

Grade 6

Teacher's Implementation Guide Skills Program Edition SY 2022-2023

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Teacher's Implementation Guide

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Manifesto

Our Manifesto

WE BELIEVE that quality math education is important for all students, to help them develop into creative problem solvers, critical thinkers, life-long learners, and more capable adults.

WE BELIEVE that math education is about more than memorizing equations or performing on tests—it's about delivering the deep conceptual learning that supports ongoing growth and future development.

WE BELIEVE all students learn math best when teachers believe in them, expect them to participate, and encourage them to own their learning.

WE BELIEVE teachers are fundamental to student success and need powerful, flexible resources and support to build dynamic cultures of collaborative learning.

WE BELIEVE our learning solutions and services can help accomplish this, and that by working together with educators and communities we serve, we guide the way to better math learning.

LONG + LIVE + MATH



At Carnegie Learning, we choose the path that has been proven most effective by research and classroom experience. We call that path the Carnegie Learning Way. Follow this code to take a look inside.

Acknowledgments

Middle School Math Solution Authors

- Sandy Bartle Finocchi, Chief Mathematics Officer
- Amy Jones Lewis, Senior Director of Instructional Design
- Kelly Edenfield, Instructional Designer
- Josh Fisher, Instructional Designer

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- Thank you to all the Texas educators and education professionals who supported the review process and provided feedback for this resource.

Mathematics is so much more than rules and algorithms. It is learning to reason, to make connections, and to make sense of the world. We believe in Learning by Doing[™]—students need to actively engage with the content if they are to benefit from it. Your classroom environment will determine what type of discourse, questioning, and sharing will take place. Students deserve a safe place to talk, to make mistakes, and to build deep understanding of mathematics. My hope is that these instructional materials help you shift the mathematical authority in your class to your students. Be mindful to facilitate conversations that enhance trust and reduce fear.

Sandy Bartle Finocchi, Chief Mathematics Officer

My hope is that you know that your students are capable of thinking like mathematicians. This book is designed to give them the opportunity to struggle with challenging tasks, to talk about math with their classmates, and to make and fix mistakes. I hope that you use this book to build this capacity in your students—to ask the necessary questions to uncover what students already know and connect it to what they are learning, to encourage creative thinking, and to give just enough support to keep students on the right path.

.

Amy Jones Lewis, Senior Director of Instructional Design

At Carnegie Learning, we have created an organization whose mission and culture is defined by student success. Our passion is creating products that make sense of the world of mathematics and ignite a passion in students. Our hope is that students will enjoy our resources as much as we enjoyed creating them.

Barry Malkin, CEO

The Carnegie Learning Way

At Carnegie Learning, we choose the path that has been proven most effective by research and classroom experience. We call that path the **Carnegie Learning Way.**

Our Instructional Approach

Carnegie Learning's instructional approach is a culmination of the collective knowledge of our researchers, instructional designers, cognitive learning scientists, and master practitioners. It is based on a scientific understanding of how people learn, as well as an understanding of how to apply the science to the classroom. At its core, our instructional approach is based on three simple, key components:



ENGAGE

Activate student thinking by tapping into prior knowledge and real-world experiences. Provide an introduction that generates curiosity and plants the seeds for deeper learning.



DEVELOP

Build a deep understanding of mathematics through a variety of activities. Students encounter real-world problems, sorting activities, Worked Examples, and peer analysis in an environment where collaboration, conversations, and questioning are routine practices.



DEMONSTRATE

Reflect on and evaluate what was learned. Ongoing formative assessment underlies the entire learning experience, driving real-time adjustments, next steps, insights, and measurements.



Our Research

Carnegie Learning has been deeply immersed in research ever since it was founded by cognitive and computer scientists from Carnegie Mellon University. Our research extends far beyond our own walls, playing an active role in the constantly evolving field of cognitive and learning science. Our internal researchers collaborate with a variety of independent research organizations, tirelessly working to understand more about how people learn, and how

learning is best facilitated. We supplement this information with feedback and data from our own products, teachers, and students, to continuously evaluate and elevate our instructional approach and its delivery.

Our Support

We're all in. In addition to our books and software, implementing Carnegie Learning in your classroom means you get access to an entire ecosystem of ongoing classroom support, including:

Professional Learning: Our team of Master Math Practitioners is always there for you, from implementation to math academies to a variety of other options to help you hone your teaching practice.

Texas Support Center: We've customized a Support Center just for you and your students. The Texas Support Center provides articles and videos to help you implement the Texas Math Solution, from the basics to get you started to more targeted support to guide you as you scaffold instruction for all learners in your classroom. Visit **www.CarnegieLearning.com/texas-help** to explore online and to access content that you can also share with your students and their caregivers.

MyCL: This is the central hub that gives you access to all of the products and resources that you and your students will need. Visit MyCL at **www.CarnegieLearning.com/login**.

LONG + LIVE + MATH: When you join this community of like-minded math educators, suddenly you're not alone. You're part of a collective, with access to special content, events, meetups, book clubs, and more. Because it's a community, it's constantly evolving! Visit **www.longlivemath.com** to get started.

Scan this code to visit the Texas Support Center and look for references throughout the Front Matter to learn more about the robust resources you will find in the Support Center.



Our Blend of Learning

The Texas Math Solution delivers instructional resources that make learning math attainable for all students. Learning Together and Learning Individually resources work in parallel to engage students with various learning experiences they need to understand the mathematics at each grade level.

For **Learning Together**, the student textbook is a consumable resource that empowers students to become creators of their mathematical knowledge. This resource is designed to support teachers in facilitating active learning so that students feel confident in sharing ideas, listening to each other, and learning together.

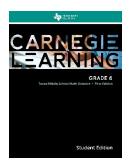
Over the course of a year, based on the recommended pacing, teachers will spend approximately 60% of their instructional time teaching whole-class activities as students learn together.

For **Learning Individually**, the Skills Practice provides students the opportunity to engage with problems that target each lesson's skills, concepts, and applications. This resource is designed to target discrete skills for development and mastery, therefore, scaffolding and extension opportunities are provided in the problem sets.

An additional Learning Individually resource is MATHia[®], an intelligent software that provides just-in-time support and tracks student progress against fine-grained skills to deliver the right content they need to become proficient with the mathematics.

Over the course of the year, based on the recommended pacing, teachers will spend approximately 40% of their instructional time monitoring students as they work and learn individually.

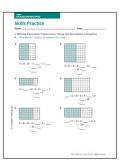
Learning Together



TEXTBOOK

I am a record of student thinking, reasoning, and problem solving. My lessons allow students to build new knowledge based upon prior knowledge and experiences, apply math to real-world situations, and learn together in a collaborative classroom.

My purpose is to create mathematical thinkers who are active learners that participate in class.



Learning Individually

SKILLS PRACTICE

I am targeted practice of each lesson's skills, mathematical concepts, and applications for each topic in the student textbook.

My purpose is to provide additional problem sets for teachers to assign as needed for additional practice or remediation.

MATHia

I am designed to empower students to learn individually at their own pace with sophisticated AI technology that personalizes their learning experiences, while giving teachers real-time insights to monitor student progress.

My purpose is to coach students alongside teachers as students learn, practice, do, and look forward.



Visit the Texas Support Center for additional information on the Learning Individually resources.

Table of Contents

Module 1: Composing and Decomposing

Topic 1: Factors and Multiples

- 1 Taking Apart Numbers and Shapes Writing Equivalent Expressions Using the Distributive Property
- 2 Searching for Common Ground Identifying Common Factors and Common Multiples
- 3 Composing and Decomposing Numbers Least Common Multiple and Greatest Common Factor

Topic 2: Positive Rational Numbers

- 1 Rocket Strips Dividing a Whole into Fractional Parts
- 2 Getting Closer Benchmark Fractions
- 3 Did You Get the Part? Multiplying Fractions
- 4 Yours IS to Reason Why! Fraction by Fraction Division

Topic 3: Shapes and Solids

- 1 Consider Every Side Constructing Triangles Given Sides
- 2 Turning a One-Eighty! Triangle Sum Theorem
- 3 All About That Base ... and Height Area of Triangles and Quadrilaterals
- 4 Length, Width, and Depth Deepening Understanding of Volume

Topic 4: Decimals

- 1 You Have a Point Plotting, Comparing, and Ordering Rational Numbers
- 2 Get in Line Adding and Subtracting Decimals
- 3 Product Placement Multiplying Decimals
- 4 Dividend in the House Dividing Whole Numbers and Decimals

