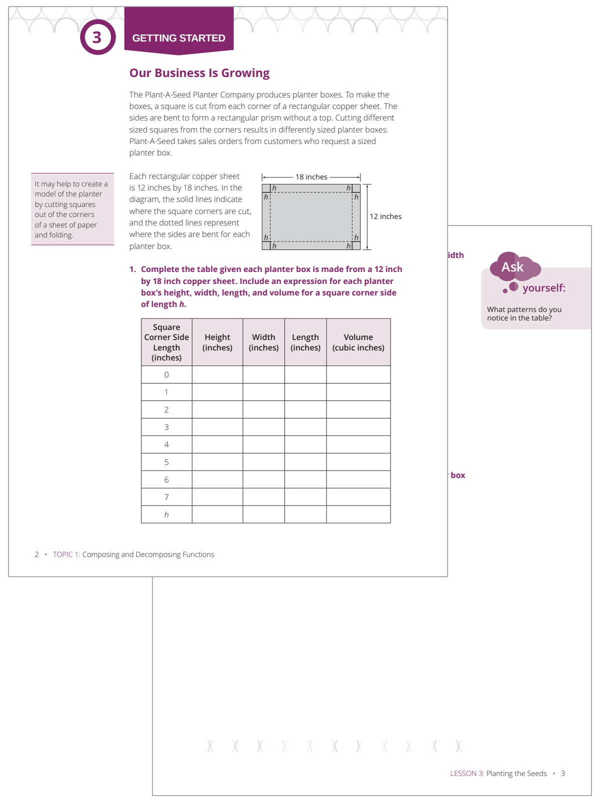
Communicate continuously with students about the learning goals of the lesson to encourage self-monitoring of their learning.

Visit the Texas Support Center for additional guidance on how to foster a classroom environment that promotes collaboration and communication.

## Activating Student Thinking

Your students enter each class with varying degrees of experience and mathematical success. The focus of the Getting Started is to tap into prior knowledge and real-world experiences, to generate curiosity, and to plant seeds for deeper learning. Pay particular attention to the strategies students use, for these strategies reveal underlying thought processes and present opportunities for connections as students proceed through the lesson.

**3. Getting Started**
Each lesson begins with Getting Started. When working on Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.



3 GETTING STARTED

### Our Business Is Growing

The Plant-A-Seed Planter Company produces planter boxes. To make the boxes, a square is cut from each corner of a rectangular copper sheet. The sides are bent to form a rectangular prism without a top. Cutting different sized squares from the corners results in differently sized planter boxes. Plant-A-Seed takes sales orders from customers who request a sized planter box.

It may help to create a model of the planter by cutting squares out of the corners of a sheet of paper and folding.

Each rectangular copper sheet is 12 inches by 18 inches. In the diagram, the solid lines indicate where the square corners are cut, and the dotted lines represent where the sides are bent for each planter box.



**1. Complete the table given each planter box is made from a 12 inch by 18 inch copper sheet. Include an expression for each planter box's height, width, length, and volume for a square corner side of length *h*.**

| Square Corner Side Length (inches) | Height (inches) | Width (inches) | Length (inches) | Volume (cubic inches) |
|---|---|---|---|---|
| 0 | | | | |
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | | | | |
| $h$ | | | | |

2 • TOPIC 1: Composing and Decomposing Functions

Ask yourself:
What patterns do you notice in the table?

LESSON 3: Planting the Seeds • 3

FM-14 • Lesson Structure

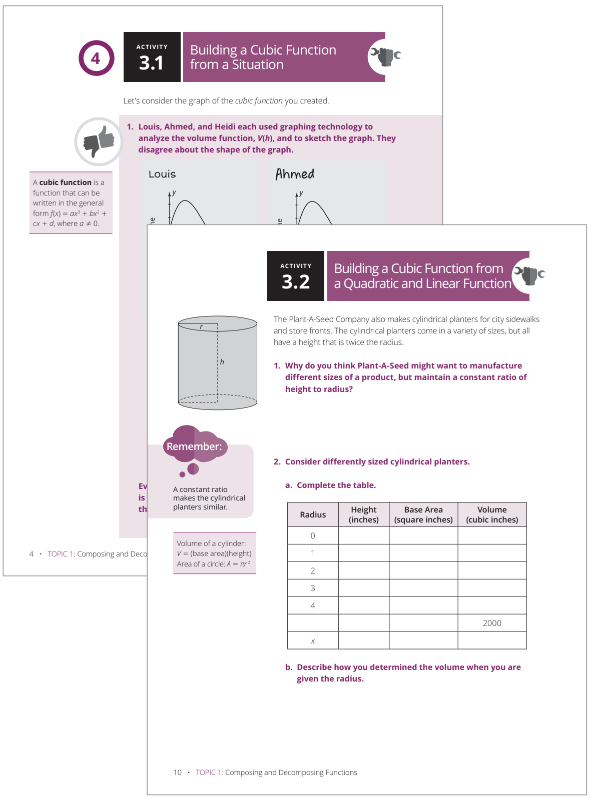
## Supporting Emergent Bilingual Students

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they complete the Getting Started activities in each lesson.

Mathematics is the science of patterns. So, we encourage students throughout this course to notice, test, and interpret patterns in a variety of ways—to put their "mental tentacles" to work in every lesson, every activity. Our hope is that this book encourages you to do the same for your students, and create an environment in your math classroom where productive and persistent learners develop and thrive.

Josh Fisher, Instructional Designer

# DEVELOP




### 4. Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about answer-getting. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, worked examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

Lesson Structure • FM-15

## Aligning Teaching to Learning

Students learn when they are actively engaged in a task: reasoning about the math, writing their solutions, justifying their strategies, and sharing their knowledge with peers.

Support productive struggle by allowing students time to engage with and persevere through the mathematics.

Support student-to-student discourse as well as whole-class conversations that elicit and use evidence of student thinking.

## Supporting Emergent Bilingual Students

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they engage in mathematical discourse throughout each lesson.

## DEMONSTRATE

### Ongoing Formative Assessment Drives Instruction

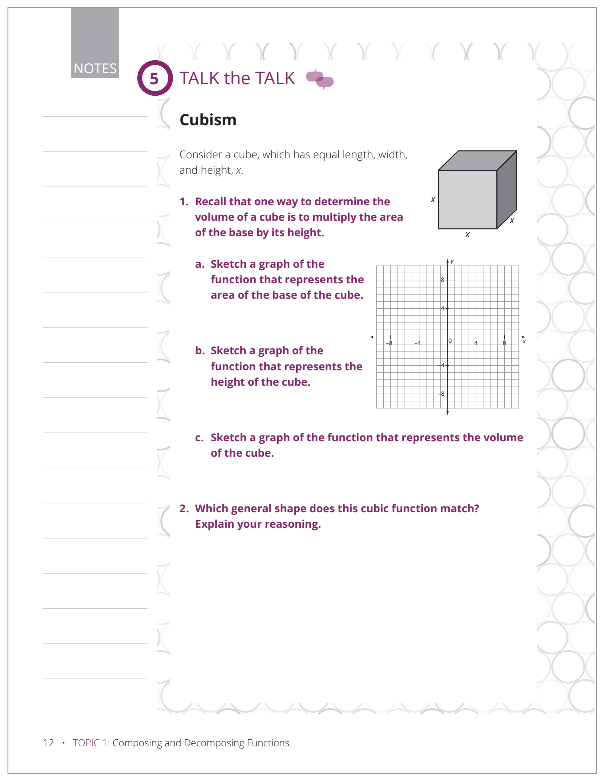
For students to take responsibility for their own learning, they need to be encouraged to self-assess. Students can use the Talk the Talk to monitor their own progress towards mastering the learning goals. Listen and review their answers and explanations and provide feedback to help them improve their understanding.

As you plan the next lesson, consider the connections you can make to build off the strengths or fill any gaps identified from this formative assessment.

**5. Talk the Talk**
Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.



NOTES

5 TALK the TALK

**Cubism**

Consider a cube, which has equal length, width, and height, *x*.

1. Recall that one way to determine the volume of a cube is to multiply the area of the base by its height.
   a. Sketch a graph of the function that represents the area of the base of the cube.
   b. Sketch a graph of the function that represents the height of the cube.
   c. Sketch a graph of the function that represents the volume of the cube.
2. Which general shape does this cubic function match? Explain your reasoning.

12 • TOPIC 1: Composing and Decomposing Functions

FM-16 • Lesson Structure

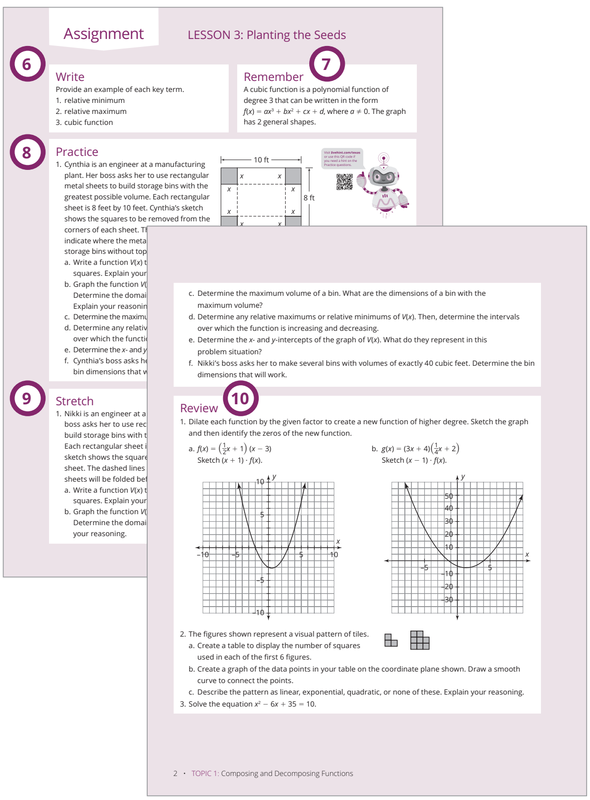
## Student Lesson Overview Videos

Each lesson has a corresponding lesson overview video(s) for students to utilize and reference to support their learning. The videos provide an overview of key concepts, strategies, and/or worked examples from the lessons.

# Assignment

An intentionally designed Assignment follows each lesson.

**ASSIGNMENT**




**6. Write**
Reflect on your work and clarify your thinking.

**7. Remember**
Take note of the key concepts from the lesson.

**8. Practice**
Use the concepts learned in the lesson to solve problems.

**9. Stretch**
Ready for a challenge?

**10. Review**
Remember what you've learned by practicing concepts from previous lessons and topics.

Assignment • FM-17

There is one Assignment per lesson. Lessons often span multiple days. Be thoughtful about which portion of the Assignment students can complete based on that day's progress.

