



**TEXAS MATH
SOLUTION**

Grade 7

Teacher's Implementation Guide

Skills Program Edition

SY 2022-2023

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Teacher's Implementation Guide

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Our Manifesto

WE BELIEVE that quality math education is important for all students, to help them develop into creative problem solvers, critical thinkers, life-long learners, and more capable adults.

WE BELIEVE that math education is about more than memorizing equations or performing on tests—it's about delivering the deep conceptual learning that supports ongoing growth and future development.

WE BELIEVE all students learn math best when teachers believe in them, expect them to participate, and encourage them to own their learning.

WE BELIEVE teachers are fundamental to student success and need powerful, flexible resources and support to build dynamic cultures of collaborative learning.

WE BELIEVE our learning solutions and services can help accomplish this, and that by working together with educators and communities we serve, we guide the way to better math learning.

LONG + LIVE + MATH



At Carnegie Learning, we choose the path that has been proven most effective by research and classroom experience. We call that path the Carnegie Learning Way. Follow this code to take a look inside.

Acknowledgments

Middle School Math Solution Authors

- Sandy Bartle Finocchi, Chief Mathematics Officer
- Amy Jones Lewis, Senior Director of Instructional Design
- Kelly Edenfield, Instructional Designer
- Josh Fisher, Instructional Designer

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“Mathematics is so much more than rules and algorithms. It is learning to reason, to make connections, and to make sense of the world. We believe in Learning by Doing™—students need to actively engage with the content if they are to benefit from it. Your classroom environment will determine what type of discourse, questioning, and sharing will take place. Students deserve a safe place to talk, to make mistakes, and to build deep understanding of mathematics. My hope is that these instructional materials help you shift the mathematical authority in your class to your students. Be mindful to facilitate conversations that enhance trust and reduce fear.”

Sandy Bartle Finocchi, Chief Mathematics Officer

“My hope is that you know that your students are capable of thinking like mathematicians. This book is designed to give them the opportunity to struggle with challenging tasks, to talk about math with their classmates, and to make and fix mistakes. I hope that you use this book to build this capacity in your students—to ask the necessary questions to uncover what students already know and connect it to what they are learning, to encourage creative thinking, and to give just enough support to keep students on the right path.”

Amy Jones Lewis, Senior Director of Instructional Design

“At Carnegie Learning, we have created an organization whose mission and culture is defined by student success. Our passion is creating products that make sense of the world of mathematics and ignite a passion in students. Our hope is that students will enjoy our resources as much as we enjoyed creating them.”

Barry Malkin, CEO

The Carnegie Learning Way

At Carnegie Learning, we choose the path that has been proven most effective by research and classroom experience. We call that path the **Carnegie Learning Way**.

Our Instructional Approach

Carnegie Learning’s instructional approach is a culmination of the collective knowledge of our researchers, instructional designers, cognitive learning scientists, and master practitioners. It is based on a scientific understanding of how people learn, as well as an understanding of how to apply the science to the classroom. At its core, our instructional approach is based on three simple, key components:



ENGAGE

Activate student thinking by tapping into prior knowledge and real-world experiences.

Provide an introduction that generates curiosity and plants the seeds for deeper learning.



DEVELOP

Build a deep understanding of mathematics through a variety of activities.

Students encounter real-world problems, sorting activities, Worked Examples, and peer analysis—in an environment where collaboration, conversations, and questioning are routine practices.



DEMONSTRATE

Reflect on and evaluate what was learned.

Ongoing formative assessment underlies the entire learning experience, driving real-time adjustments, next steps, insights, and measurements.



Our Research

Carnegie Learning has been deeply immersed in research ever since it was founded by cognitive and computer scientists from Carnegie Mellon University. Our research extends far beyond our own walls, playing an active role in the constantly evolving field of cognitive and learning science. Our internal researchers collaborate with a variety of independent research organizations, tirelessly working to understand more about how people learn, and how learning is best

facilitated. We supplement this information with feedback and data from our own products, teachers, and students, to continuously evaluate and elevate our instructional approach and its delivery.