Module 2: Relating Quantities

Topic 1: Ratios

- 1 It's All Relative Introduction to Ratio and Ratio Reasoning
- 2 Going Strong Comparing Ratios to Solve Problems
- 3 Oh, Yes, I Am the Muffin Man Determining Equivalent Ratios
- 4 A Trip to the Moon Using Tables to Represent Equivalent Ratios
- 5 They're Growing! Graphs of Ratios
- 6 One Is Not Enough Using and Comparing Ratio Representations

Topic 2: Percents

- 1 We Are Family! Percent, Fraction, and Decimal Equivalence
- 2 Warming the Bench Using Estimation and Benchmark Percents
- 3 The Forest for the Trees Determining the Part and the Whole in Percent Problems

Topic 3: Unit Rates and Conversions

- 1 Several Ways to Measure Using Ratio Reasoning to Convert Units
- 2 What Is the Best Buy? Introduction to Unit Rates
- 3 Seeing Things Differently Multiple Representations of Unit Rates

Module 3: Moving Beyond Positive Quantities

Topic 1: Signed Numbers and the Four Quadrants

- 1 Human Number Line Introduction to Negative Numbers
- 2 Magnificent Magnitude Absolute Value
- 3 What's in a Name? Rational Number System
- 4 Four Is Better Than One Extending the Coordinate Plane

Topic 2: Operating with Integers

- 1 Math Football Using Models to Understand Integer Addition
- 2 Walk the Line Adding Integers, Part I
- 3 Two-Color Counters Adding Integers, Part II
- 4 What's the Difference? Subtracting Integers
- 5 Equal Groups Multiplying and Dividing Integers

Module 4: Determining Unknown Quantities

Topic 1: Expressions

- 1 Relationships Matter Evaluating Numeric Expressions
- 2 Into the Unknown Introduction to Algebraic Expressions
- 3 Second Verse, Same as the First Equivalent Expressions
- 4 Are They Saying the Same Thing? Verifying Equivalent Expressions
- 5 DVDs and Songs Using Algebraic Expressions to Analyze and Solve Problems

Topic 2: Equations and Inequalities

- 1 First Among Equals Reasoning with Equal Expressions
- 2 Bar None Solving One-Step Addition Equations
- 3 Play It In Reverse Solving One-Step Multiplication Equations
- 4 The Real Deal Solving Equations to Solve Problems
- 5 Greater Than Most Solving Inequalities with Inverse Operations

Topic 3: Graphing Quantitative Relationships

- 1 Every Graph Tells a Story Independent and Dependent Variables
- 2 The Power of the Horizontal Line Using Graphs to Solve Problems
- 3 Planes, Trains, and Paychecks Multiple Representations of Equations
- 4 Time for Triathlon Training Relating Distance, Rate, and Time
- 5 There Are Many Paths ... Problem Solving on the Coordinate Plane

Topic 4: Financial Literacy: Accounts, Credit, and Careers

- 1 Knowledge You Can Bank On Checking Accounts
- 2 You Are a Real Card! Debit Cards vs. Credit Cards
- 3 Financial Report Card Understanding Credit Reports
- 4 The Possibilities Are Endless Career Exploration
- 5 Student Aid 101 Paying for College

Module 5: Describing Variability of Quantities

Topic 1: The Statistical Process

- 1 What's Your Question? Understanding the Statistical Process
- 2 Get in Shape Analyzing Numerical Data Displays
- 3 Follow Me on Histogram Using Histograms to Display Data

Topic 2: Numerical Summaries of Data

- 1 In the Middle Analyzing Data Using Measures of Center
- 2 Box It Up Displaying the Five-Number Summary
- 3 Dealing with Data Collecting, Displaying, and Analyzing Data

End of Course Topic

Formative Assessment

- 1 Living Room Rug Performance Task
- 2 Pizza Party Performance Task
- 3 School Ski Trip Performance Task

Glossary

Instructional Design

In a word, every single piece of Carnegie Learning's Texas Math Solution is **intentional.** Our instructional designers work alongside our master math practitioners, cognitive scientists, and researchers to intentionally design, draft, debate, test, and revise every piece, incorporating the latest in learning science.

Intentional Mathematics Design

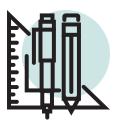
Carnegie Learning's Texas Math Solution is thoroughly and thoughtfully designed to ensure students build the foundation they'll need to experience ongoing growth in mathematics.

Mathematical Coherence: The arc of mathematics develops coherently, building understanding by linking together within and across grades, so students can learn concepts more deeply and apply what they've learned to more complex problems going forward.

Mathematical Process Standards: Carnegie Learning is organized around the Mathematical Process Standards to encourage experimentation, creativity, and false starts, which is critical if we expect students to tackle difficult problems in the real world, and persevere when they struggle.

Multiple Representations: Carnegie Learning recognizes the importance of connecting multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

Transfer: Carnegie Learning focuses on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.



Texas Math Solution Overview

The instructional materials in the Carnegie Learning Texas Math Solution emphasize active learning and making sense of the mathematics. We ask deep questions that require students to thoroughly understand the mathematical concepts they are learning. We think about how to guide students to connect interrelated ideas in a holistic way to integrate students' understanding with their developing habits of mind.

What are the Carnegie Learning Texas Math Solution guiding principles?

The Texas Math Solution has been strongly influenced by scientific research into the learning process and student motivations for academic success. Its guiding principles are active learning, discourse through collaboration, and personalized learning.

Active Learning: The research makes it clear that students need to actively engage with content in order to benefit from it. Studies show that as instruction moves up the scale from entirely passive to fully interactive, learning becomes more robust. All of the activities we provide for the classroom encourage students to be thoughtful about their work, to consider hypotheses and conclusions from different perspectives, and to build a deep understanding of mathematics. The format of the student text, as a consumable workbook, supports active instruction.

Discourse through Collaborative Learning: Effective collaboration encourages students to articulate their thinking, resulting in self-explanation. Reviewing other students' approaches and receiving feedback on their own provides further metacognitive feedback. Collaborative problem-solving encourages an interactive instructional model, and we have looked to research to provide practical guidance for making collaboration work. The collaborative activities within our lessons are designed to promote active dialogue centered on structured activities.

Personalized Learning: One of the ways to build intrinsic motivation is to relate activities to students' existing interests. Research has proven that problems that capture student interests are more likely to be taken seriously. In the textbook, problems often begin with the students' intuitive understanding of the world and build to an abstract concept, rather than the other way around.

How is the mathematical content delivered to promote productive mathematical processes?

Students deserve math learning that develops them into creative problem solvers, critical thinkers, life-long learners, and more capable adults, while teachers deserve instructional resources that will support them in bringing learning to life. There are three organizing principles that guide these instructional resources.

Seeing Connections: Activities make use of models—e.g., real-world situations, graphs, diagrams, and worked examples—to help students see and make connections between different topics. In each lesson, learning is linked to prior knowledge and experiences so that students build their new understanding on the firm foundation of what they already know. We help students move from concrete representations and an intuitive understanding of the world to more abstract representations and procedures. Activities thus focus on real-world situations to demonstrate the usefulness of mathematics.

Exploring Structure: Questions are phrased in a way that promotes analysis, develops higher-order-thinking skills, and encourages the seeking of mathematical relationships. Students inspect a given figure, equation, or data set, and in each case, they are asked to discern a pattern or structure. We want students to become fluent in seeing how the structure of each representation—verbal, graphic, numerical, and algebraic—reveals properties of the relationship it defines. We want students to become fluent at composing and decomposing expressions, equations, and data sets. As students gain proficiency in manipulating structure, they become capable of comparing, contrasting, composing, decomposing, transforming, solving, representing, clarifying, and defining the characteristics of figures, equations leading to functions, and data sets.

Reflecting and Communicating: A student-centered approach focuses on students thinking about and discussing mathematics as active participants in their own learning. Through articulating their thinking in conversations with a partner, in a group, or as a class, students integrate each piece of new knowledge into their existing cognitive structure. They use new insights to build new connections. Through collaborative activities and the examination of peer work—both within their groups and from examples provided in the lessons—students give and receive feedback, which leads to verifying, clarifying, and/or improving the strategy.

Texas Math Solution Year at a Glance

This Year at a Glance highlights the sequence of topics and the number of blended instructional days (1 day is a 45-minute instructional session) allocated for Grade 6 in the Texas Math Solution. The pacing information also includes time for assessments, providing you with an instructional map that covers 180 days of the school year. As you set out at the beginning of the year, we encourage you to still modify this plan as necessary.

Want More Support Designing Your Long-Term Plan?

You can find this Year at a Glance and additional guidance on planning intentionally and flexibly on the Texas Support Center at **www.CarnegieLearning.com/texas-help**.