The **Stretch** section is not necessarily appropriate for all learners. Assign this to students who are ready for more advanced concepts.

The **Review** section provides spaced practice of concepts from the previous lesson and topic and of the fluency skills important for the course.

# Topic Summary

A Topic Summary is provided for students at the end of each topic. The Topic Summary lists all key terms of the topic and provides a summary of each lesson. Each lesson summary defines key terms and reviews key concepts, strategies, and/or worked examples.

# Extending Linear Relationships Summary

### KEY TERMS

- Gaussian elimination
- solution of a system of linear inequalities
- linear programming
- matrix (matrices)
- dimensions
- square matrix
- matrix element
- matrix multiplication
- identity matrix
- multiplicative inverse of a matrix
- matrix equation
- coefficient matrix
- variable matrix
- constant matrix
- absolute value
- reflection
- line of reflection
- argument of a function
- linear absolute value equation
- linear absolute value inequality
- equivalent compound inequality

LESSON 1

## Gauss in Das Haus

Recall that a system o
substitution method o
one solution, no soluti

A system of two equat
using methods similar
have one solution, two
equations on the same

For example, you can s
then verify the solution

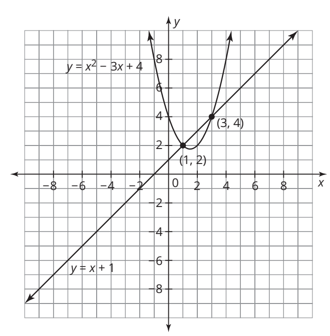
$$\begin{cases} y = x + 1 \\ y = x^2 - 3x + 4 \end{cases}$$

$$\begin{aligned} x^2 - 3x + 4 &= x + 1 \\ x^2 - 4x + 3 &= 0 \\ (x - 3)(x - 1) &= 0 \end{aligned}$$

$x = 3$ or $x = 1$

Substitute $x = 3$ into the linear equation.
$y = 3 + 1 = 4$
Substitute $x = 1$ into the linear equation.
$y = 1 + 1 = 2$

The solutions to the system are (3, 4) and (1, 2).




To solve a system of three linear equations using substitution, the first step is to solve for one variable in one of the equations. Then substitute this expression for that variable in the other two equations. The two new equations will then have only two unknown variables and can be solved using either substitution or linear combinations.

The goal of **Gaussian elimination** is to use linear combinations to isolate one variable for each equation. When using this method, you can:

- swap the positions of two equations.
- multiply an equation by a nonzero constant.
- add one equation to the multiple of another.

For example, you can solve the system $\begin{cases} x + 5y - 6z = 24 \\ -x - 4y + 5z = -21 \\ 5x - 4y + 2z = 21 \end{cases}$ using Gaussian elimination.

Add the first and second equation and replace the second equation.

$$\begin{array}{r} x + 5y - 6z = 24 \\ -x - 4y + 5z = -21 \\ \hline y - z = 3 \end{array} \rightarrow \begin{cases} x + 5y - 6z = 24 \\ y - z = 3 \\ 5x - 4y + 2z = 21 \end{cases}$$

Multiply the first equation by −5 and add it to the third equation. Replace the third equation.

$$\begin{array}{r} -5x - 25y + 30z = -120 \\ 5x - 4y + 2z = 21 \\ \hline -29y + 32z = -99 \end{array} \rightarrow \begin{cases} x + 5y - 6z = 24 \\ y - z = 3 \\ -29y + 32z = -99 \end{cases}$$

Multiply the second equation by 29 and add it to the third equation. Replace the third equation.

$$\begin{array}{r} 29y - 29z = 87 \\ -29y + 32z = -99 \\ \hline 3z = -12 \end{array} \rightarrow \begin{cases} x + 5y - 6z = 24 \\ y - z = 3 \\ 3z = -12 \end{cases}$$

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Multiply the third equation by $\frac{1}{3}$ and add it to the second equation. Replace the second equation.

$$\begin{array}{r} y - z = \ \ 3 \\ z = -4 \\ \hline y = -1 \end{array} \quad \rightarrow \quad \begin{cases} x + 5y - 6z = 24 \\ y = -1 \\ 3z = -12 \end{cases}$$

Multiply the second equation by −5 and add it to the first equation. Replace the first equation.

$$\begin{array}{r} x + 5y - 6z = 24 \\ -5y \qquad = \ \ 5 \\ \hline x \qquad - 6z = 29 \end{array} \quad \rightarrow \quad \begin{cases} x - 6z = 29 \\ y = -1 \\ 3z = -12 \end{cases}$$

Multiply the third equation by 2 and add it to the first equation. Replace the first equation.

$$\begin{array}{r} x - 6z = \ \ 29 \\ 6z = -24 \\ \hline x \qquad = \ \ 5 \end{array} \quad \rightarrow \quad \begin{cases} x = 5 \\ y = -1 \\ 3z = -12 \end{cases}$$

Multiply the third equation by $\frac{1}{3}$. Replace the third equation.

$$\begin{array}{r} \frac{1}{3}(3z = -12) \\ z = -4 \end{array} \quad \rightarrow \quad \begin{cases} x = 5 \\ y = -1 \\ z = -4 \end{cases}$$

The solution to the system is $(x, y, z) = (5, -1, -4)$.

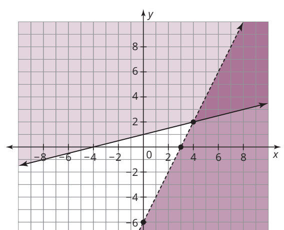
## LESSON 2 Make the Best of It

The **solution of a system of inequalities**, is the intersection of the solutions to each inequality. Every point in the intersection satisfies all inequalities in the system.

For example, consider the system $\begin{cases} y < 2x - 6 \\ y \geq \frac{1}{4}x + 1 \end{cases}$.

Graph the system of linear inequalities on one coordinate plane.

The solution to the system of linear inequalities is the overlapping region of the solutions to each inequality in the system. The points (6, 4) and (8, 8) are both solutions to the system of linear inequalities.



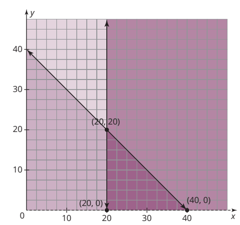

In **linear programming**, the vertices of the solution region of the system of linear inequalities are substituted into an equation to find the maximum or minimum value.

For example, consider the situation in which Anabelle works as a lifeguard and as tutor over the summer. She can work no more than 40 hours each week. Her lifeguarding job requires that she work at least 20 hours each week. If Anabelle earns $15 for each hour she lifeguards and $25 for each hour she tutors, how many hours should she work at each job to maximize her earnings?

The situation can be modeled by a system of linear inequalities. Let $x$ represent the number of hours lifeguarding and let $y$ represent the number of hours tutoring.

$$\begin{cases} x + y \leq 40 \\ x \geq 20 \\ x > 0 \\ y > 0 \end{cases}$$

The vertices of the solution region are (20, 0), (20, 20), and (40, 0).




Substitute the coordinates into an equation representing Anabelle's total earnings for both jobs.

$15x + 25y = E$
$15(20) + 25(0) = 300$
$15(20) + 20(20) = 700$
$15(40) + 25(0) = 600$

Anabelle should work 20 hours as a lifeguard and 20 hours as a tutor to maximize her earnings, which would be $700.

## LESSON 3 Systems Redux

The **identity matrix**, $I$, is a square matrix such that for any matrix $A$, $AI = A$. The elements of the identity matrix are all zeros except for a diagonal from the upper left to the lower right where the elements are all ones. The **multiplicative inverse of a matrix**, $A$, labeled as $A^{-1}$, is a matrix such that when matrix $A$ is multiplied by it the result is the identity matrix. In symbols, $A \cdot A^{-1} = I$. Technology can be used to find the inverse of a square matrix. Not every square matrix has an inverse, and non-square matrices do not have inverses.

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# Problem Types You Will See

Lessons include a variety of problem types to engage students in reasoning about the math.

## Worked Examples

Research shows students learn best when they are actively engaged with a task. Many students need a model to know how to engage effectively with Worked Examples. Students need to be able to question their understanding, make connections with the steps, and ultimately self-explain the progression of the steps and the final outcome. Worked Examples provide a means for students to view each step taken to solve the example problem. The questions that follow are designed to serve as a model for self-questioning and self-explanations. They represent and mimic an internal dialogue about the mathematics and the strategies. This approach doesn't allow students to skip over the example without interacting with it, thinking about it, and responding to the questions. This approach will help students develop the desired habits of mind for being conscientious about the importance of steps and their order.

**PROBLEM TYPES YOU WILL SEE**

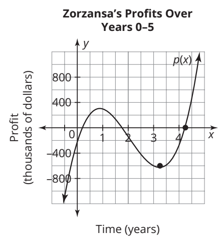
**Worked Example**

**When you see a Worked Example:**
- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

**Ask Yourself:**
- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

**Worked Example**

You can determine the average rate of change of Zorzansa's profit for the time interval (3.25, 4.25).




Substitute the input and output values into the average rate of change formula.

Evaluate the expression.

$$\frac{f(b) - f(a)}{b - a} = \frac{f(4.25) - f(3.25)}{4.25 - 3.25}$$

$$= \frac{0 - (-600)}{1}$$

$$= \frac{600}{1} = 600$$

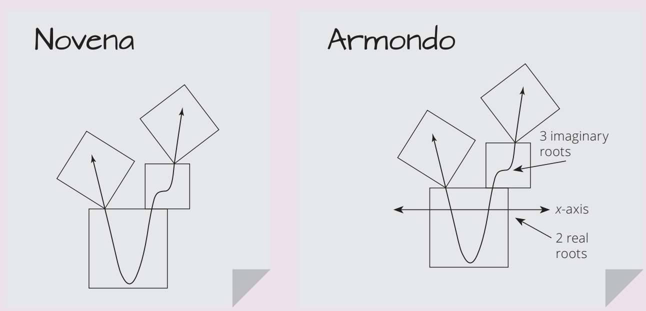
The average rate of change for the time interval (3.25, 4.25) is approximately $600,000 per year.

**Who's Correct?**

**When you see a Who's Correct icon:**
- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or not correct.

**Ask Yourself:**
- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

**4. Novena created this graph of a fourth degree polynomial. Armondo said that she is incorrect, that it is a fifth degree polynomial. Who is correct? For the student who is incorrect, explain the error in their thinking.**




FM-18 • Problem Types You Will See

## Who's Correct?

"Who's Correct?" problems are an advanced form of correct vs. incorrect responses. In this problem type, students are not given who is correct. Students have to think more deeply about what the strategies really mean and whether each of the solutions makes sense. Students will determine what is correct and what is incorrect, and then explain their reasoning. These types of problems will help students analyze their own work for errors and correctness.

