

## Grade 6

## Student Textbook Skills Program Edition SY 2022-2023

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## Manifesto

## Our Manifesto

WE BELIEVE that quality math education is important for all students, to help them develop into creative problem solvers, critical thinkers, life-long learners, and more capable adults.

WE BELIEVE that math education is about more than memorizing equations or performing on tests-it's about delivering the deep conceptual learning that supports ongoing growth and future development.

WE BELIEVE all students learn math best when teachers believe in them, expect them to participate, and encourage them to own their learning.

WE BELIEVE teachers are fundamental to student success and need powerful, flexible resources and support to build dynamic cultures of collaborative learning.

WE BELIEVE our learning solutions and services can help accomplish this, and that by working together with educators and communities we serve, we guide the way to better math learning.

## LONG + LIVE + MATH

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Mathematics is so much more than memorizing rules. It is learning to reason, to make connections, and to make sense of the world. We believe in Learning by Doing ${ }^{(T \mathrm{M})}$ —you need to actively engage with the content if you are to benefit from it. The lessons were designed to take you from your intuitive understanding of the world and build on your prior experiences to then learn new concepts. My hope is that these instructional materials help you build a deep understanding of math.

Sandy Bartle Finocchi, Chief Mathematics Officer

My hope is that as you work through this course, you feel capable-capable of exploring new ideas that build upon what you already know, capable of struggling through challenging problems, capable of thinking creatively about how to fix mistakes, and capable of thinking like a mathematician.

Amy Jones Lewis, Senior Director of Instructional Design

At Carnegie Learning, we have created an organization whose mission and culture is defined by your success. Our passion is creating products that make sense of the world of mathematics and ignite a passion in you. Our hope is that you will enjoy our resources as much as we enjoyed creating them.

Barry Malkin, CEO

Module 1: Composing and Decomposing
Topic 1: Factors and Multiples
1 Taking Apart Numbers and Shapes
Writing Equivalent Expressions Using the Distributive Property
2 Searching for Common Ground
Identifying Common Factors and Common Multiples
3 Composing and Decomposing Numbers
Least Common Multiple and Greatest Common Factor
Topic 2: Positive Rational Numbers
1 Rocket StripsDividing a Whole into Fractional Parts
2 Getting Closer
Benchmark Fractions
3 Did You Get the Part?
Multiplying Fractions
4 Yours IS to Reason Why!Fraction by Fraction Division
Topic 3: Shapes and Solids
1 Consider Every SideConstructing Triangles Given Sides
2 Turning a One-Eighty!
Triangle Sum Theorem
3 All About That Base .....  and Height
Area of Triangles and Quadrilaterals
4 Length, Width, and Depth
Deepening Understanding of Volume

## Topic 4: Decimals

1
You Have a Point
Plotting, Comparing, and Ordering Rational Numbers

## Module 2: Relating Quantities

## Topic 1: Ratios

1 It's All Relative

2 Going Strong
Comparing Ratios to Solve Problems
3 Oh, Yes, I Am the Muffin Man
Determining Equivalent Ratios
4 A Trip to the Moon
Using Tables to Represent Equivalent Ratios
5 They're Growing!
Graphs of Ratios
6 One Is Not Enough
Using and Comparing Ratio Representations

## Topic 2: Percents

1 We Are Family!
Percent, Fraction, and Decimal Equivalence
2 Warming the Bench
Using Estimation and Benchmark Percents
3 The Forest for the Trees
Determining the Part and the Whole in Percent Problems

## Topic 3: Unit Rates and Conversions

1 Several Ways to Measure Using Ratio Reasoning to Convert Units<br>2 What Is the Best Buy?<br>Introduction to Unit Rates<br>3 Seeing Things Differently Multiple Representations of Unit Rates