Texas Grade 6: Year at a Glance

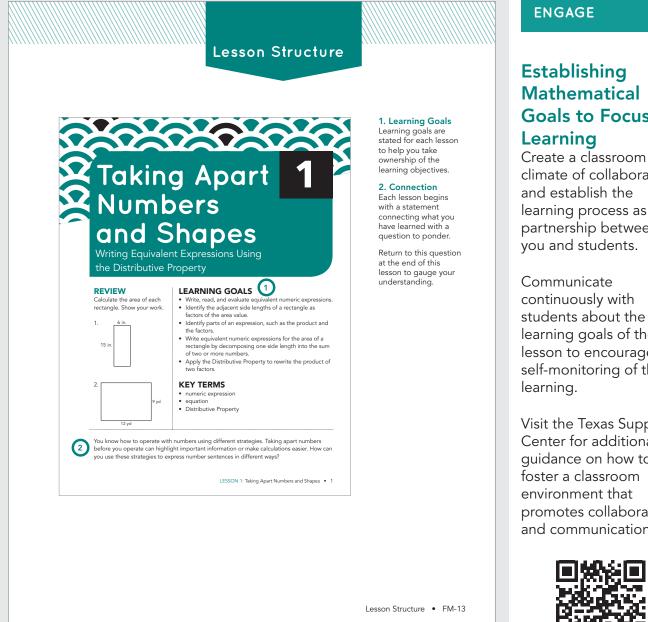
*1 Day Pacing = 45-minute Session

Module	Торіс	Pacing	TEKS				
Process Standards are embedded in every module: 6.1A, 6.1B, 6.1C, 6.1D, 6.1E, 6.1F, 6.1G							
1 Composing &	1: Factors & Multiples	6	6.7A, 6.7D				
	2: Positive Rational Numbers	10	6.2D, 6.2E, 6.3A, 6.3B, 6.3E, 6.4F, 6.5C				
	3: Shapes and Solids	11	6.8A, 6.8B, 6.8C, 6.8D				
Decomposing	4: Decimals	9	5.3K, 6.2C, 6.2D, 6.3E, 6.8D				
		36					
	1: Ratios	21	6.4A, 6.4B, 6.4C, 6.4D, 6.4E, 6.5A, 6.6C				
2	2: Percents	10	6.2D, 6.4E, 6.4F, 6.4G, 6.5B, 6.5C				
Relating Quantities	3: Unit Rates & Conversions	10	6.4B, 6.4D, 6.4H, 6.5A				
		41					
3	1: Signed Numbers & Four Quadrants	13	6.2A, 6.2B, 6.2C, 6.2D, 6.11A				
Moving Beyond	2: Operating with Integers	12	6.3C, 6.3D				
Positive Quantities		25					
	1: Expressions	13	6.3D, 6.7A, 6.7B, 6.7C, 6.7D				
4	2: Equations & Inequalities	15	6.3D, 6.7D, 6.8C, 6.9A, 6.9B, 6.9C, 6.10A, 6.10B				
Determining	3: Graphing Quantitative Relationships	14	6.6A, 6.6B, 6.6C, 6.11A				
Unknown Quantities	4: Financial Literacy	8	6.14A, 6.14B, 6.14C, 6.14D, 6.14E, 6.14F, 6.14G, 6.14H				
		50					
5	1: The Statistical Process	9	6.12A, 6.12B, 6.12D, 6.13A, 6.13B				
Describing Variability	2: Numerical Summaries of Data	10	6.12A, 6.12B, 6.12C, 6.12D, 6.13A				
of Quantities		19					
End of Course	Performance Tasks	9	6.3E, 6.4B, 6.5A, 6.5B, 6.9A, 6.10A				
Formative Assessment		9					
	Total Days:	180					

Connecting Content and Practice

Lesson Structure

Each lesson of the Texas Math Solution has the same structure. This consistency allows both you and your students to track your progress through each lesson. Key features of each lesson are noted.



Goals to Focus

climate of collaboration and establish the learning process as a partnership between you and students.

continuously with students about the learning goals of the lesson to encourage self-monitoring of their

Visit the Texas Support Center for additional quidance on how to foster a classroom environment that promotes collaboration and communication.



Activating Student Thinking

Your students enter each class with varying degrees of experience and mathematical success. The focus of the Getting Started is to tap into prior knowledge and realworld experiences, to generate curiosity, and to plant seeds for deeper learning.

Pay particular attention to the strategies students use, for these strategies reveal underlying thought processes and present opportunities for connections as students proceed through the lesson.

Supporting Emergent Bilingual Students

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they complete the Getting Started activities in each lesson.



3. Getting Started Each lesson begins with a Getting Started. When working on the Getting Started, was what you know	3 Getting Started				
	Break It Down to Build It Up				
use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.	Callie is building a rectangular walkway up to her house. The width of the walkway is 5 feet and the length is 27 feet. She needs to calculate the area of the walkway to determine the amount of materials needed to build it. 1. Mark and label 2 different ways you could divide an area model to determine the area of the walkway. 2. Determine the areas of each of the subdivided parts of your models.				
	3. What is the total area of the walkway?				
	2 • TOPIC 1: Factors and Multiples				
L					
FM-14 • Lesson Structure					

Mathematics is the science of patterns. So, we encourage students throughout this course to notice, test, and interpret patterns in a variety of ways—to put their "mental tentacles" to work in every lesson, every activity. Our hope is that this book encourages you to do the same for your students, and create an environment in your math classroom where productive and persistent learners develop and thrive.

Josh Fisher, Instructional Designer

4	ACTIVITY 1.1 Connecting Area Models and the Distributive Property	4. Activities You are going to bu a deep understand of mathematics thre a variety of activitie in an environment
	The numeric expression of 5 × 27 represents the area of the walkway from the Getting Started. A numeric expression is a mathematical phrase that contains numbers and operations.	where collaboration and conversations a important and expe
	equal to the expression 155.	t tare er ways could but you will also le
	show that two or more quantities are the same as one another. 1. Reflect on the different ways you can rewrite the product and	one of why those strategic factors work and how they write a are connected to
	of 5 and 27. Select one of your area models to complete the example.	ation? other strategies yo ation already know. equation
	length of 27? 1000 split the into	Ilkeifyou Remember: cone of elifyou Remember: infactors elifyou factors elifyou answer-getting two The answer-getting
	What are the factors of each = (5 ·) + (5 ·) regis	
	What is the area of each =+	of learning, so take risks.
	What is the total area? =	There is often than one way to solve a problem
	LESSON 1: Taking Apart Numbers and	d Shapes • 3 d Shapes • 3
		Be prepared to share your solution and methods with your classmates.
		Lesson Structure • F

DEVELOP

Aligning Teaching to Learning

Students learn when they are actively engaged in a task: reasoning about the math, writing their solutions, justifying their strategies, and sharing their knowledge with peers.

Support productive struggle by allowing students time to engage with and persevere through the mathematics.

Support student-tostudent discourse as well as whole-class conversations that elicit and use evidence of student thinking.

Supporting Emergent Bilingual Students

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they engage in mathematical discourse throughout each lesson.



DEMONSTRATE

Ongoing Formative Assessment Drives Instruction

For students to take responsibility for their own learning, they need to be encouraged to self-assess. Students can use the Talk the Talk to monitor their own progress towards mastering the learning goals. Listen and review their answers and explanations and provide feedback to help them improve their understanding.

As you plan the next lesson, consider the connections you can make to build off the strengths or fill any gaps identified from this formative assessment.

you an opportunity to reflect on the main ideas of the lesson.	NOTES
 Be honest with yourself. 	5 TALK the TALK The Floor Is Yours
 Ask questions to clarify anything you 	You can apply the Distributive Property to solve real-world problems.
don't understand.Show what	Consider the situation.
you know! Don't forget to revisit the question posed	Tyler is setting up the gym floor for an after-school program. He wants to include a rectangular area for playing volleyball and another for dodgeball. He also wants to have an area for kids who like to play board games or just sit and read. The gym floor is already 50 feet by 84 feet or 4200 square feet.
on the lesson opening page to gauge your understanding.	 Create a diagram to show how you would split up the gym floor. Represent your diagram using the Distributive Property and write an explanation for the areas assigned to each activity.
	6 • TOPIC 1: Factors and Multiples

Student Lesson Overview Videos

Each lesson has a corresponding lesson overview video(s) for students to utilize and reference to support their learning. The videos provide an overview of key concepts, strategies, and/or worked examples from the lessons.

Assignment

An intentionally designed assignment follows each student lesson.

There is one Assignment per lesson. Lessons often span multiple days. Be thoughtful about which portion of the Assignment students can complete based on that day's progress.