### Thumbs Up

**When you see a Thumbs Up icon:**

- Take your time to read through the correct solution.
- Think about the connections between steps.

**Ask Yourself:**

- Why is this method correct?
- Have I used this method before?

Augie

The cubic function $f(x) = (x - 3)(x - 1)(x + 4)$ has the three zeros given. I can verify this by solving the equations $x - 3 = 0$, $x - 1 = 0$, and $x + 4 = 0$.

### Thumbs Down

**When you see a Thumbs Down icon:**

- Take your time to read through the incorrect solution.
- Think about what error was made.

**Ask Yourself:**

- Where is the error?
- Why is it an error?
- How can I correct it?

Emily

A cubic function must have three zeros. I know this from the Fundamental Theorem of Algebra. However, the number of real and imaginary zeros can vary. The function may have 0, 1, 2, or 3 imaginary zeros.

Problem Types You Will See • FM-19

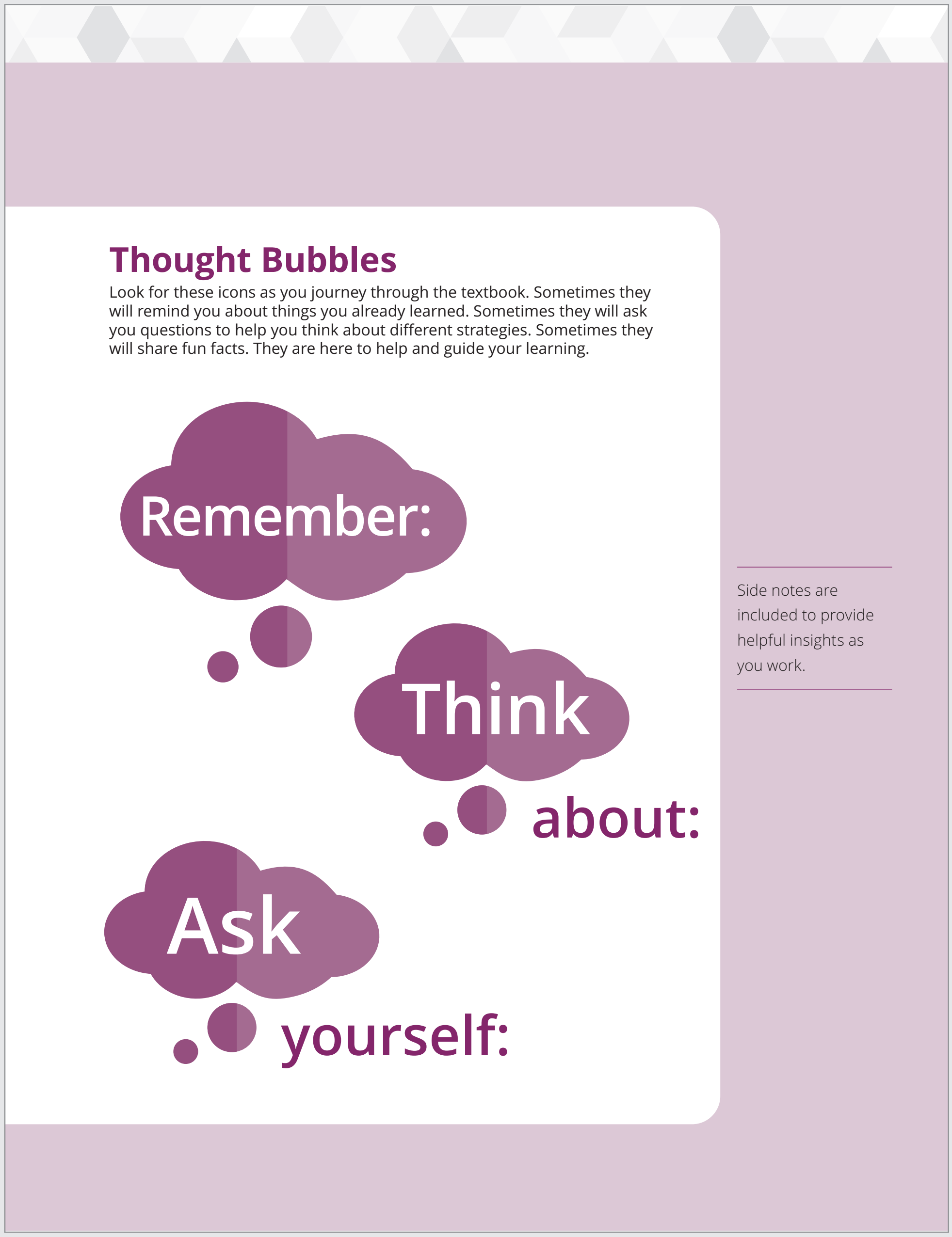
## Thumbs Up/ Thumbs Down

Thumbs Up problems provide a framework that allows students the opportunity to analyze viable methods and problem-solving strategies. Questions are presented to help students think deeper about the various strategies, and to focus on an analysis of correct responses. Research shows that only providing positive examples does not eliminate some of the things students may think; it is also efficient to show negative examples. From the incorrect responses, students learn to determine where the error in calculation is, why the method is an error, and also how to correct the method to correctly calculate the solution.

# Promoting Self-Reflection

**Thought Bubbles**

Thought bubbles are embedded throughout the Texas Math Solution promote productive reflection by reminding students to stop and think. This feature is used in a variety of ways: it may remind students to recall a previous mathematical concept, help students develop expertise to think through problems, and occasionally, present a fun fact.



## Thought Bubbles

Look for these icons as you journey through the textbook. Sometimes they will remind you about things you already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are here to help and guide your learning.

Remember:

Think about:

Ask yourself:

Side notes are included to provide helpful insights as you work.

> A mathematician is an artist who works with patterns. I think the beauty of mathematics lies in the new connections you can make to express the patterns around you, no matter your age. The art is in the process, not the outcome. When we can get students to see the beauty of the mathematics, and equip them with the tools to express themselves mathematically, then we can truly create critical thinkers.

Victoria Fisher, Instructional Designer

# Mathematical Process Standards

MATHEMATICAL PROCESS STANDARDS

### Texas Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The “I can” expectations listed below align with the TEKS Mathematical Process Standards and encourage students to develop their mathematical learning and understanding.

**► Apply mathematics to problems arising in everyday life, society, and the workplace.**

I can:

- use the mathematics that I learn to solve real-world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

**► Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.**

I can:

- explain what a problem “means” in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

FM-20 • Mathematical Process Standards

**Note**

Each lesson provides opportunities for students to think, reason, and communicate their mathematical understanding. However, it is your responsibility as a teacher to recognize these opportunities and incorporate these practices into your daily rituals. Expertise is a long-term goal, and students must be encouraged to apply these practices to new content throughout their school career.

## Supporting Students to Use Mathematical Tools

Visit the Texas Support Center for strategies to support students as they use mathematical tools, including formula charts and reference sheets.

**Note**

When you are facilitating each lesson, listen carefully and value diversity of thought, redirect students' questions with guiding questions, provide additional support with those struggling with a task, and hold students accountable for an end product. When students share their work, make your expectations clear, require that students defend and talk about their solutions, and monitor student progress by checking for understanding.

There is one more page of mathematical process standards that is not provided here, but is available in the Student Textbook Front Matter.

### ▶ Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

### ▶ Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

I can:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

### ▶ Create and use representations to organize, record, and communicate mathematical ideas.

I can:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.



## Supporting ALL Learners

Visit the Texas Support Center for facilitation strategies to support ALL students as the engage in the Mathematical Process Standards.

# Academic Glossary

## ACADEMIC GLOSSARY

There are important terms you will encounter throughout this book. It is important that you have an understanding of these words as you get started on your journey through the mathematical concepts. Knowing what is meant by these terms and using these terms will help you think, reason, and communicate your ideas.

Visit the Students & Caregivers Portal on the Texas Support Center at **www.CarnegieLearning.com/**texas-help to access the Mathematics Glossary for this course anytime, anywhere.

### ANALYZE

**Definition**

To study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

**Ask Yourself**

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

**Related Phrases**

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

### EXPLAIN YOUR REASONING

**Definition**

To give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

**Ask Yourself**

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

**Related Phrases**

- Show your work
- Explain your calculation
- Justify
- Why or why not?

Academic Glossary • FM-23

## Language Expectations

It is critical for students to possess an understanding of the language of their text. Students must learn to read for different purposes and write about what they are learning. Encourage students to become familiar with the key words and the questions they can ask themselves when they encounter these words.

It is our recommendation to be explicit about your expectations of language used and the way students write responses throughout the text. Encourage students to answer questions with complete sentences. Complete sentences help students reflect on how they arrived at a solution, make connections between topics, and consider what a solution means both mathematically as well as in context.

## Supporting Students at Varying Levels of Language Proficiency

Visit the Texas Support Center for guidance on how to leverage the Academic Glossary to support students at varying levels of language proficiency.

## Ask Yourself

The Ask Yourself questions help students develop the proficiency to explain to themselves the meaning of problems.

## Real-World Context

Real-world contexts confirm concrete examples of mathematics. The scenarios in the lessons help students recognize and understand that quantitative relationships seen in the real world are no different that quantitative relationships in mathematics. Some problems begin with a real-world context to remind students that the quantitative relationships they already use can be formalized mathematically. Other problems will use real-world situations as an application of mathematical concepts.

### REPRESENT

**Related Phrases**

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

**Definition**

To display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

**Ask Yourself**

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

### ESTIMATE

**Related Phrases**

- Predict
- Approximate
- Expect
- About how much?

**Definition**

To make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

**Ask Yourself**

- Does my reasoning make sense?
- Is my solution close to my estimation?

### DESCRIBE

**Related Phrases**

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

**Definition**

To represent or give an account of in words. Describing communicates mathematical ideas to others.