## Module 3: Moving Beyond Positive Quantities

## Topic 1: Signed Numbers and the Four Quadrants

1 Human Number Line Introduction to Negative Numbers

2 Magnificent Magnitude Absolute Value

3 What's in a Name? Rational Number System

4 Four Is Better Than One
Extending the Coordinate Plane

## Topic 2: Operating with Integers

1 Math Football
Using Models to Understand Integer Addition
2 Walk the Line Adding Integers, Part I

3 Two-Color Counters
Adding Integers, Part II
4 What's the Difference?
Subtracting Integers
5 Equal Groups
Multiplying and Dividing Integers

## Module 4: Determining Unknown Quantities

## Topic 1: Expressions

1 Relationships Matter
Evaluating Numeric Expressions
2 Into the Unknown
Introduction to Algebraic Expressions
3 Second Verse, Same as the First
Equivalent Expressions
4 Are They Saying the Same Thing?
Verifying Equivalent Expressions
5 DVDs and Songs
Using Algebraic Expressions to Analyze and Solve Problems

## Topic 2: Equations and Inequalities

1 First Among Equals
Reasoning with Equal Expressions
2 Bar None
Solving One-Step Addition Equations
3 Play It In Reverse
Solving One-Step Multiplication Equations
4 The Real Deal
Solving Equations to Solve Problems
5 Greater Than Most
Solving Inequalities with Inverse Operations

## Topic 3: Graphing Quantitative Relationships

1 Every Graph Tells a StoryIndependent and Dependent Variables
2 The Power of the Horizontal Line
Using Graphs to Solve Problems
3 Planes, Trains, and Paychecks
Multiple Representations of Equations
4 Time for Triathlon Training
Relating Distance, Rate, and Time
5 There Are Many Paths .....
Problem Solving on the Coordinate Plane
1 Knowledge You Can Bank On Checking Accounts
2 You Are a Real Card!
Debit Cards vs. Credit Cards
3 Financial Report Card Understanding Credit Reports
4 The Possibilities Are Endless
Career Exploration
5 Student Aid 101Paying for College
Topic 4: Financial Literacy: Accounts, Credit, and Careers
Module 5: Describing Variability of Quantities
Topic 1: The Statistical Process
1 What's Your Question?
Understanding the Statistical Process
2 Get in Shape
Analyzing Numerical Data Displays
3 Follow Me on Histogram
Using Histograms to Display Data
Topic 2: Numerical Summaries of Data
1 In the Middle
Analyzing Data Using Measures of Center
2 Box It Up
Displaying the Five-Number Summary
3 Dealing with Data
Collecting, Displaying, and Analyzing Data
End of Course Topic
Formative Assessment
1 Living Room Rug
Performance Task
2 Pizza Party
Performance Task
3 School Ski Trip
Performance Task
Glossary


## 1. Learning Goals

 Learning goals are stated for each lesson to help you take ownership of the learning objectives.
## 2. Connection

 Each lesson begins with a statement connecting what you have learned with a question to ponder.Return to this question at the end of this lesson to gauge your understanding.
3. Getting Started Each lesson begins with a Getting Started. When working on the Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.


The numeric expression of $5 \times 27$ represents the area of the walkway from the Getting Started. A numeric expression is a mathematical phrase that contains numbers and operations.

The equation $5 \times 27=135$ shows that the expression $5 \times 27$ is equal to the expression 135 .

An equation is a mathematical sentence that uses an equals sign to show that two or more quantities are the same as one another.

1. Reflect on the different ways you can rewrite the product of 5 and 27. Select one of your area models to complete the example.

How did you split the side

$$
5 \times 27=5
$$

$\qquad$ $+$ $\qquad$ length of 27 ?

What are the factors of each smaller region?

What is the area of each smaller region?
$\qquad$


What is the total area? $\qquad$

## 4. Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

## Remember:

- It's not just about answer-getting. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, Worked Examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

## 5. Talk the Talk

 Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

## Assignment



## 6. Write

Reflect on your work and clarify your thinking.

## 7. Remember

Take note of the key concepts from the lesson.

## 8. Practice

Use the concepts learned in the lesson to solve problems.

## 9. Stretch

Ready for a challenge?

## 10. Review

Remember what you've learned by practicing concepts from previous lessons and topics.