The **Stretch** section is not necessarily appropriate for all learners. Assign this to students who are ready for more advanced concepts.

The **Review** section provides spaced practice of concepts from the previous lesson and topic and of the fluency skills important for the course.

Assignment 6. Write Reflect on your work Assignment LESSON 1: Taking Apart Numbers and Shape and clarify your thinking. Remember 7 6 Write 7. Remember Take note of the key concepts from the lesson. Practice 8. Practice 8 题 Use the concepts learned in the lesson to solve problems. 9. Stretch Ready for a challenge? 10. Review 4.6(12+4) 5. 10 + 4(2 + 20) 6.7(4 + 19) Remember what you've learned by practicing Stretch concepts from previous lessons and topics. Review 10 1. Width = 5 feet Length = $\frac{2}{3}$ foot 2. Width = 10 feet Length = $\frac{2}{3}$ foot 3. Width = 15 inche Length = $\frac{2}{3}$ inch 4. Width = 20 inche Length = $\frac{2}{3}$ inch 2 • TOPIC 1: Factors and Multiples

Assignment • FM-17

Topic Summary

A Topic Summary is provided for students at the end of each topic. The Topic Summary lists all key terms of the topic and provides a summary of each lesson. Each lesson summary defines key terms and reviews key concepts, strategies, and/or worked examples.

Facto Sumi	ors and mary	d Mul	ltiple	S	
KEY TERMS					
 numeric express equation Distributive Prop base power 	• commo	n factor y prime : common	 multiple Commutative least common multiple (LC) 	on	
LESSON T	aking Apart N	umbers and	d Shapes		
	sion is a mathematical p ematical sentence that e another.				
The equation 5 $ imes$ 2	27 = 135 shows that the	e expression 5 $ imes$ 2	?7 is equal to the e	expression 135.	
here are many way roperties oplied for nd c, a(b -	ys to rewrite equivalent	expressions using			
You can rea For example four times four times		e area of rectang		learning about factors.	
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For example, you can use prime factorization to determine common factors of 56 and 42. Start by writing each number as a product of its prime factors.

.

 $56 = 2 \cdot 2 \cdot 2 \cdot 7$

 $42 = 2 \cdot 3 \cdot 7$

Organize the prime factors into a table, where only shared factors are listed in the same column.

Number	Prime Factors						
56	2	2	2		7		
42	2			3	7		

The common factors of the two numbers are the numbers that are in both rows and the product of the numbers that are in both rows.

The common factors of 56 and 42 are 2, 7, and 14.

The greatest common factor (GCF) is the largest factor two or more numbers have in common. In the example of 56 and 42, the greatest common factor is 14. Two numbers that do not have any common factors other than 1 are relatively prime.

The **Commutative Property**, when applied for multiplication, states that for any numbers a and b, the product $a \cdot b$ is equal to the product $b \cdot a$.

You can use rectangles to determine multiples and common multiples. A **multiple** is the product of a given whole number and another whole number. One way to think about the area model is to analyze the collection of rows and columns in a rectangle.

In the example shown, the addition of each new row creates a multiple of 8. The addition of each new column creates a multiple of 6.

LESSON

While 48 is least comm (other than : You can use the LCM. Yc factors that the product $2 \cdot 2 \cdot 2 \cdot 3$ LCM(56, 42) You can cor

Composing and Decomposing Numbers

You can solve real-world problems that involve common factors or common multiples by thinking about the question you are trying to answer. Remember that common factors help you think about how to divide, or share things equally, and common multiples help you think about how things with different cycles can occur at the same time.

For example, a local bus arrives at the stop near Aaron's house every 15 minutes. An express bus arrives at the same stop every 9 minutes. Aaron sees both a local and an express bus arrive at the stop at 10 a.m. What is the next time that he would expect to see both buses arrive at the stop?

The problem is asking about when the two different cycles of the buses will occur again at the same time, so you can use the least common multiple of 15 and 9 to answer the question.

The multiples of 15 are 15, 30, 45, 60, 75, . . . The multiples of 9 are 9, 18, 27, 36, 45, 54, . .

The least common multiple of 15 and 9 is 45, therefore the two buses should arrive at the stop at the same time every 45 minutes. The next time Aaron would expect to see both buses at the stop is 10.45 A.M.

In another example, Ramona is filling window box planters that will be sold to benefit a local charity. She has 56 pansies, 42 tulips, and 28 marigolds. What is the greatest number of planters she can fill if she wants to use all of the flowers and have the same number of each type of flower in each planter? How many of each flower type will be in a planter?

The problem is asking you to share each type of flower among an equal number of groups, so you can use the greatest common factor of 56, 42, and 28 to answer the question.

You can use prime factorization to determine the prime factors for each type of flower.

Pansies: $56 = 2 \times 2 \times 2 \times 7$

Tulips: $42 = 2 \times 3 \times 7$

Marigolds: $28 = 2 \times 2 \times 7$

The greatest common factor of 56, 42, and 28 is 14. Therefore, Ramona can fill 14 planters. Each planter will contain 4 pansies, 3 tulips, and 2 marigolds.

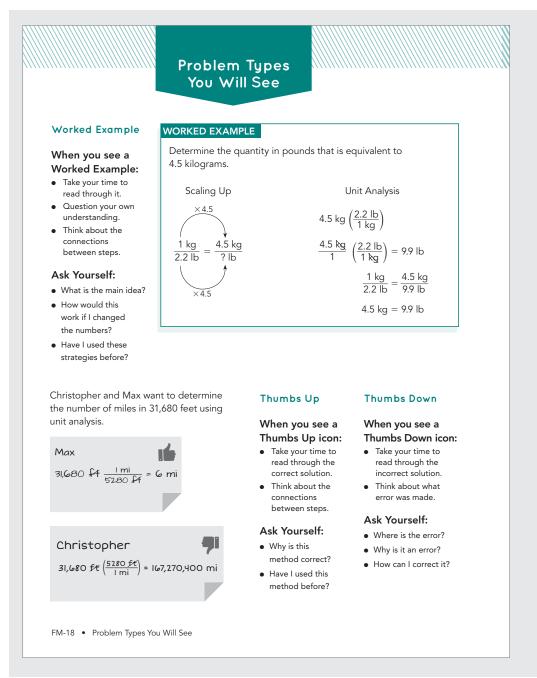
4 • TOPIC 1: Factors and Multiples

Problem Types You Will See

Lessons include a variety of problem types to engage students in reasoning about the math.

Worked Examples

Worked Examples help students develop their skills as they question their understanding, make connections with the steps, and ultimately explain the progression of the steps towards the final outcome. They represent and mimic an internal dialogue about the mathematics and the strategies, and the questions that follow them are designed to serve as a model for self-questioning and self-explanationswhile making sure that students don't skip over a Worked Example without interacting with it, thinking about it, and responding to its accompanying questions. This approach aids students as they develop their desired habits of mind for being conscientious about the importance of steps and their order.



Thumbs Up / Thumbs Down

Thumbs Up problems give students the opportunity to analyze viable methods and problem-solving strategies. Questions are presented to help students consider the various strategies in depth and to focus on an analysis of correct responses. Because research shows that providing only positive examples is less effective for eliminating common student misconceptions than also showing negative examples, incorrect responses are provided alongside the correct responses. From the incorrect responses, students learn to determine where the error in calculation is, why the method is wrong or is being used wrong, and also how to correct the method to calculate the solution properly.

Tim and Dan love cereal, but don't want to spend a lot of money. After scanning the aisle in the grocery store for the lowest prices, the boys make the following statements.

- Tim says, "I found Sweetie Oat Puffs for \$0.14 per ounce. That's the cheapest cereal in the aisle!"
- Dan replies, "It's not cheaper than Sugar Hoops! The unit price for that is 6.25 oz per dollar."

Who is correct? Explain your reasoning.



Who's Correct?

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or not correct.

Ask Yourself:

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

Who's Correct?

Who's Correct? problems are an advanced form of correct vs. incorrect responses. In this problem type, students are not told who is correct. Students have to think more deeply about what the strategies really mean and whether each of the solutions makes sense. Students will determine what is correct and what is incorrect, and then explain their reasoning. These types of problems will help students analyze their own work for errors and correctness.

Problem Types You Will See • FM-19

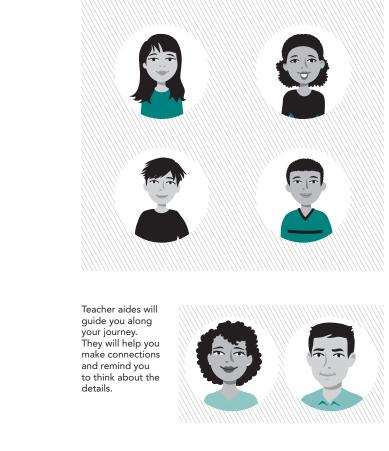
Promoting Self-Reflection

The Crew

Characters are embedded throughout the Texas Math Solution to remind students to stop and think in order to promote productive reflection. They are used in a variety of ways: to remind students to recall a previous mathematical concept, help students develop expertise to think through problems, and occasionally, present a fun fact.