**Ask Yourself**

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

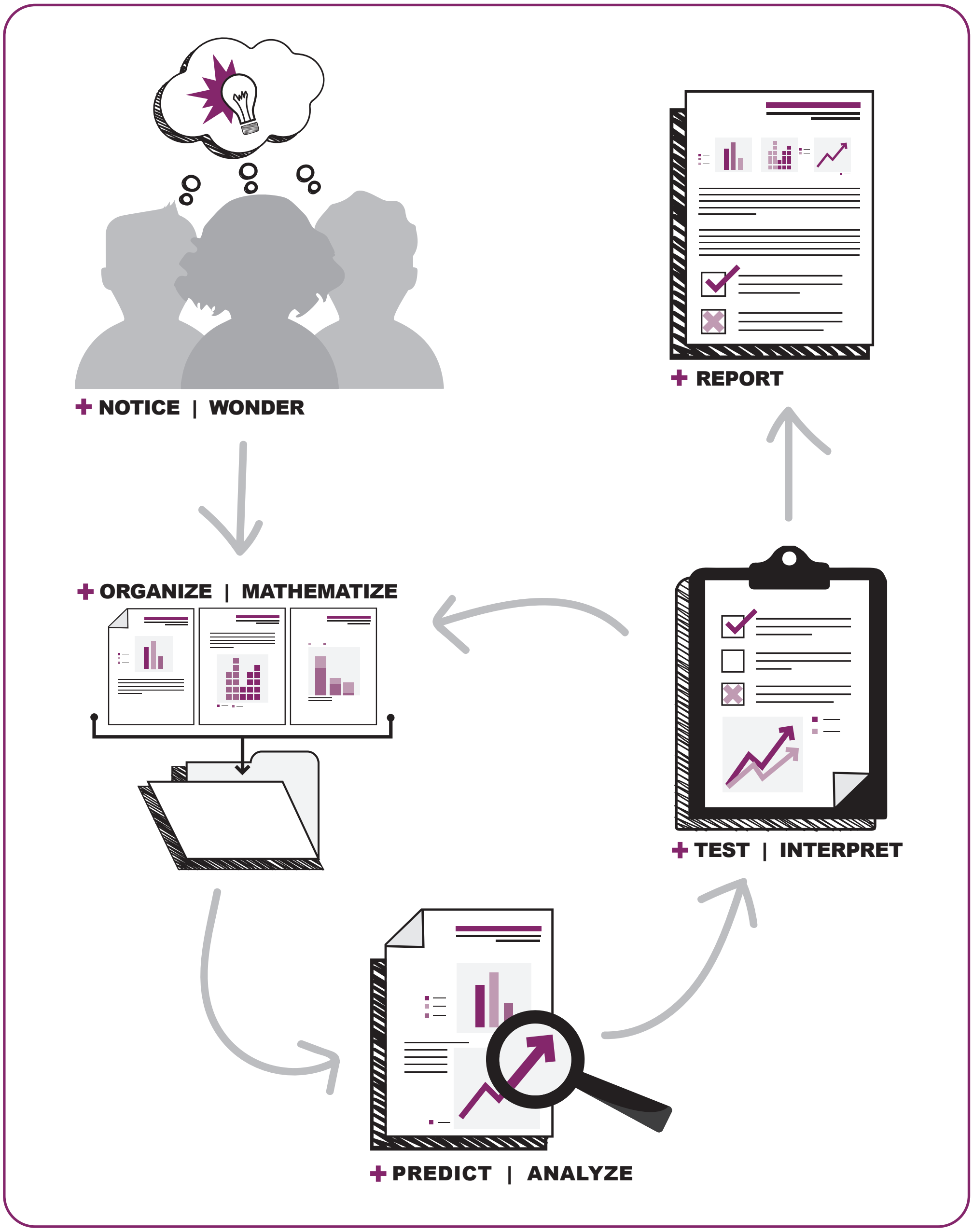
FM-24 • Academic Glossary

## Mathematics Glossary

A course-specific mathematics glossary is available for students to utilize and reference during their learning. Definitions and examples of key terms are provided in the glossary.

# The Modeling Process

Modeling is the process of choosing appropriate mathematical tools to analyze and understand real-world phenomena and to make decisions accordingly. The Modeling Process provides a structure to help students become better problem solvers. In the textbook, students will encounter activities that explicitly guide them through the four steps of the Modeling Process. As they progress through high school mathematics, they should start to use this process intuitively.




**Notice and Wonder**
Gather information, notice patterns, and formulate mathematical questions about what you notice.

**Organize and Mathematize**
Organize your information and represent it using mathematical notation.

**Predict and Analyze**
Extend the patterns created, complete operations, make predictions, and analyze the mathematical results.

**Test and Interpret**
Interpret your results and test your mathematical predictions in the real world. Make adjustments as necessary.

FACILITATING STUDENT LEARNING

# Teacher's Implementation Guide

The Teacher's Implementation Guide (TIG) is designed to fully support a wide-range of teachers implementing our materials: from first-year teachers to 30-year veterans and from first-time Carnegie Learning users to master practitioners.

One goal in developing the TIG was to make our instructional design apparent to the users.

The lessons of each topic were written to be accessible to the full range of learners. With every instructional decision you make, keep in mind your mathematical objectives for the topic and module and the course. Plan each lesson by thinking about how you will create access for your particular group of students, maintain access and pace throughout the lesson, and assess their understanding along the way. We recommend that you do the math in each topic before implementing the activities with your specific group of students.

## WHAT MAKES THIS TIG USEFUL?

### Effective Lesson Design

Each lesson has a consistent structure for teachers and students to follow. The learning experiences are engaging and effective for students.

### Pacing

Each course is designed to be taught in a 180-day school year. Pacing suggestions are provided for each lesson. Each day in the pacing guide is equivalent to about a 45-minute instructional session.

### Instructional Supports

Guiding questions are provided for teachers to use as they're circulating the room, as well as differentiation strategies, common student misconceptions, and student look-fors.

### Clearly Defined Mathematics

The content and instructional goals are clearly described at the module, topic, lesson, and activity levels.

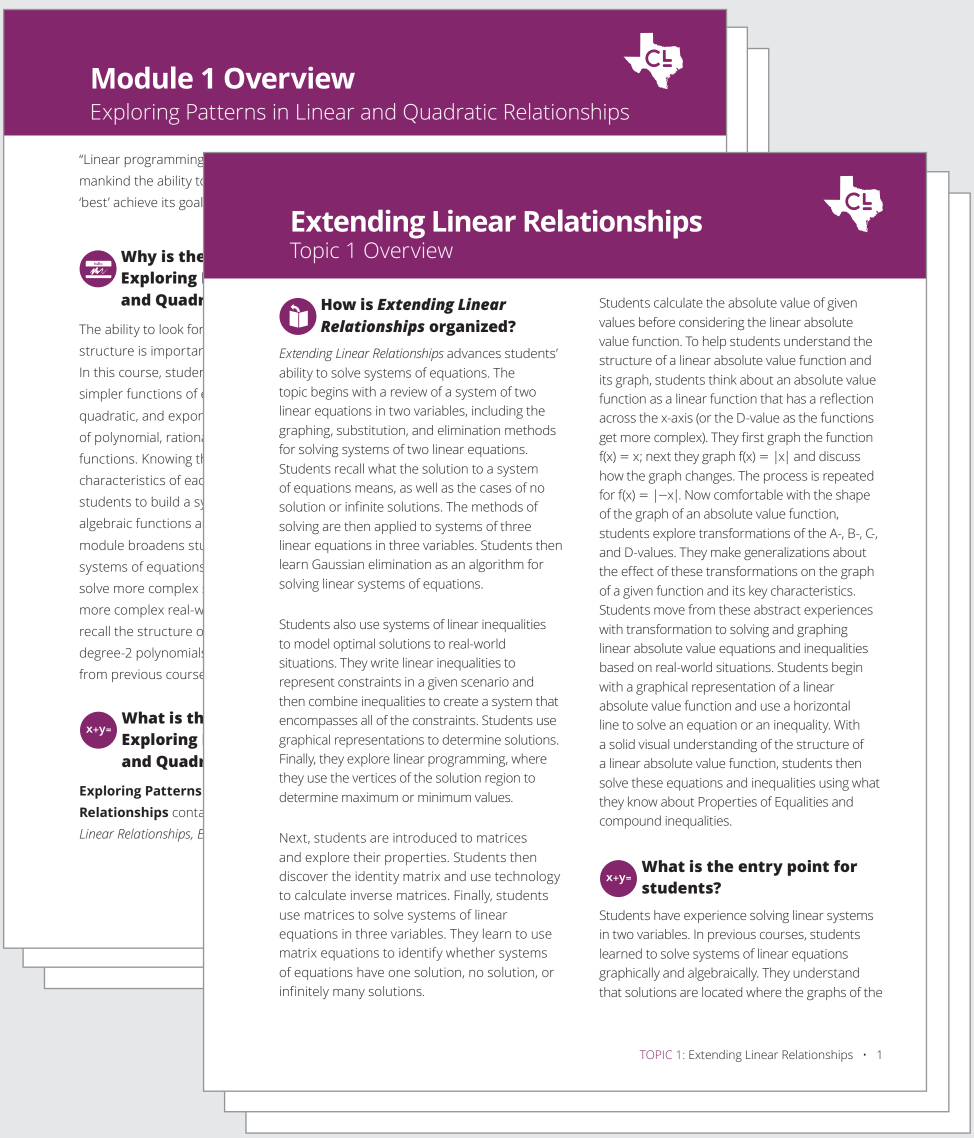
The TIG is critical to understanding how the mathematics that students encounter should be realized in the classroom. The TIG describes the depth of understanding that students need to develop for each standard and a pathway for all learners to be successful. It provides differentiation strategies to support students who struggle, to extend certain activities for students who are advanced in their understanding of the content, and to support emergent bilingual students.

Visit the Texas Support Center at **www.CarnegieLearning.com/texas-help** for additional resources to support you anytime, anywhere.

# Module and Topic Overviews

"Teachers must first develop their ideas about where the curriculum program is going mathematically (curriculum vision) before deciding whether the curriculum materials will help them reach that mathematical goal (curriculum trust)" (Drake & Sherin, 2009, p. 325).

You are responsible for teaching the essential concepts associated with a particular course. You need to understand how activities within lessons build to achieve understanding within topics, and how topics build to achieve understanding throughout the course. In the Texas Math Solution, Carnegie Learning seeks to establish a shared curriculum vision with you.



Module 1 Overview

Exploring Patterns in Linear and Quadratic Relationships

Extending Linear Relationships

Topic 1 Overview

**How is *Extending Linear Relationships* organized?**

*Extending Linear Relationships* advances students' ability to solve systems of equations. The topic begins with a review of a system of two linear equations in two variables, including the graphing, substitution, and elimination methods for solving systems of two linear equations. Students recall what the solution to a system of equations means, as well as the cases of no solution or infinite solutions. The methods of solving are then applied to systems of three linear equations in three variables. Students then learn Gaussian elimination as an algorithm for solving linear systems of equations.

Students also use systems of linear inequalities to model optimal solutions to real-world situations. They write linear inequalities to represent constraints in a given scenario and then combine inequalities to create a system that encompasses all of the constraints. Students use graphical representations to determine solutions. Finally, they explore linear programming, where they use the vertices of the solution region to determine maximum or minimum values.

Next, students are introduced to matrices and explore their properties. Students then discover the identity matrix and use technology to calculate inverse matrices. Finally, students use matrices to solve systems of linear equations in three variables. They learn to use matrix equations to identify whether systems of equations have one solution, no solution, or infinitely many solutions.