## Problem Types You Will See

## Worked Example

## When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.


## Ask Yourself:

- What is the main idea?
- How would this work if I changed the numbers?


## WORKED EXAMPLE

Determine the quantity in pounds that is equivalent to 4.5 kilograms.

Scaling Up

$\frac{1 \mathrm{~kg}}{2.2 \mathrm{lb}}=\frac{4.5 \mathrm{~kg}}{? \mathrm{lb}}$


Unit Analysis

$$
4.5 \mathrm{~kg}\left(\frac{2.2 \mathrm{lb}}{1 \mathrm{~kg}}\right)
$$

$$
\frac{4.5 \mathrm{~kg}}{1}\left(\frac{2.2 \mathrm{lb}}{1 \mathrm{~kg}}\right)=9.9 \mathrm{lb}
$$

$$
\frac{1 \mathrm{~kg}}{2.2 \mathrm{lb}}=\frac{4.5 \mathrm{~kg}}{9.9 \mathrm{lb}}
$$

$$
4.5 \mathrm{~kg}=9.9 \mathrm{lb}
$$

- Have I used these strategies before?

Christopher and Max want to determine the number of miles in 31,680 feet using unit analysis.

| Max |
| :--- |
| $31,680 \mathrm{fy}$ |
| 5280 ff |$=6 \mathrm{mi}$

Christopher
$31,680 \mathrm{ft}\left(\frac{5280 \mathrm{ft}}{1 \mathrm{mi}}\right)=167,270,400 \mathrm{mi}$

Thumbs Up

## When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.


## Ask Yourself:

- Why is this method correct?
- Have I used this method before?

Thumbs Down
When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.


## Ask Yourself:

- Where is the error?
- Why is it an error?
- How can I correct it?

Tim and Dan love cereal, but don't want to spend a lot of money. After scanning the aisle in the grocery store for the lowest prices, the boys make the following statements.

- Tim says, "I found Sweetie Oat Puffs for $\$ 0.14$ per ounce. That's the cheapest cereal in the aisle!"
- Dan replies, "It's not cheaper than Sugar Hoops! The unit price for that is 6.25 oz per dollar."

Who is correct? Explain your reasoning.

## Who's Correct?

## When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or not correct.


## Ask Yourself:

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?


## The Crew

The Crew is here to help you on your journey. Sometimes they will remind you about things you already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are members of your groupsomeone you can rely on!


Teacher aides will guide you along your journey. They will help you make connections and remind you to think about the details.


# Mathematical <br> Process Standards 

## Texas Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The "I can" expectations listed below align with the TEKS Mathematical Process Standards and encourage students to develop their mathematical learning and understanding.

- Apply mathematics to problems arising in everyday life, society, and the workplace.

I can:

- use the mathematics that I learn to solve real world problems.
- interpret mathematical results in the contexts of a variety of problem situations.
- Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

I can:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.
- Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.
- Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

I can:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.
- Create and use representations to organize, record, and communicate mathematicalideas.

I can:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.
- Analyze mathematical relationships to connect and communicate mathematicalideas.

I can:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure
- look for general methods and more efficient ways to solve problems.


## - Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I can:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.


## Academic Glossary

Visit the Students \&
Caregivers Portal on the Texas Support Center at www.
CarnegieLearning. com/texas-help to access the Mathematics Glossary for this course anytime, anywhere.

## Related Phrases

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match


## Related Phrases

- Show your work
- Explain your calculation
- Justify
- Why or why not?

There are important terms you will encounter throughout this book. It is important that you have an understanding of these words as you get started on your journey through the mathematical concepts. Knowing what is meant by these terms and using these terms will help you think, reason, and communicate your ideas.

## ANALYZE

## Definition

To study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

## Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?


## EXPLAIN YOUR REASONING

## Definition

To give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

## Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?


## REPRESENT

## Related Phrases

## Definition

To display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

## Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table
- Is my representation accurate?


## ESTIMATE

Related Phrases

## Definition

To make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

## Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?


## DESCRIBE

## Definition

To represent or give an account of in words. Describing communicates mathematical ideas to others.

## Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?


## Related Phrases

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?


## Resources for Students and Caregivers

## Student Lesson Overview Videos

Each lesson has a corresponding lesson overview video(s) for you to use and reference as you are learning. The videos provide an overview of key concepts, strategies, and/or worked examples from the lessons.


## Topic Summary

A Topic Summary is provided at the end of each topic. The Topic Summary lists all key terms of the topic and provides a summary of each lesson. Each lesson summary defines key terms and reviews key concepts, strategies, and/or worked examples.


## Mathematics Glossary

## A course-specific mathematics glossary is available to utilize and reference while you are learning. Use the glossary to locate definitions and examples of math key terms.



## absolute value

The absolute value, or magnitude, of a number is its distance from zero on a number line.

## Example

The absolute value of -3 is the same as the absolute value of 3 because they are both a distance of 3 from zero on a number line.

$|-3|=|3|$
account balance
Amount of money in an account at a given time.

## Addition Property of Equality

The Addition Property of Equality states that if two values $a$ and $b$ are equal, when you add the same value $c$ to each, the sums are equal.

## Examples

$12=12$ and $12+7=12+7$
If $a=b$, then $a+c=b+c$.
additive inverses
Two numbers with the sum of zero are called additive inverses.

## Examples

$-19+19=0 \quad a+(-a)=0$
additive reasoning
Additive reasoning focuses on the use of addition and subtraction for comparisons.

## Examples

Vicki is 40 years old and Ben is 10 years old. In 5 years, Vicki will be 45 and Ben will be 15. Vicki will always be 30 years older than Ben. This is additive reasoning.
algebraic expression
An algebraic expression is a mathematical phrase that has at least one variable, and it can contain numbers and operation symbols.

## Examples

$a \quad 2 a+b \quad x y \quad \frac{4}{p} \quad z^{2}$

## algorithm

An algorithm is a process or description of steps you can follow to complete a mathematical calculation.

Annual Percentage Yield (APY)
Annual Percentage Yield (APY) is a percentage that is paid to customers based on the account balance in an account for a year.

## Example

For example, Yvonne opens a checking account with an average monthly balance of $\$ 600$. The account has a $2.5 \%$ APY.
$(600)(0.025)=15$
Yvonne will earn \$15 on this checking account over the course of the year.

## Module Family and Caregiver Guides

Each module guide will provide a different highlight of the academic glossary, description and examples of TEKS Mathematical Process Standards, and an overview of a different component of our instructional approach known as The Carnegie Learning Way. Also included is a module overview of content, specific key terms, visual representations, and strategies you are learning in each topic of the module.

The purpose of the Family and Caregiver Guides is to bridge student learning in the classroom to student learning at home. Our goal is to empower you and your family to understand the concepts and skills learned in the classroom so that you can review, discuss, and solidify the understanding of these key concepts together. Videos will be available on the Students \& Caregivers Portal on the Texas Support Center to provide added support.


## Topic Family Guides

Each topic contains a Family Guide that provides an overview of the math of the topic and answers the questions, "Where have we been?" and "Where are we going?" Additional components of the Family Guide are, as follows: an example of a math model or strategy taught in the topic, definitions of a few key terms, busting of a math myth, and questions families and caregivers can ask you to support your learning.

We recognize that learning outside of the classroom is crucial to student success at school. While we don't expect families and caregivers to be math teachers, the Family Guides are designed to assist families and caregivers as they talk to you about what you are learning. Our hope is that both you and your family will read and benefit from these guides.


## Students and Caregivers Portal

Research has proven time and again that family engagement greatly improves a student's likelihood of success in school.

The Students \& Caregivers Portal on the Texas Support Center provides:

- Getting to Know Carnegie Learning video content to provide an introduction to the instructional materials and research.
- Articles and quick tip videos offering strategies for how families and caregivers can support student learning.
- Access to instructional resources to support students and caregivers.
To access new content and resources, visit the Students and Caregivers Portal on the Texas Support Center at https:// www.CarnegieLearning.com/texas-help/students-caregivers/