The Crew is here to help you on your journey. Sometimes they will remind you about things you already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are members of your group someone you can rely on!



FM-20 • The Crew

Mathematical Process Standards

Mathematical Process Standards



Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The "I can" expectations listed below align with the TEKS Mathematical Process Standards and encourage students to develop their mathematical learning and understanding.

► Apply mathematics to problems arising in everyday life, society, and the workplace.

l can:

- use the mathematics that I learn to solve real world problems.
- interpret mathematical results in the contexts of a variety of problem situations.
- ► Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

l can:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Mathematical Process Standards • FM-21

Note

Each lesson provides opportunities for students to think, reason, and communicate their mathematical understanding. However, it is your responsibility as a teacher to recognize these opportunities and incorporate these practices into your daily rituals. Expertise is a long-term goal, and students must be encouraged to apply these practices to new content throughout their school career.

Supporting Students to Use Mathematical Tools

Visit the Texas Support Center for strategies to support students as they use mathematical tools, including formula charts and reference sheets.



Note

When you are facilitating each lesson, listen carefully and value diversity of thought, redirect students' questions with guiding questions, provide additional support with those struggling with a task, and hold students accountable for an end product. When students share their work, make your expectations clear, require that students defend and talk about their solutions. and monitor student progress by checking for understanding.

Consider having students create "I can" statements to promote their self-reflection.

There is one more page of mathematical process standards that is not provided here, but is available in the Student Textbook Front Matter.





Supporting ALL Learners

Visit the Texas Support Center for facilitation strategies to support ALL students as they engage in the Mathematical Process Standards.

Academic Glossary

Academic Glossary

Visit the Students & Caregivers Portal on the Texas Support Center at www. CarnegieLearning. com/texas-help to access the Mathematics Glossary for this course anytime, anywhere.

Related Phrases

• Examine

- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

There are important terms you will encounter throughout this book. It is important that you have an understanding of these words as you get started on your journey through the mathematical concepts. Knowing what is meant by these terms and using these terms will help you think, reason, and communicate your ideas.

ANALYZE

Definition

To study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

To give details or describe how to determine an answer or solution. Explaining

Ask Yourself

Ask Yourself

Is my explanation logical?

- Do I see any patterns?
- Have I seen something like this before?

your reasoning helps justify conclusions.

• How should I organize my thoughts?

• How can I justify my answer to others?

• Does my reasoning make sense?

• What happens if the shape, representation, or numbers change?

Related Phrases

- Show your work
- Explain your calculation

FM-24 • Academic Glossary

- Justify
- Why or why not?

EXPLAIN YOUR REASONING

Language Expectations It is critical for

It is critical for students to possess an understanding of the language of their text. Students must learn to read for different purposes and write about what they are learning. Encourage students to become familiar with the key words and the questions they can ask themselves when they encounter these words.

lt is our

recommendation to be explicit about your expectations of language use and the way students write responses throughout the text. Encourage students to answer questions with complete sentences. Complete sentences help students reflect on how they arrived at a solution, make connections between topics, and consider what a solution means both mathematically as well as in context.

Supporting Students at Varying Levels of Language Proficiency

Visit the Texas Support Center for guidance on how to leverage the Academic Glossary to support students at varying levels of language proficiency.



Ask Yourself

The Ask Yourself questions help students develop the proficiency to explain to themselves the meaning of problems.

Real-World Context

Real-world contexts confirm concrete examples of mathematics. The scenarios in the lessons help students recognize and understand that quantitative relationships seen in the real world are no different than quantitative relationships in mathematics. Some problems begin with a real-world context to remind students that the quantitative relationships they already use can be formalized mathematically. Other problems will use real-world situations as an application of mathematical concepts.

REPRESENT

Definition

To display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

ESTIMATE

Definition

To make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

DESCRIBE

Definition

To represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

Related Phrases

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equationComplete the table
 - Complete the table

Related Phrases

- Predict
- Approximate
- Expect
- About how much?

Related Phrases

Demonstrate

- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the
- disadvantages?What is similar?
- What is different?
- Academic Glossary FM-25

Mathematics Glossary

A course-specific mathematics glossary is available for students to utilize and reference during their learning. Definitions and examples of key terms are provided in the glossary.

Facilitating Student Learning

Teacher's Implementation Guide

The Teacher's Implementation Guide (TIG) is designed to fully support a wide-range of teachers implementing our materials: from first-year teachers to 30-year veterans and from first-time Carnegie Learning users to master practitioners.

One goal in developing the TIG was to make our instructional design apparent to the users.

The lessons of each topic were written to be accessible to the full range of learners. With every instructional decision you make, keep in mind your mathematical objectives for the topic and module and the course. Plan each lesson by thinking about how you will create access for your particular group of students, maintain access and pace throughout the lesson, and assess their understanding along the way. We recommend that you do the math in each topic before implementing the activities with your specific group of students.

What makes this TIG useful?

Effective Lesson Design: Each lesson has a consistent structure for teachers and students to follow. The learning experiences are engaging and effective for students.

Pacing: Each course is designed to be taught in a 180-day school year. Pacing suggestions are provided for each lesson. Each day in the pacing guide is equivalent to about a 45-minute instructional session.

Instructional Supports: Guiding questions are provided for teachers to use as they're circulating the room, as well as differentiation strategies, common student misconceptions, and student look-fors.

Clearly Defined Mathematics: The content and instructional goals are clearly described at the module, topic, lesson, and activity levels.

The TIG is critical to understanding how the mathematics that students encounter should be realized in the classroom. The TIG describes the depth of understanding that students need to develop for each standard and a pathway for all learners to be successful. It provides differentiation strategies to support students who struggle, to extend certain activities for students who are advanced in their understanding of the content, and to support emergent bilingual students. Visit the Texas Support Center at **www. CarnegieLearning. com/texas-help** for additional resources to support you anytime, anywhere.



Module and Topic Overviews

You are responsible for teaching the essential concepts associated with a particular course. You need to understand how activities within lessons build to achieve understanding within topics, and how topics build to achieve understanding throughout the module and course. In the Texas Math Solution, Carnegie Learning seeks to establish a shared curriculum vision with you.

Module Overview

Each module begins with an overview that describes the reasoning behind the name, the mathematics being developed, the connections to prior learning, and the connections to future learning.

Topic Overview

A Topic Overview describes how the topic is organized, the entry point for students, how a student will demonstrate understanding, why the mathematics is important, how the activities promote expertise in the mathematical process standards, materials needed for the topic, examples of visual representations or strategies used, and more detailed information to help with pacing.

Module 1 Overview Composing and Decomposing

"Understanding of and proficiency with measurement should flourish in the middle grades, especially in conjunction with other parts of the mathematics curriculum."—Navigating through Measurement, page 4

Why is this Module named Composing and Decomposing?

Throughout Grade 6, students reason, look for structure, and identify similarities across mathematica

operations, proport equations and relati and data, and persc **Composing and De** work by deepening of numbers and sha relationships. Stude numbers and shape more complicated c large numbers and smaller numbers and smaller numbers and perform calculations more flexible with h numbers, they will b structure, which in t

Students learn to ap decomposing (takin (putting together) o already understood.

to develop strategie

across mathematica

Factors and Multiples Topic 1 Overview

have developed some number sense; they

have broken down numbers into sums, differences, products, and quotients.

Now, students discover that numbers

are composed of numerical expressions

How are the key concepts of Factors and Multiples developed?

In Factors and Multiples, students extend their knowledge of area and numbers to compose and decompose areas that represent numeric expressions. Students decompose numbers into factors and apply the Distributive Property to compute products efficiently. They use the Distributive Property to express sums of two numbers as a product of two factors. They use the Commutative Property to express equivalent expressions. Students use factor trees to determine all of the prime factors for a given number, and they use tables to organize prime factors for two or more numbers. Students then use their knowledge of factors to determine the greatest common factors and least common multiples. Students use whole number exponents and prime factorization to

What is the entry point for students?

Students enter grade 6 with experience using area models, both tiling areas with unit squares and representing multiplication. The Factors and Multiples topic draws on this to formalize the Distributive Property and to decompose numeric expressions. Students' prior work with

generate equivalent numerical expressions.

factor pairs supports their new learning about least common multiples and greatest common factors.