Students calculate the absolute value of given values before considering the linear absolute value function. To help students understand the structure of a linear absolute value function and its graph, students think about an absolute value function as a linear function that has a reflection across the x-axis (or the D-value as the functions get more complex). They first graph the function $f(x) = x$; next they graph $f(x) = |x|$ and discuss how the graph changes. The process is repeated for $f(x) = |-x|$. Now comfortable with the shape of the graph of an absolute value function, students explore transformations of the A-, B-, C-, and D-values. They make generalizations about the effect of these transformations on the graph of a given function and its key characteristics. Students move from these abstract experiences with transformation to solving and graphing linear absolute value equations and inequalities based on real-world situations. Students begin with a graphical representation of a linear absolute value function and use a horizontal line to solve an equation or an inequality. With a solid visual understanding of the structure of a linear absolute value function, students then solve these equations and inequalities using what they know about Properties of Equalities and compound inequalities.

**What is the entry point for students?**

Students have experience solving linear systems in two variables. In previous courses, students learned to solve systems of linear equations graphically and algebraically. They understand that solutions are located where the graphs of the

TOPIC 1: Extending Linear Relationships • 1

### Module Overview

Each module begins with an overview that describes the reasoning behind the name, the mathematics being developed, the connections to prior learning, the connections to future learning.

### Topic Overview

A Topic Overview describes how the topic is organized, the entry point for students, how a student will demonstrate understanding, why the mathematics is important, how the activities promote expertise in the mathematical process standards, what materials are needed, examples of new tools and notations, and more detailed information to help with pacing.

# Facilitation Notes

For each lesson, you are provided with detailed facilitation notes to fully support your planning process. This valuable resource provides point-of-use support that serves as your primary resource for planning, guiding, and facilitating student learning.

**1. Materials**
Materials required for the lesson are identified.

**2. Lesson Overview**
The Lesson Overview sets the purpose and describes the overarching mathematics of the lesson, explaining how the activities build and how the concepts are developed.

**3. TEKS Addressed**
The focus TEKS for each lesson are listed. Carnegie Learning recognizes that some lessons could list several TEKS based on the skills needed to complete the activities, however, the TEKS listed are what the lesson is focused on developing and mastering.

**4. ELPS Addressed**
The English Language Proficiency Standards for each lesson are listed. As you plan, consider these ELPS and determine the instructional strategies that you will use to meet these ELPS.

**5. Essential Ideas**
These statements are derived from the standards and state the concepts students will develop.

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## Systems Redux

Solving Matrix Equations

**MATERIALS** (1)
Technology that can operate with matrices

(2) **Lesson Overview**
Students are introduced to identity and inverse matrices. They express a system of equation as a matrix eqution. Students relate solving a matrix equation to solving a linear equation, and then use technology to solve a matrix equation. As a culminating activity, they model a scenario with a system of equations, convert it to a matrix equation, solve the matrix equation using technology, and interpret the solution in terms of the scenario.

(3) **Algebra 2**
**Systems of Equations and Inequalities**
**(3) The student applies mathematical processes to formulate systems of equations and inequalities, uses a variety of methods to solve, and analyzes reasonableness of solutions. The student is expected to:**

(B) solve systems of three linear equations in three variables by using Gaussian elimination, technology with matrices, and substitution.

(4) **ELPS**
1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

(5) **Essential Ideas**

- The multiplicative identity matrix, $I$, is a square matrix such that for any matrix $A$, $A \cdot I = A$.
- The multiplicative inverse of a matrix of a square matrix $A$ is designated as $A^{-1}$, and is a matrix such that $A \cdot A^{-1} = I$.
- Matrices can be used to solve a system of equations in the form $Ax + By + Cz = D$ by writing the system as a matrix equation in the form $A \cdot X = B$, where $A$ represents the coefficient matrix, $X$ represents the variable matrix, and $B$ represents the constant matrix.
- Technology can be used to solve matrix equations.

LESSON 3: Systems Redux • 1

## Lesson Structure and Pacing: 2 Days

6

### Day 1

7

**Engage**

**Getting Started: Black and Gold Track What Sold**

Students analyze two different representations of information about a fundraiser. One representation uses running text to list the number of items sold in two different months and the other representation presents the same information in matrix form. They respond to questions by referencing the data and comparing the representations.

**Develop**

**Activity 3.1: Matrices and Their Inverses**

The terms *matrix, dimensions, square matrix, matrix element, matrix multiplication, identity matrix*, and *multiplicative inverse* are defined. Using a real-world context, students explore the components of a matrix. They multiply matrices and use matrix multiplication to determine whether two matrices are inverses.

### Day 2

**Activity 3.2: Solving Systems with Matrices**

Students are introduced to matrices as a way to represent and solve systems of equations. They analyze a series of worked examples that demonstrate how to express a system of equations as a matrix equation, how to use technology to solve matrix equations, and how to recognize whether a matrix equation has many solutions or no solutions. Students conclude the activity by solving systems of equations with matrices.

**Demonstrate**

**Talk the Talk: Show Us Your Stuff**

Students encounter a scenario that can be represented using a system of equations. They write a system of equations representing the situation, express the system as a matrix equation, and then use technology with matrices to solve the matrix equation. Students also interpret their solution in the context of the problem situation.

2 • TOPIC 1: Extending Linear Relationships

### 6. Lesson Structure

This section highlights how the parts of the lesson fit within the instructional design: Engage, Develop, and Demonstrate. A summary of each activity included.

### 7. Pacing

Lessons often span more than one 45-minute class period. Suggested pacing is provided for each lesson so that the entire course can be completed in a school year.

### 8. Facilitation Notes by Activity

A detailed set of guidelines walks the teacher through implementing the Getting Started, Activities, and Talk the Talk portions of the lesson. These guidelines include an activity overview, grouping strategies, guiding questions, possible student misconceptions, differentiation strategies, student look-fors, and an activity summary.

### 9. Activity Overview

Each set of Facilitation Notes begins with an overview that highlights how students will actively engage with the task to achieve the learning goals.

### 10. Differentiation Strategies

To extend an activity for students who are ready to advance beyond the scope of the activity, additional challenges are provided.

### 11. White Space

The white space in each margin is intentional. Use this space to make additional planning notes or to reflect on the implementation of the lesson.

---

8 **Getting Started: Black and Gold Track What Sold** — ENGAGE

9 **Facilitation Notes**

In this activity, students analyze two different representations of information about a fundraiser. One representation uses running text to list the number of items sold in two different months and the other representation presents the same information in matrix form. They respond to questions by referencing the data and comparing the representations.

Have students work with a partner or in a group to read the problems and answer Questions 1 through 7. Share responses as a class.

10 **Differentiation strategy**

To extend the activity, provide students the first paragraph and Mr. Black's records, but do not show them Ms. Gold's records. Ask them to organize the information, and then compare their strategies with Ms. Gold's organization.

**Questions to ask**

- What are the advantages to Ms. Gold's organization?
- How did you identify the data required to respond to this question?
- What order of operations did you use to answer Question 3?
- Is there another order that could have been used to answer Question 3? If so, explain the process.
- What is another question you could answer by referring to the data?

11

12 **Summary**

Data organized in rows and columns allows for easier identification and interpretation of the information.

**Activity 3.1**
**Matrices and Their Inverses** — DEVELOP

**Facilitation Notes**

In this activity, the terms *matrix, dimensions, square matrix, matrix element, matrix multiplication, identity matrix,* and *multiplicative inverse* are defined. Using a real-world context, students explore the components of a matrix. They multiply matrices and use matrix multiplication to determine whether two matrices are inverses.

LESSON 3: Systems Redux • 3

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### 12. Summary

The summary brings the activity to closure. This statement encapsulates the big mathematical ideas of the particular activity.

Have students work with a partner or in a group to read the introduction and complete Questions 1 and 2. Share responses as a class.

**Differentiation strategy**

13 To support students who struggle, remind them that they used rectangular arrays to model multiplication problems in elementary school. They also labeled these arrays using the notation *row* × *column*.

**Questions to ask**

- What is a matrix?
- How is a matrix like a table? How is a matrix different than a table?
- When identifying the dimensions of a matrix or the location of an element in a matrix, is the column or row expressed first?
- How is a 2 × 5 matrix different than a 5 × 2 matrix?
- What does the 0 in matrix *B* represent?

Have students work with a partner or in a group to complete Questions 3 through 5. Share responses as a class.

14 **Questions to ask**

- Explain why it is necessary to have five values in the second matrix.
- Explain the relationship between a row in the first matrix and the column in the second matrix.
- Why does the resulting matrix only have four elements?
- Why does it make sense that the resulting matrix has four rows?

Ask a student to read the information following Question 5. Discuss as a class.

**Questions to ask**

- How does the $a_{pq}$ notation relate to the example?
- Explain how each element in the results matrix is the result of both multiplication and addition.

Have students work with a partner or in a group to complete Questions 6 through 8. Share responses as a class.

**Questions to ask**

- How can you tell by the dimensions whether or not you can multiply two matrices?
- Provide an example of the dimensions of two matrices that can be multiplied.

**Differentiation strategy**

15 To scaffold support for all students, provide directions on how to use technology to multiply matrices.

Have students work with a partner or in a group to complete Questions 9 and 10. Share responses as a class.

4 • TOPIC 1: Extending Linear Relationships

## 13. Differentiation Strategies

To scaffold instruction, suggestions are provided for additional scaffolding or alternative methods of instruction to help struggling students fully engage in the lesson.

## 14. Questions to Ask

The overarching questioning strategies throughout each lesson promote analysis and higher-order thinking skills beyond simple yes or no responses. These questions can be used to gather information, probe thinking, make the mathematics explicit, and encourage reflection and justification as students are working together or when they are sharing responses as a class. These questions are an embedded formative assessment strategy to provide feedback as students are actively engaged in learning.

## 15. Differentiation Strategies

To assist all students, instructional strategies are provided that benefit the full range of learners.

### 16. As Students Work, Look For

These notes provide specific language, strategies, and/or errors to look and listen for as you circulate and monitor students working in pairs or groups. You can incorporate these ideas when students share their responses with the class.

### 17. Misconceptions

Common student misconceptions are provided in places where students may overgeneralize mathematical relationships or have confusion over the vocabulary used. Suggestions are provided to address the given misconception.

16

**As students work, look for**

Whether they use mental math or write out all the operations when calculating the value of each element in the result matrix.

**Questions to ask**

- If the product matrix $A \cdot A^{-1}$ does not equal the identity matrix I, what does this tell you about the two matrices?
- How did you calculate each element in the product matrix?

## Summary

A matrix is an array of numbers, known as matrix elements, composed into rows and columns. You can determine an element $a_{pq}$ of the product matrix by multiplying each element in row p of the first matrix by an element from column $q$ in the second matrix and calculating the sum of the products. The product of a matrix and its multiplicative inverse is the identity matrix.