Our Support

We're all in. In addition to our books and software, implementing Carnegie Learning in your classroom means you get access to an entire ecosystem of ongoing classroom support, including:

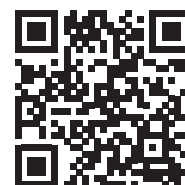
Professional Learning: Our team of Master Math Practitioners is always there for you, from implementation to math academies to a variety of other options to help you hone your teaching practice.

Texas Support Center: We've customized a Support Center just for you and your students. The Texas Support Center provides articles and videos to help you implement the Texas Math Solution, from the basics to get you started to more targeted support to guide you as you scaffold instruction for all learners in your classroom. Visit www.CarnegieLearning.com/texas-help to explore online and to access content that you can also share with your students and their caregivers.

MyCL: This is the central hub that gives you access to all of the products and resources that you and your students will need. Visit MyCL at www.CarnegieLearning.com/login.

LONG + LIVE + MATH: When you join this community of like-minded math educators, suddenly you're not alone. You're part of a collective, with access to special content, events, meetups, book clubs, and more. Because it's a community, it's constantly evolving! Visit www.longlivemath.com to get started.

Scan this code to visit the Texas Support Center and look for references throughout the Front Matter to learn more about the robust resources you will find in the Support Center.



Our Blend of Learning

The Texas Math Solution delivers instructional resources that make learning math attainable for all students. Learning Together and Learning Individually resources work in parallel to engage students with various learning experiences they need to understand the mathematics at each grade level.

For **Learning Together**, the student textbook is a consumable resource that empowers students to become creators of their mathematical knowledge. This resource is designed to support teachers in facilitating active learning so that students feel confident in sharing ideas, listening to each other, and learning together.

Over the course of a year, based on the recommended pacing, teachers will spend approximately 60% of their instructional time teaching whole-class activities as students learn together.

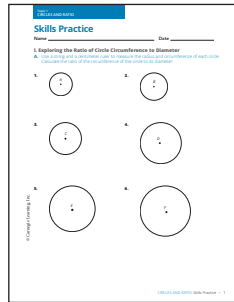
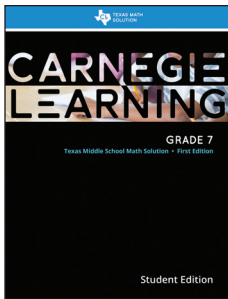
For **Learning Individually**, the Skills Practice provides students the opportunity to engage with problems that target each lesson's skills, concepts, and applications. This resource is designed to target discrete skills for development and mastery, therefore, scaffolding and extension opportunities are provided in the problem sets.

An additional Learning Individually resource is MATHia®, an intelligent software that provides just-in-time support and tracks student progress against fine-grained skills to deliver the right content they need to become proficient with the mathematics.

Over the course of the year, based on the recommended pacing, teachers will spend approximately 40% of their instructional time monitoring students as they work and learn individually.

Learning Together

Learning Individually



TEXTBOOK

I am a record of student thinking, reasoning, and problem solving.

My lessons allow students to build new knowledge based upon prior knowledge and experiences, apply math to real-world situations, and learn together in a collaborative classroom.

My purpose is to create mathematical thinkers who are active learners that participate in class.

SKILLS PRACTICE

I am targeted practice of each lesson's skills, mathematical concepts, and applications for each topic in the student textbook.

My purpose is to provide additional problem sets for teachers to assign as needed for additional practice or remediation.

MATHia

I am designed to empower students to learn individually at their own pace with sophisticated AI technology that personalizes their learning experiences, while giving teachers real-time insights to monitor student progress.

My purpose is to coach students alongside teachers as students learn, practice, do, and look forward.



Visit the Texas Support Center for additional information on the Learning Individually resources.

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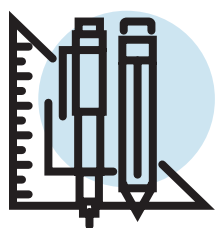
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Glossary

Instructional Design

In a word, every single piece of Carnegie Learning's Texas Math Solution is **intentional**. Our instructional designers work alongside our master math practitioners, cognitive scientists, and researchers to intentionally design, draft, debate, test, and revise every piece, incorporating the latest in learning science.