CĿ

How does a student demonstrate understanding?

CĿ

Students will demonstrate an understanding of the standards in *Factors and Multiples* when they can:

- Apply properties of operations to compose and decompose numbers and shapes to understand the relationship between factors and multiples.
- Create equivalent expressions using the Commutative and Distributive Properties.
- Identify the factors of two whole numbers and determine the greatest common factor.
- Identify the multiples of two whole numbers and determine the least common multiple.
- Generate equivalent numerical expressions using whole number exponents and prime factorization.

Why is Factors and Multiples important?

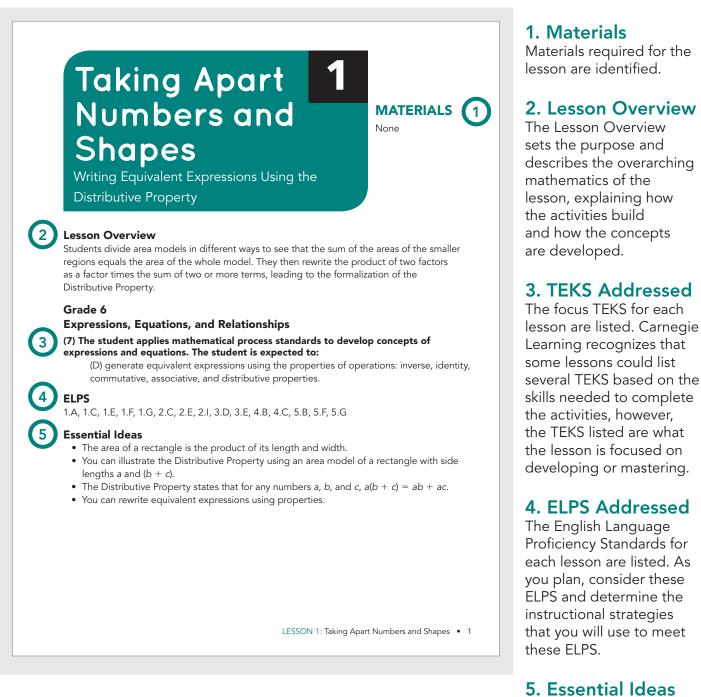
Factors and Multiples focuses on composing and decomposing numbers and expressions. Students will apply the same properties and terminology to algebraic expressions in **Determining Unknown Quantities**, where they will use the properties of operations to

TOPIC 1: Factors and Multiples • 1

"Teachers must first develop their ideas about where the curriculum program is going mathematically (curriculum vision) before deciding whether the curriculum materials will help them reach that mathematical goal (curriculum trust)" (Drake & Sherin, 2009, p. 325).

Facilitation Notes

For each lesson, you are provided with detailed facilitation notes to fully support your planning process. This valuable resource provides point-of-use support that serves as your primary resource for planning, guiding, and facilitating student learning.



These statements are derived from the standards and state the concepts students will develop.

6. Lesson **Structure**

This section highlights how the parts of the lesson fit within the instructional design: Engage, Develop, and Demonstrate. A summary of each activity is included.

7. Pacing

Lessons often span more than one 45-minute class period. Suggested pacing is provided for each lesson so that the entire course can be completed in a 180-day school year.



Engage

Getting Started: Break It Down to Build It Up

Students divide area models for the product 5 imes 27 in two different ways. They calculate the areas of the subdivided parts before determining the area of the whole model. Develop

Activity 1.1: Connecting Area Models and the Distributive Property

Students rewrite the product of two factors as a factor times the sum of two or more terms, leading to the formalization of the Distributive Property. They decompose factors and products into equivalent representations.

Demonstrate

Talk the Talk: The Floor Is Yours

Students design the floor plan in a gymnasium for different after-school activities. They represent their model using the Distributive Property and then explain their rationale.

2 • TOPIC 1: Factors and Multiples

Getting Started: Break It Down to Build It Up

ENGAGE

Facilitation Notes

In this activity, students divide area models for the product 5 \times 27 in two different ways. They calculate the areas of the subdivided parts before determining the area of the whole model.

Ask a student to read the situation aloud. Have students complete Question 1 individually. Share responses as a class.

As students work, look for

- Whether students use a vertical, horizontal, or slanted line to divide the area model.
- Splitting 27 into numbers that make the computation of area easier.
- Correct dimensions for each of the smaller regions in the area model.

Questions to ask

- What is an area model?
- Did you split the length to obtain specific values that add up to 27? If so, explain your thinking.

Misconceptions

Students may decide to make a diagonal line to split the area. While correct, discuss that their decision makes two trapezoids, or two triangles, instead of rectangles, and it is much more efficient to use rectangles. Also, rectangles are required to model the Distributive Property.

Have students complete Questions 2 and 3 individually. Share responses as a class.

Questions to ask

- What was the same about each of your area calculations? Why is that the case?
- Why does everyone get the same total area even though they split the walkway differently?

Summary

You can divide an area model into smaller regions. The sum of the areas of each region is the total area of the model.

LESSON 1: Taking Apart Numbers and Shapes • 3

Position yourself to take full advantage of the richness of the mathematics addressed in the textbook. The Facilitation Notes provide guidance to reach each student from their current level of understanding to advance to the next stage. Place yourself in the position of the student by experiencing the textbook activities prior to class. Realize your role in the classroom—empower your students! Step back and let them do the math with confidence in their role as learner and your role as facilitator of learning.

Janet Sinopoli, Instructional Designer

8. Facilitation Notes by Activity

A detailed set of guidelines walks the teacher through implementing the Getting Started, Activities, and Talk the Talk portions of the lesson. These quidelines include an activity overview, grouping strategies, guiding questions, possible student misconceptions, differentiation strategies, student lookfors, and an activity summary.

9. Activity Overview

Each set of Facilitation Notes begins with an overview that highlights how students will actively engage with the task to achieve the learning goals.

10. Differentiation Strategies

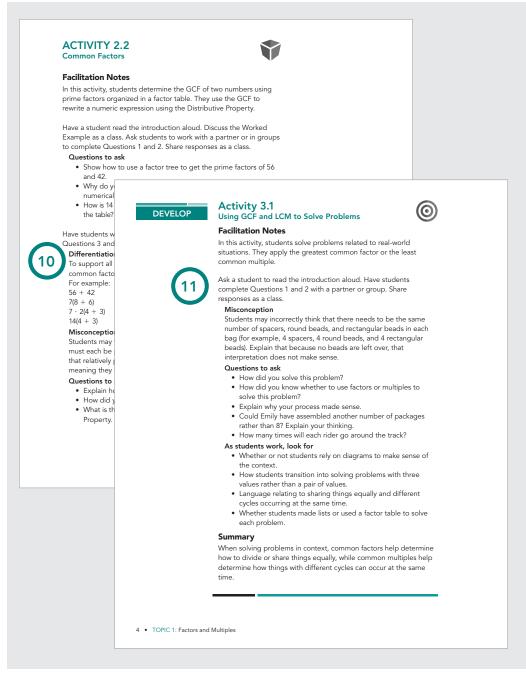
To assist all students, instructional strategies are provided that benefit the full range of learners.

11. Grouping Strategies

Suggestions appear to help chunk each activity into manageable pieces and establish the cadence of the lesson.

Learning is social. Whether students work in pairs or in groups, the critical element is that they are engaged in discussion. Carnegie Learning believes, and research supports, that studentto-student discourse is a motivating factor; it increases student learning and supports ongoing formative assessment. Additionally, it provides students with opportunities to have mathematical authority.

Working collaboratively can, when done well, encourage students to articulate their thinking (resulting in self-explanation) and also provides metacognitive feedback (by reviewing other students' approaches and receiving feedback on their own).

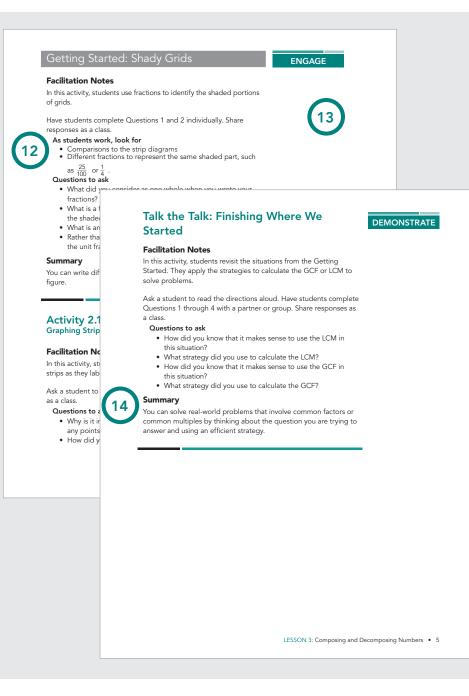


The student discussion is then transported to a classroom discussion facilitated by the teacher to guarantee all necessary mathematics is addressed, once again, with the same benefits of discussion.