## Activity 3.2

**Solving Systems with Matrices**

### Facilitation Notes

In this activity, students are introduced to matrices as a way to represent and solve systems of equations. They analyze a series of Worked Examples that demonstrate how to express a system of equations as a matrix equation, how to use technology to solve matrix equations, and how to recognize whether a matrix equation has many solutions or no solutions. Students conclude the activity by solving systems of equations with matrices.

Ask a student to read the introduction and analyze the Worked Example as a class. Have students work with a partner or in a group to complete Question 1.

17

**Misconception**

Students may be confused by the equation $A \cdot X = B$ thinking it is referring to the same variables as in the referenced equation $Ax + By + Cz = D$. Clarify this misunderstanding by explaining that $A$, $X$, and $B$ represent matrices, while $A$, $B$, $C$ and $D$ represent constants, and $x$, $y$, and $z$ represent individual variables.

**Questions to ask**

- How is a coefficient matrix formed?
- Why does the $X$ matrix have three elements?
- Why do you think $X$ is a $3 \times 1$ matrix?

LESSON 3: Systems Redux • 5

### Note: Alternative Grouping Strategies

Differentiation strategies that provide other grouping strategies, such as whole class participation and the jigsaw method, are sometimes recommended for specific activities. These are listed as Differentiation Strategies.

More information about grouping strategies is available online in the Texas Support Center at **www.CarnegieLearning.com/texas-help**.

- Use the dimensions of $A$ and $X$ to explain why $B$ has the dimensions it has.

Have students work with a partner or in a group to complete Questions 2 and 3. Share responses as a class.

**Questions to ask**

- Explain how the matrix equation $A \cdot X = B$ is another way to represent the system of equations.
- How do you account for missing variables in a system of equations when you represent the system as a matrix equation?
- How would you correct Antoine's work?

Have students work with a partner or in a group to complete Question 4. Share responses as a class.

18

**Questions to ask**

- How is a coefficient matrix formed?
- Why does the $X$ matrix have three elements?
- Why do you think $X$ is a $3 \times 1$ matrix?
- Use the dimensions of $A$ and $X$ to explain why $B$ has the dimensions it has.

Have students read the Worked Example and then work with a partner or in a group to complete Questions 4. Share responses as a class.

**Questions to ask**

- According to the Worked Example, what additional step can the technology do for you when solving a matrix equation?
- How does this error message relate to the system of equations?

Have students read the worked example following Question 4 and complete Questions 5 through 6.

**Differentiation strategy**

To scaffold support, provide examples of systems of equations in two variables for them to solve using the linear combination method to help them relate to the concepts of many solutions and no solutions in the worked example.

$$x + 2y = 8 \qquad x + 2y = 8$$
$$3x + 6y = 24 \qquad 3x + 6y = 23$$

**Questions to ask**

- How can you tell when a matrix equation has many solutions or no solutions?
- Can you identify whether a matrix equation has many solutions or no solutions without the use of technology? Explain.
- How did you build an equation to get the results you wanted?

Have students work with a partner or in a group to complete Question 7.

6 • TOPIC 1: Extending Linear Relationships

## 18. Grouping Strategies

Suggestions appear to help chunk each activity into manageable pieces and establish the cadence of the lesson.

Learning is social. Whether students work in pairs or in groups, the critical element is that they are engaged in discussion. Carnegie Learning believes, and research supports, that student-to-student discourse is a motivating factor; it increases student learning and supports ongoing formative assessment. Additionally, it provides students with opportunities to have mathematical authority.

Working collaboratively can, when done well, encourage students to articulate their thinking (resulting in self-explanation) and also provides metacognitive feedback (by reviewing other students' approaches and receiving feedback on their own).

The student discussion is then transported to a classroom discussion facilitated by the teacher to guarantee all necessary mathematical is addressed, once again, with the same benefits of discussion.

## Note

Differentiation strategies are provided that will ensure all students acquire the knowledge of the activity. These strategies provide flexibility within the lesson to allow for varying student acquisition and demonstration of learning. These strategies provide suggestions for all students, including those with learning strengths or learning gaps.

## Note

The Talk the Talk helps you to assess student learning and to make decisions about helpful connections you need to make in future lessons.

DEMONSTRATE

## Talk the Talk: Show Us Your Stuff

### Facilitation Notes

In this activity, students encounter a scenario that can be represented using a system of equations. They write a system of equations representing the situation, expresss the system as a matrix equation, and then use technology with matrices to solve the matrix equation. Students also interpret their solution in the context of the problem situation.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

**Questions to ask**

- Explain how your system of equations models the context.
- How did you rewrite your system of equations using matrices?
- Explain how you used technology to solve your matrix equation.

### Summary

A real-world situation that can be modeled by a system of linear equations may be solved using a matrix equation and technology.

8 • TOPIC 1: Extending Linear Relationships

"Position yourself to take full advantage of the richness of the mathematics addressed in the textbook. The Facilitation Notes provide guidance to reach each student from their current level of understanding to advance to the next stage. Place yourself in the position of the student by experiencing the textbook activities prior to class. Realize your role in the classroom—empower your students! Step back and let them do the math with confidence in their role as learner and your role as facilitator of learning."

Janet Sinopoli, Instructional Designer

# Supporting Emergent Bilingual Students

Emergent bilingual students often face multiple challenges in the mathematics classroom beyond language development skills, including a lack of confidence, peer-to-peer understanding, and building solid conceptual mastery. The Carnegie Learning Texas Math Solution seeks to support emergent bilingual students as they develop skills in both mathematics and language.

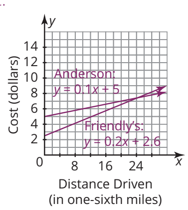
**Answers**

1. Let $x$ = the number of one-sixth miles driven.

   Let $y$ = the total cost of the taxi ride.

   $\begin{cases} y = 0.20x + 2.60 \\ y = 0.10x + 5.00 \end{cases}$

2.




3. The solution to the system of equations is $x = 24$. This is the $x$-coordinate of the point where the two graphs intersect. The $y$-coordinate of this point is between 7 and 8.

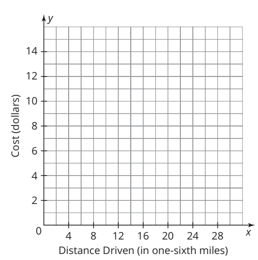
GETTING STARTED

**Which Fare Is Fair?**

You would like to take a taxi to the airport. There are two local taxi companies. Friendly's Cab Company charges $2.60 plus $0.20 per one-sixth of a mile driven. Anderson Taxi, Inc. charges $5.00 plus $0.10 per one-sixth of a mile driven.

**1. Write a system of two linear equations in two variables to represent this problem situation. Be sure to define your variables.**

**2. Graph the system of equations.**




**3. Estimate the solution to the system of equations. Justify your reasoning.**

**Remember:**

The solution to a system of linear equations occurs when the values of the variables satisfy all of the linear equations.

2 • TOPIC 1: Extending Linear Relationships

**ELL Tip**

Make sure students are familiar with what a taxi or taxicab is by comparing it to other similar transportation services.

14 • TOPIC 1: Extending Linear Relationships

Throughout instruction, ELL tips are placed for teachers at point-of-use on the mini-lesson page in the TIG. They provide additional modifications to support this special population.

These tips:
- Inform teachers of potential learning obstacles specific to the lesson.
- Provide engaging activities for learning and assessment.
- Reinforce newly acquired mathematical language to gain an increasing level of comprehension of English.
- Introduce students to language needed to understand a specific context.

Students internalize new content language by using and reusing it in meaningful ways in a variety of different speaking activities that build concept and language attainment.

## For More Support

Visit the Texas Support Center for many more resources to support you and your students who are emergent bilingual students.

# Assessments

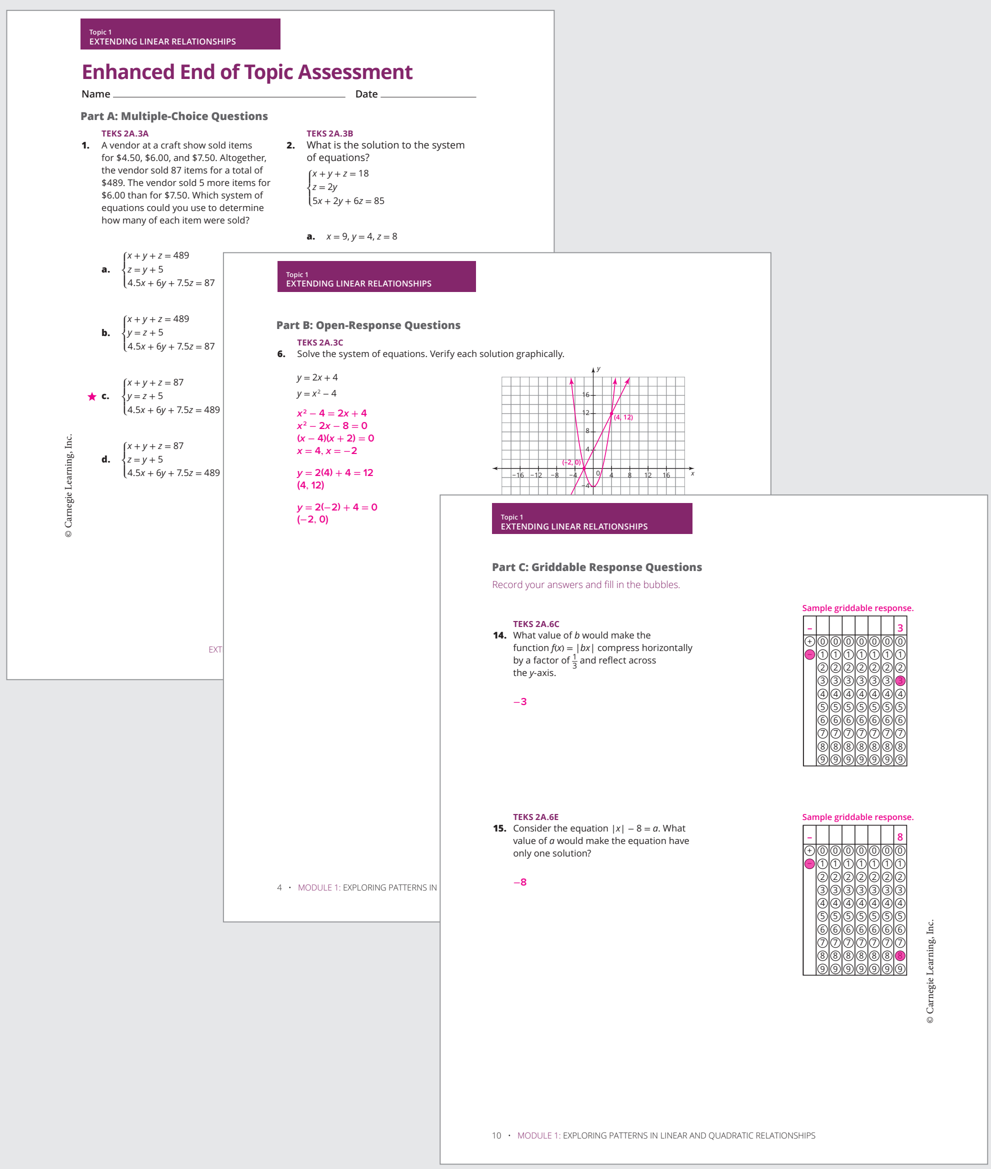
Both formative and summative assessments are an integral part of information gathering. Formative assessment tools are provided throughout each lesson, providing you with ongoing feedback of student performance and encouraging students to monitor their own progress. Ongoing formative assessment underlies the entire learning experience, driving real-time adjustments, next steps, insights, and measurements.

End of topic summative assessments are provided to measure student performance on a clearly denoted set of standards. For certain topics that extend longer than four instructional weeks, a mid-topic summative assessment is also provided.

## Enhanced End of Topic Assessment

There are three problem type sections per assessment. Multiple-choice questions, open-response questions, and griddable response questions prepare students for enhanced standardized tests.

The answer key provides teachers with the TEKS aligned to each question, as well as sample answers for open-response and griddable response questions.



# End of Course Topic

The End of Course Topic is the final topic of the course which includes a collection of problem-based performance tasks that are aligned with selected priority math standards of the course. This final topic provides students an additional opportunity to demonstrate their ability to make sense of multi-step, real-world problems, communicate their thinking, represent solutions, and justify their reasoning on content aligned with these selected math standards.

## Performance Tasks

Each performance task is a formative assessment tool that allows students to demonstrate their learning of the selected course content. At the end of each task, a section titled "Your Work Should Include" lists the categories and the corresponding max scoring points from the grading rubric.

2

PERFORMANCE TASK

### Ride Like the Wind

Ximena bikes 20 miles from her home to her favorite lunch spot and then bikes back. She bikes with the wind to lunch and against the wind on the way home. Her total travel time on the bike is 3.5 hours. When biking with no wind her average speed is 14 miles per hour.

1. Write and solve an equation to calculate the average speed of the wind. Show your work.

The next week Ximena bikes 28 miles to a different location to eat. She then bikes back to her home. She bikes against the wind to lunch and with the wind on the way home. Ximena's total travel time on the bike is 5 hours and 15 minutes, and the average speed of the wind is 10 miles per hour.

2. Write and solve an equation to calculate Ximena's average speed with no wind. Show your work.

**Your work should include:**
- A table that organizes the first set of given information. (3 points)
- An algebraic expression for the total travel time in the first situation. (2 points)
- A distance equation for the first problem situation written and solved with work shown. (3 points)
- A solution for the first problem situation. (1 point)
- A table that organizes the second set of given information. (3 points)
- An algebraic expression for the total travel time in the second situation. (2 points)
- A distance equation for the second problem situation written and solved with work shown. (3 points)
- A solution for the second problem situation. (1 point)

## Grading Rubric

The grading rubric is for students and teachers to set clear expectations for how each completed performance task will be evaluated. Students should use the rubric to guide their work and self-monitor their progress. Teachers should use the rubric to evaluate and provide feedback for the completed performance task.

RUBRIC: 18 TOTAL POINTS

| | 0 points | 1 point | 2 points | 3 points |
|---|---|---|---|---|
| **Completion of for the First Problem Situation** | No entries provided. | Some table entries are made but not all are correct. | Table is complete and most entries are correct. | Table is complete and all entries are correct. |
| **Algebraic Expression for the First Problem Situation** | No expression provided. | Expression is incorrect. | Expression is correct. | N/A |
| **Equation Written and Solved for the First Problem Situation** | No equation provided. | An incorrect equation or a correct equation with major errors in the solution steps. | A correct equation with a minor error in the solution steps. | A correct equation with accurate solution steps. |
| **Solution for the First Situation** | No solution or an incorrect solution based on the context of the problem. | A correct solution based on the context of the problem. | N/A | N/A |
| **Completion of Table for the Second Problem Situation** | No entries provided. | Some table entries are made but not all are correct. | Table is complete and most entries are correct. | Table is complete and all entries are correct. |
| **Algebraic Expression for the Second Problem Situation** | No expression provided. | Expression is incorrect. | Expression is correct. | N/A |
| **Equation Written and Solved for the Second Problem Situation** | No equation provided. | An incorrect equation or a correct equation with major errors in the solution steps. | A correct equation with a minor error in the solution steps. | A correct equation with accurate solution steps. |
| **Solution for the Second Problem Situation** | No solution or an incorrect solution based on the context of the problem. | A correct solution based on the context of the problem. | N/A | N/A |

## Teacher's Implementation Guide

The Teacher's Implementation Guide for the End of Course Topic contains a performance task overview, list of aligned TEKS and ELPS, essential ideas, facilitation notes which describe how to pace the two-day performance task, sample answer, and grading rubric.

**2**

**Performance Task**

Ride Like the Wind

**MATERIALS**

- Graphing technology
- Allow students to have access to any additional materials that may assist in the completion of this task

**Performance Task Overview**

Students are given the following information about a bike rider: distance traveled, total travel time, and average biking speed in windless conditions; the biker travels with the wind to her destination and against the wind on the way back. They create a table to organize the information and then use the information in the table to write and solve an equation to determine the speed of the wind. Students must consider an extraneous solution that does not make sense in the context of the problem.

Next, they are given new data for the rider's second trip: distance traveled, the total travel time, and the average wind speed. Students create a table to organize the information and then use the information in the table to write and solve an equation to determine the average biking speed with no wind. This time students must consider the reasonableness of their solutions in relation to the problem scenario.

**Algebra 2**
**Cubic, Cube Root, Absolute Value and Rational Functions, Equations, and Inequalities**

**(6) The student applies mathematical processes to understand that cubic, cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. The student is expected to:**

(H) formulate rational equations that model real-world situations.
(I) solve rational equations that have real solutions.
(J) determine the reasonableness of a solution to a rational equation.

**ELPS**

1.A, 1.B, 1.D, 1.E, 1.H, 2.B, 2.D, 2.E, 2.G, 3.B, 3.C, 3.D, 3.E, 3.F, 3.G, 3.H, 3.J, 4.C, 4.E, 4.F, 4.G, 4.H, 4.J, 5.A, 5.B, 5.C, 5.D, 5.F, 5.G

**Essential Ideas**

- A rational equation is an equation that contains one or more rational expressions.
- Rational functions can be used to model real-world problems.
- Rational equations can be solved using various algebraic methods.
- A distance problem is a type of problem that involves distance, rate, and time.

END OF COURSE TOPIC: Performance Task 2 • 1

SAMPLE ANSWER

1. Let $x$ represent the speed of the wind.

| | Distance Traveled | Time Traveled | Average Speed |
|---|---|---|---|
| | Miles | Hours | $\frac{\text{Miles}}{\text{Hours}}$ |
| **With the Wind** | 20 | $\frac{20}{14+x}$ | $14+x$ |
| **Against the Wind** | 20 | $\frac{20}{14-x}$ | $14-x$ |
| **Round Trip** | 40 | 3.5 | $\frac{40}{3.5} \approx 11.43$ |

$\frac{20}{14+x} + \frac{20}{14-x} = 3.5 \qquad x \neq 14, -14$

$(14+x)(14-x)\left[\frac{20}{14+x} + \frac{20}{14-x} = 3.5\right]$

$20(14-x) + 20(14+x) = 3.5(14+x)(14-x)$

$280 - 20x + 280 + 20x = 3.5(196 - x^2)$

$560 = 3.5(196 - x^2)$

$160 = 196 - x^2$

$36 = x^2$

$x = 6, -6; \qquad -6$ is an extraneous solution

The average speed of the wind is 6 miles per hour.

2. Let $x$ represent Ximena's average speed with no wind.

| | Distance Traveled | Time Traveled | Average Speed |
|---|---|---|---|
| | Miles | Hours | $\frac{\text{Miles}}{\text{Hours}}$ |
| **With the Wind** | 28 | $\frac{28}{x+10}$ | $x+10$ |
| **Against the Wind** | 28 | $\frac{28}{x-10}$ | $x-10$ |
| **Round Trip** | 56 | 5.25 | $\frac{56}{5.25} \approx 10.67$ |

$\frac{28}{x+10} + \frac{28}{x-10} = 5.25 \qquad x \neq 10, -10$

$(x+10)(x-10)\left[\frac{28}{x+10} + \frac{28}{x-10} = 5.25\right]$

$28(x-10) + 28(x+10) = 5.25(x+10)(x-10)$

$28x - 280 + 28x + 280 = 5.25(x^2 - 100)$

$56x = 5.25x^2 - 525$

$0 = 5.25x^2 - 56x - 525$

$x \approx 16.7, -6; \qquad -6$ does not make sense in relation to the problem situation

Ximena's average biking speed without any wind was approximately 16.7 miles per hour.

Similar to the other topics in this course, the End of Course Topic also has a Topic Family Guide for students and caregivers, and a Topic Overview for teachers. The End of Course Topic does not include an end of topic assessment since each performance task is a formative assessment.

Carnegie Learning recognizes that it is the classroom teachers who make the material come alive for students, transforming the way math is taught. Implementation requires integrating learning together and learning individually.

PREPARE YOURSELF

# Prepare for Learning Together

The most important first step you can take in preparing to teach with these instructional materials is to become comfortable with the mathematics.

- Read through the Module 1 Overview and the Topic 1 Overview.
- Do the math of the first Topic, and consider the facilitation notes.
- Prepare team-building activities to intentionally create a student-centered environment.

# Prepare for Learning Individually

Plan how you will utilize Skills Practice as a Learning Individually resource. Then, determine how you will introduce Skills Practice to students. Explain to them the benefits of working individually and why practice is important.

- Read through Module 1 Topic 1 Skills Practice.
- Determine which problem sets align with the activities in the corresponding student lessons.
- Based on student performance in the lesson, be prepared to assign the class, small groups of students, or individual students different problem sets to practice skills to develop mastery.

Plan how you will introduce students to MATHia. Explain to them the benefits of working individually and why practice is important.

- Test out the computers or tablets that your students will be using.
- Verify your classes have been set up in Teacher's Toolkit with correct MATHia content assigned. Or manually set up your classes in Teacher's Toolkit if applicable.
- Use the Content Browser in Teacher's Toolkit to explore the content students are assigned.
- Be prepared to demonstrate how students will access and log into MATHia.