Alternative Grouping Strategies

Differentiation strategies that provide other grouping strategies, such as whole class participation and the jigsaw method, are sometimes recommended for specific activities. These are listed as Differentiation Strategies.

More information about grouping strategies is available online in the Texas Support Center at **www.CarnegieLearning.com/texas-help**



12. As Students Work, Look For

These notes provide specific language, strategies, and/or errors to look and listen for as you circulate and monitor students working in pairs or groups. You can incorporate these ideas when students share their responses with the class.

Note

Talk the Talk helps you to assess student learning and to make decisions about helpful connections you need to make in future lessons.

13. White Space

The white space in each margin is intentional. Use this space to make additional planning notes or to reflect on the implementation of the lesson.

14. Summary

The summary brings the activity to closure. This statement encapsulates the big mathematical ideas of the particular activity.

Supporting Emergent Bilingual Students

Emergent bilingual students often face multiple challenges in the mathematics classroom beyond language development skills, including a lack of confidence, peer-to-peer understanding, and building solid conceptual mastery. The Carnegie Learning Texas Math Solution seeks to support emergent bilingual students as they develop skills in both mathematics and language.

Answers 1. See the model (\mathbf{Y}) Common Multiples 2.3 You can use rectangular arrays to determine multiples and common multiples Consider the area model for $6 \cdot 8 = 48$. 12 18 24 30 36 42 48 A multiple is the product of a given 2. See the model whole numbe and another whole number. 16 One way to think about the area model is to analyze the collection 24 of columns. The addition of each new column creates a multiple of 6. 32 - The first column is a 6×1 rectangle representing the first 40 multiple of 6, or 6. The first and second columns together are a 6 × 2 rectangle representing the second multiple of 6, or 12. The whole rectangle represents 6 × 8, or 48. 1. List the first eight multiples of 6 by labeling each column of The Commutative the area model Property, when applied for multiplication, states that for any numbers Next, think about the area model as a collection of 6 rows. The first a and b, the product row alone creates an 8 \times 1 rectangle, which represents the first multiple of 8, or 8. Including all rows, the 8 \times 6 rectangle represents $a \cdot b$ is equal to the product b · a. the sixth multiple of 8, or 48 2. List the first six multiples of 8 by labeling each row of the area model **ELL Tip** Discuss how the meaning of the 8 • TOPIC 1: Factors and Multiples everyday term commute relates to the Commutative Property. Commute means to travel; according to the Commutative **ELL Tip** Property, terms can Connect the terms multiple and multiply. The multiples of a travel or move to a number are created by taking a number and multiplying it by different order. 1, 2, 3, etc. 16 • TOPIC 1: Factors and Multiples

For More Support

Visit the Texas Support Center for many more resources to support you and your students who are emergent bilingual students.

Throughout instruction, ELL tips are placed for teachers at point-of-use on the mini-lesson page in the TIG. They provide additional modifications to support this special population.

These tips:

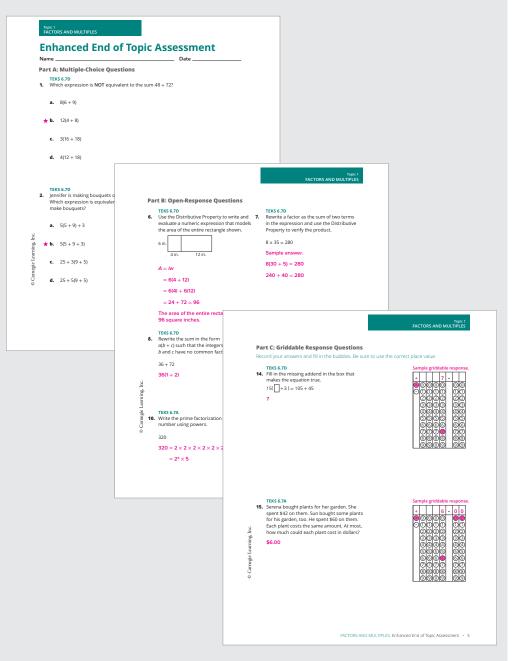
- Inform teachers of potential learning obstacles specific to the lesson.
- Provide engaging activities for learning and assessing.
- Reinforce newly acquired mathematical language to gain an increasing level of comprehension of English.
- Introduce students to language needed to understand a specific context.

Students internalize new content language by using and reusing it in meaningful ways in a variety of different speaking activities that build concept and language attainment.



Assessments

Formative assessment tools are provided throughout each lesson, providing you with ongoing feedback of student performance and encouraging students to monitor their own progress. End of topic summative assessments are provided to measure student performance on a clearly denoted set of standards. For certain topics that extend longer than four instructional weeks, a mid-topic summative assessment is also provided.



Enhanced End of Topic Assessment

There are three problem type sections per assessment. Multiple-choice questions, openresponse questions, and griddable response questions prepare students for enhanced standardized tests.

The answer key provides teachers with the TEKS aligned to each question, as well as sample answers for open-response and griddable response questions.



Supporting Students to Use Mathematical Tools

Visit the Texas Support Center for strategies to support students as they use mathematical tools, including formula charts and reference sheets.

End of Course Topic

The End of Course Topic is the final topic of the course which includes a collection of problem-based performance tasks that are aligned with selected priority math standards of the course. This final topic provides students an additional opportunity to demonstrate their ability to make sense of multi-step, real-world problems, communicate their thinking, represent solutions, and justify their reasoning on content aligned with these selected math standards.

Performance Tasks

Each performance task is a formative assessment tool that allows students to demonstrate their learning of the selected course content. At the end of each task, a section titled "Your Work Should Include" lists the categories and the corresponding maximum scoring points from the grading rubric.

Living Room R Charlotte is buying a re	ectangular rug to			
width : length ratio of 1 and the length is 20 ft.				living room floor is 15 ft
Charlotte eliminate she eliminate right		t are a square or	closest to a squ	uare. Which rug(s) did
Which rug(s) might	t Charlotte buy s	o that the ratios	of the rug and	the floor are the same?
Create a graph and	d a double numb	er line to suppo	rt your answer.	
 If area rugs are sol with the same wid 				
	Rug	Width (ft)	Length (ft)	
	1	4	6	
	2	5	7	
	4	8	10	
	5	9	12	
	6	10	10	
	7	10	14	
	8	11	17	
	y y	IZ	16	
Your work should incl A graph showing w A double number l An explanation usi Your calculations (3	vidth : length ration ine showing widt ng math terms of points)	th: length ratio (3 f how you detern		ers (3 points)
	ame ratio as the	floor (2 points)		

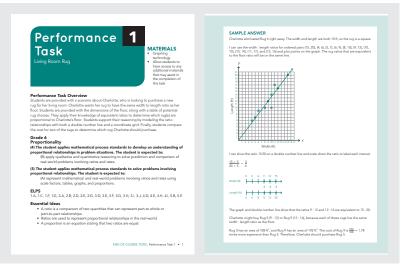
Grading Rubric

The grading rubric is for students and teachers to set clear expectations for how each completed performance task will be evaluated. Students should use the rubric to guide their work and selfmonitor their progress. Teachers should use the rubric to evaluate and provide feedback for the completed performance task.

	0 points	1 point	2 points	3 points
Graph	No graph is shown.	The graph is incomplete or incorrect.	The graph has minor errors.	The graph is complete and correct.
Double Number Line	number line is fruit ber line is number line has		number line has	The double number line is complete and correct.
Explanation	No explanation is given.	Explanation given uses no math terms.	Explanation corresponding to one of the answers is complete and includes math term(s).	Explanation corresponding to all questions is complete and includes math term(s).
Calculations	No calculations are shown.	Calculations are shown, but include significant errors.	Calculations are shown, but include minor errors.	Calculations are shown that are complete and correct.
Statement	Rug sizes are not stated or are incorrect.	Only one rug size is stated or correct.	Both correct rug sizes are stated.	N/A

Teacher's Implementation Guide

The Teacher's Implementation Guide for the End of Course Topic contains a performance task overview, list of aligned TEKS and ELPS, essential ideas, facilitation notes which describe how to pace the two-day performance task, sample answer, and grading rubric.



Similar to the other topics in this course, the End of Course Topic also has a Topic Family Guide for students and caregivers, and a Topic Overview for teachers. The End of Course Topic does not include an end of topic assessment since each performance task is a formative assessment.

Getting Ready

Carnegie Learning recognizes that it is the classroom teachers who make the material come alive for students, transforming the way math is taught. Implementation requires integrating learning together and learning individually.

Prepare for Learning Together

PREPARE YOURSELF

The most important first step you can take in preparing to teach with these instructional materials is to become comfortable with the mathematics.

- Read through the Module 1 Overview and the Topic 1 Overview.
- Do the math of the first Topic, and consider the facilitation notes.
- Prepare team-building activities to intentionally create a student-centered environment.

Prepare for Learning Individually

Plan how you will utilize Skills Practice as a Learning Individually resource. Then, determine how you will introduce Skills Practice to students. Explain to them the benefits of of working individually and why practice is important.

- Read through Module 1 Topic 1 Skills Practice.
- Determine which problem sets align with the activities in the corresponding student lessons.
- Based on student performance in the lesson, be prepared to assign the class, small groups of students, or individual students different problem sets to practice skills to develop mastery.

Plan how you will introduce students to MATHia. Explain to them the benefits of working individually and why practice is important.

- Test out the computers or tablets that your students will be using.
- Verify your classes have been set up in Teacher's Toolkit with correct MATHia content assigned. Or manually set up your classes in Teacher's Toolkit if applicable.
- Use the Content Browser in Teacher's Toolkit to explore the content students are assigned.
- Be prepared to demonstrate how students will access and log into MATHia.

Prepare the Environment

The classroom is often considered the third teacher. Consider how to create a learning environment that engages students and fosters a sense of ownership. The use of space in your classroom should be flexible and encourage open sharing of ideas.

- Consider how your students are going to use the consumable book. It is the student's record of their learning. Many teachers have students move an entire topic to a three-ring binder as opposed to carrying the entire book.
- Arrange your desks so students can talk and collaborate with each other.
- Prepare a toolkit for groups to use as they work together and share their reasoning (read the materials list in each Topic Overview).
- Consider where you will display student work, both complete and in-progress.
- Create a word wall of key terms used in the textbook.

Prepare the Learners

If you expect students to work well together, they need to understand what it means to collaborate and how it will benefit them. It is important to establish classroom guidelines and structure groups to create a community of learners.

- Facilitate team-building activities and encourage students to learn each others' names.
- Set clear expectations for how the class will interact:
 - Their text is a record of their learning and is to be used as a reference for any assignments or tests you give.
 - They will be doing the thinking, talking, and writing in your classroom.
 - They will be working and sharing their strategies and reasoning with their peers.
 - Mistakes and struggles are normal and necessary.

Prepare the Support

- Prepare a letter to send home on the first day. Visit the Texas Support Center for a sample letter.
- Encourage families and caregivers to read the introduction of the student book.
- Ensure that families and caregivers receive the module Family and Caregiver guide at the start of each module. They should also receive the topic Family Guide at the start of the first topic and each subsequent topic.
- Consider a Family Math Night some time within the first few weeks of the school year.
- Encourage families and caregivers to explore the Students & Caregivers Portal on the Texas Support Center at www.CarnegieLearning.com/texas-help/students-caregivers.

PREPARE YOUR CLASSROOM

PREPARE YOUR STUDENTS

PREPARE FAMILIES AND CAREGIVERS



Students and Caregivers Portal

Research has proven time and again that family engagement greatly improves a student's likelihood of success in school.

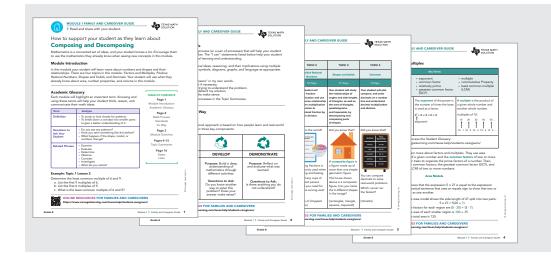
The Students & Caregivers Portal on the Texas Support Center provides:

- Getting to Know Carnegie Learning video content to provide an introduction to the instructional materials and research.
- Articles and quick tip videos offering strategies for how families and caregivers can support student learning.
 Visit the Texas Support Center regularly to access new content and resources for students and caregivers as they learn mathematics in a variety of environments outside of the classroom.

Module Family and Caregiver Guides

Each module has a Family and Caregiver Guide available through the Students & Caregivers Portal on the Texas Support Center. Each module guide will provide a different highlight of the academic glossary, description and examples of TEKS Mathematical Process Standards, and an overview of a different component of our instructional approach known as The Carnegie Learning Way. Also included is a module overview of content, specific key terms, visual representations, and strategies students are learning in each topic of the module.

The purpose of the Family and Caregiver Guide is to bridge student learning in the classroom to student learning at home. The goal is to empower families to understand the concepts and skills learned in the classroom so that families can review, discuss, and solidify the understanding of these key concepts together. Videos will also be available on the Students & Caregivers Portal to provide added support.

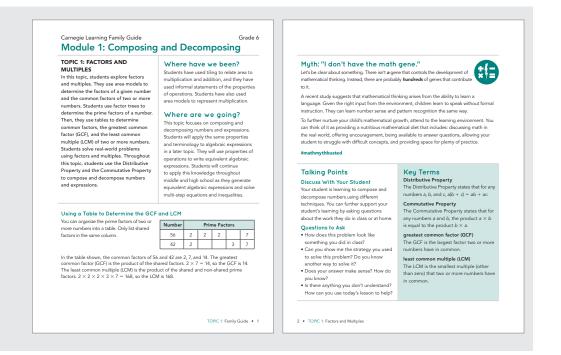




Topic Family Guides

Each topic contains a Family Guide that provides an overview of the mathematics of the topic, how that math is connected to what students already know, and how that knowledge will be used in future learning. It provides families and caregivers an example of a math model or strategy their student is learning in the topic, busting of a math myth, questions to ask their student to support their learning, and a few of the key terms their student will learn.

We recognize that learning outside of the classroom is crucial to students' success at school. While we don't expect families and caregivers to be math teachers, the Family Guides are designed to assist caregivers as they talk to their students about what they are learning. Our hope is that both the students and their caregivers will read and benefit from the guides.



We're here for you.

The Carnegie Learning Texas Support Team is available to help with any issue at help@ carnegielearning.com.

Monday–Friday 8:00 am–8:00 pm CT

via email, phone, or live chat.

Our expert team provides support for installations, networking, and technical issues, and can also help with general questions related to pedagogy, classroom management, content, and curricula.

You Might Be Wondering...

Why are the student books consumable?

The Student Textbook contains all of the resources students need to complete the Learning Together component of the course. Students are to actively engage in this textbook, topic by topic, creating a record of their learning as they go. There is room to record answers, take notes, draw diagrams, and fix mistakes.

Why do we believe in our brand of blended: Learning Together and Learning Individually?

There has been a lot of research on the benefits of learning collaboratively. Independent practice is necessary for students to become fluent and automatic in a skill. A balance of these two pieces provides students with the opportunity to develop a deep conceptual understanding through collaboration with their peers, while demonstrating their understanding independently.

Why don't we have a Worked Example at the start of every lesson?

Throughout the Texas Math Solution, we do provide Worked Examples. Sweller and Cooper (1985) argue that Worked Examples are educationally efficient because they reduce working memory load. Ward and Sweller (1990) found that alternating between problem solving and viewing Worked Examples led to the best learning. Students often read Worked Examples with the intent to confirm that they understand the individual steps. However, the educational value of the Worked Example often lies in thinking about how the steps connect to each other and how particular steps might be added, omitted, or depending on context.

Where are the colorful graphics to get students' attention?

Color and visuals make for stronger student engagement, right? Not quite. Our instructional materials have little extraneous material. This approach follows from research showing that "seductive details" used to spice up the presentation of material often have a negative effect on student learning (Mayer et al., 2001; Harp & Meyer, 1998). Students may not know which elements of an instructional presentation are essential and which are intended simply to provide visual interest. So, we focus on the essential materials. While we strive to make our educational materials attractive and engaging to students, research shows that only engagement based on the mathematical content leads to learning.

Why so many words?

For students to deeply learn the math, they need to work through it. They also need to develop their work and demonstrate that they really understand it. Math isn't just about solving equations or formulas—it's about thinking, working through ideas, and seeing how the math relates to the real world.

Notes:

If you have questions, reach out to us for support. Our team of master practitioners have been where you are. We made mistakes and we learned from them. We want to help you. We have many professional development options. Whether we come to your school for a workshop, join you in your classroom for modeling or coaching, or you join us online for a webinar or an entire course, our goal is to make sure you feel supported and prepared to use the tasks you'll find in this book to their fullest!

Kasey Bratcher, Senior VP of Professional Learning