PREPARE
YOUR CLASSROOM

# Prepare the Environment

The classroom is often considered the third teacher. Consider how to create a learning environment that engages students and fosters a sense of ownership. The use of space in your classroom should be flexible and encourage open sharing of ideas.

- Consider how your students are going to use the consumable book. It is the student's record of their learning. Many teachers have students move an entire topic to a three-ring binder as opposed to carrying the entire book.
- Arrange your desks so students can talk and collaborate with each other.
- Prepare a toolkit for groups to use as they work together and share their reasoning (read the materials list in each Topic Overview).
- Consider where you will display student work, both complete and in-progress.
- Create a word wall of key terms used in the textbook.

PREPARE
YOUR STUDENTS

# Prepare the Learners

If you expect students to work well together, they need to understand what it means to collaborate and how it will benefit them. It is important to establish classroom guidelines and structure groups to create a community of learners.

- Facilitate team-building activities and encourage students to learn each others' names.
- Set clear expectations for how the class will interact:
    - Their text is a record of their learning and is to be used as a reference for any assignments or tests you give.
    - They will be doing the thinking, talking, and writing in your classroom.
    - They will be working and sharing their strategies and reasoning with their peers.
    - Mistakes and struggles are normal and necessary.

PREPARE FAMILIES
AND CAREGIVERS

# Prepare the Support

- Prepare a letter to send home on the first day. Visit the Texas Support Center for a sample letter.
- Encourage families and caregivers to read the introduction of the textbook.
- Ensure that families and caregivers receive the module Family and Caregiver Guide at the start of each module. They should also receive the Family Guide at the start of the first topic and each subsequent topic.
- Consider a Family Math Night some time within the first few weeks of the school year.
- Encourage families and caregivers to explore the Students & Caregivers Portal on the Texas Support Center at **www.CarnegieLearning.com/texas-help/students-caregivers**.

# Students and Caregivers Portal

Research has proven time and again that family engagement greatly improves a student's likelihood of success in school.

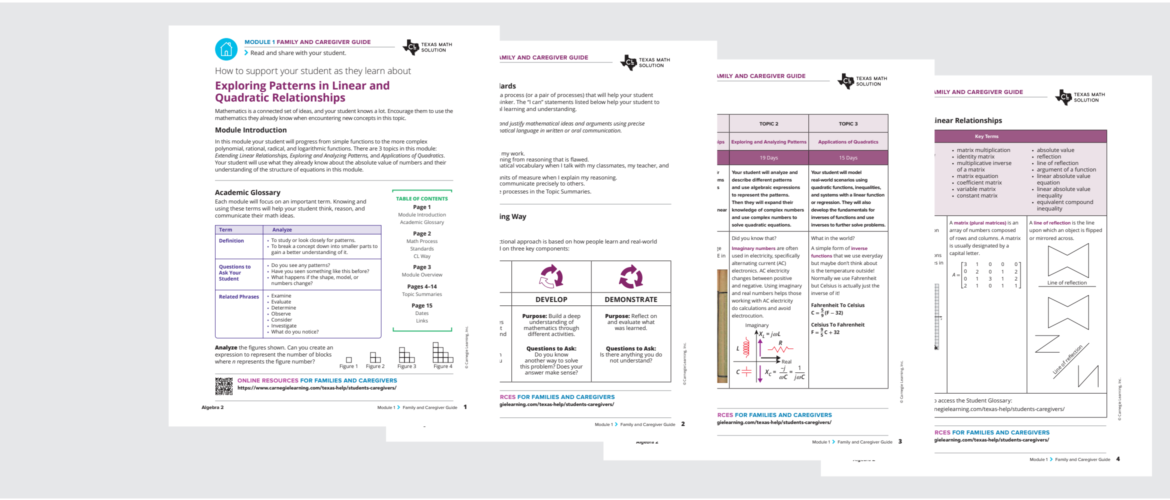
The Students & Caregivers Portal on the Texas Support Center provides:

- Getting to Know Carnegie Learning video content to provide an introduction to the instructional materials and research.
- Articles and quick tip videos offering strategies for how families and caregivers can support student learning. **Visit the Texas Support Center regularly to access new content and resources for students and caregivers as they learn mathematics in a variety of environments outside of the classroom**.

## MODULE FAMILY AND CAREGIVER GUIDES

Each module has a Family and Caregiver Guide available through the Students & Caregivers Portal on the Texas Support Center. Each module guide of the course will provide a different highlight of the academic glossary, description and examples of TEKS Mathematical Process Standards, and an overview of a different component of our instructional approach known as The Carnegie Learning Way. Also included is a module overview of content, specific key terms, visual representations, and strategies students are learning in each topic of the module.

The purpose of the Family and Caregiver Guide is to bridge student learning in the classroom to student learning at home. The goal is to empower families and caregivers to understand the concepts and skills learned in the classroom so that families can review, discuss, and solidify the understanding of these key concepts together. Videos will also be available on the Students & Caregivers Portal to provide added support.



## TOPIC FAMILY GUIDES

Each topic contains a Family Guide that provides an overview of the mathematics of the topic, how that math is connected to what students already know, and how that knowledge will be used in future learning. It also incorporates an illustration of math from the real world, a sample standardized test question, talking points, and a few of the key terms that students will learn.

We recognize that learning outside of the classroom is crucial to students' success at school. While we don't expect families and caregivers to be math teachers, the Family Guides are designed to assist families and caregivers as they talk to their students about what they are learning. Our hope is that both the students and their caregivers will read and benefit from the guides.

Carnegie Learning Family Guide — Algebra II

### Module 1: Extending Linear Relationships

**TOPIC 1: EXTENDING LINEAR RELATIONSHIPS**

Students begin this topic by reviewing what they know about systems of linear equations. They apply this knowledge to solve systems involving a linear and a quadratic equation and systems of three linear equations in three variables. Students also use systems of linear inequalities and linear programming to model optimal solutions to real-world situations. They use matrices to solve systems of linear equations in three variables.

Next, they calculate the absolute value of given values before considering the linear absolute value function. Students first graph the function $f(x) = x$, and then graph $f(x) = |x|$ and $f(x) = |-x|$, discussing how each graph changed. Students explore transformations of the function before moving on to solve and graph linear absolute value equations and inequalities based on real-world situations.

**Where have we been?**

Students enter this topic with a wide range of experiences with linear functions. Students have set up and solved systems of equations since late middle school and early high school. They have investigated properties of real numbers, including the multiplicative identity and multiplicative inverse. In this topic, students will extend these properties to a new object—a matrix.

**Where are we going?**

Although derived from linear relationships, linear absolute value functions are more complex than the linear functions students have dealt with previously. They share enough characteristics with linear functions to be

**Linear Absolute Value Func**

The coordinate plane shows the graph of the absolute value function $f(x) = -2|x - 1| + 4$.

The graph increases to a vertex and then decreases and is symmetric across a vertical line through the vertex.

**Systems**

Your body is an amazing collection of different systems. Your cardiovascular system pumps blood throughout your body, your skeletal system provides shape and support, and your nervous system controls communication between your senses and your brain. Your skin, including your hair and fi ngernails, is a system all by itself—the integumentary system—and it protects all of your body's other systems. You also have a digestive system, endocrine system, excretory system, immune system, muscular system, reproductive system, and respiratory system.

**Talking Points**

Absolute value is an important topic to know about for college admissions tests.

Here is an example of a sample question:

**What are the values of $n$ and $p$ so that $-n|2p - 6| > 0$?**

For the product to be greater than 0, the factors must be either both greater than 0 or both less than 0.

Since one of the factors is an absolute value, the factors cannot be both less than 0, so they are both greater than 0.

This means that $n$ must be less than 0, and $p$ cannot be equal to 3.

The solution is all values such that $n < 0$ and $p \neq 3$.

**Key Terms**

**linear programming**
Linear programming is a branch of mathematics that determines the maximum and minimum value of linear expressions on a region produced by a system of linear inequalities.

**matrix**
A matrix (plural matrices) is an array of numbers composed of rows and columns.

**absolute value**
The absolute value of a number is its distance from zero on the number line.

**line of reflection**
A line of reflection is the line that the graph is reflected across.

**linear absolute value equation**
An equation in the form $|x + a| = c$ is a linear absolute value equation.

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# YOU MIGHT BE WONDERING. . .

### Why do we believe in our brand of blended: Learning Together and Learning Individually?

There has been lots of research on the benefits of learning collaboratively. Independent practice is necessary for students to become fluent and automatic in a skill. A balance of these two pieces provides students with the opportunity to develop a deep conceptual understanding through collaboration with their peers, while demonstrating their understanding independently.

### Why don't we have a Worked Example at the start of every lesson?

Throughout the Texas Math Solution, we do provide worked examples. Sweller and Cooper (1985) argue that worked examples are educationally efficient because they reduce working memory load. Ward and Sweller (1990) found that alternating between problem solving and viewing worked examples led to the best learning. Students often read worked examples with the intent to confirm that they understand the individual steps. However, the educational value of the worked example often lies in thinking about how the steps connect to each other and how particular steps might be added, omitted, or changed, depending on context.

### Where are the colorful graphics to get students' attention?

Color and visuals make for stronger student engagement, right? Not quite. Our instructional materials have little extraneous material. This approach follows from research showing that "seductive details" used to spice up the presentation of material often have a negative effect on student learning (Mayer et al., 2001; Harp & Meyer, 1998). Students may not know which elements of an instructional presentation are essential and which are intended simply to provide visual interest. So, we focus on the essential materials. While we strive to make our educational materials attractive and engaging to students, research shows that only engagement based on the mathematical content leads to learning.

### Why is the book so big?

The student textbook contains all of the resources students need to complete the Learning Together component of the course. Students are to actively engage in this textbook, topic-by-topic, creating a record of their learning as they go. There is room to record answers, take notes, draw diagrams, and fix mistakes. Visit the Texas Support Center at **https://www.CarnegieLearning.com/texas-help/** for tips on managing your textbooks.

### CUSTOMER SUPPORT

The Carnegie Learning Texas Support Team is available to help with any issue at help@carnegielearning.com.

**Monday–Friday**
**8:00 am–8:00 pm CT**
via email, phone, or live chat

Our expert team provides support for installations, networking, and technical issues, and can also help with general questions related to pedagogy, classroom management, content, and curricula.

# Notes

> If you have questions, reach out to us for support. Our team of master practitioners have been where you are. We made mistakes and we learned from them. We want to help you. We have many professional development options. Whether we come to your school for a workshop, join you in your classroom for modeling or coaching, or you join us online for a webinar or an entire course, our goal is to make sure you feel supported and prepared to use the tasks you'll find in this book to their fullest!
>
> Kasey Bratcher, Senior VP of Professional Learning