Intentional Mathematics Design

Carnegie Learning's Texas Math Solution is thoroughly and thoughtfully designed to ensure students build the foundation they'll need to experience ongoing growth in mathematics.

Mathematical Coherence: The Process Standards arc of mathematics develops coherently, building understanding by linking together within and across grades, so students can learn concepts more deeply and apply what they've learned to more complex problems going forward.

Mathematical Process Standards: Carnegie Learning is organized around the Mathematical Process Standards to encourage experimentation, creativity, and false starts, which is critical if we expect students to tackle difficult problems in the real world, and persevere when they struggle.

Multiple Representations: Carnegie Learning recognizes the importance of connecting multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

Transfer: Carnegie Learning focuses on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.

Texas Math Solution Overview

The instructional materials in the Carnegie Learning Texas Math Solution emphasize active learning and making sense of the mathematics. We ask deep questions that require students to thoroughly understand the mathematical concepts they are learning. We think about how to guide students to connect interrelated ideas in a holistic way to integrate students' understanding with their developing habits of mind.

What are the Carnegie Learning Texas Math Solution guiding principles?

The Texas Math Solution has been strongly influenced by scientific research into the learning process and student motivations for academic success. Its guiding principles are active learning, discourse through collaboration, and personalized learning.

Active Learning: The research makes it clear that students need to actively engage with content in order to benefit from it. Studies show that as instruction moves up the scale from entirely passive to fully interactive, learning becomes more robust. All of the activities we provide for the classroom encourage students to be thoughtful about their work, to consider hypotheses and conclusions from different perspectives, and to build a deep understanding of mathematics. The format of the student text, as a consumable workbook, supports active instruction.

Discourse through Collaborative Learning: Effective collaboration encourages students to articulate their thinking, resulting in self-explanation. Reviewing other students' approaches and receiving feedback on their own provides further metacognitive feedback. Collaborative problem-solving encourages an interactive instructional model, and we have looked to research to provide practical guidance for making collaboration work. The collaborative activities within our lessons are designed to promote active dialogue centered on structured activities.

Personalized Learning: One of the ways to build intrinsic motivation is to relate activities to students' existing interests. Research has proven that problems that capture student interests are more likely to be taken seriously. In the textbook, problems often begin with the students' intuitive understanding of the world and build to an abstract concept, rather than the other way around.

How is the mathematical content delivered to promote productive mathematical processes?

Students deserve math learning that develops them into creative problem solvers, critical thinkers, life-long learners, and more capable adults, while teachers deserve instructional resources that will support them in bringing learning to life. There are three organizing principles that guide these instructional resources.

Seeing Connections: Activities make use of models—e.g., real-world situations, graphs, diagrams, and worked examples—to help students see and make connections between different topics. In each lesson, learning is linked to prior knowledge and experiences so that students build their new understanding on the firm foundation of what they already know. We help students move from concrete representations and an intuitive understanding of the world to more abstract representations and procedures. Activities thus focus on real-world situations to demonstrate the usefulness of mathematics.

Exploring Structure: Questions are phrased in a way that promotes analysis, develops higher-order-thinking skills, and encourages the seeking of mathematical relationships. Students inspect a given figure, equation, or data set, and in each case, they are asked to discern a pattern or structure. We want students to become fluent in seeing how the structure of each representation—verbal, graphic, numerical, and algebraic—reveals properties of the relationship it defines. We want students to become fluent at composing and decomposing expressions, equations, and data sets. As students gain proficiency in manipulating structure, they become capable of comparing, contrasting, composing, decomposing, transforming, solving, representing, clarifying, and defining the characteristics of figures, equations leading to functions, and data sets.

Reflecting and Communicating: A student-centered approach focuses on students thinking about and discussing mathematics as active participants in their own learning. Through articulating their thinking in conversations with a partner, in a group, or as a class, students integrate each piece of new knowledge into their existing cognitive structure. They use new insights to build new connections. Through collaborative activities and the examination of peer work—both within their groups and from examples provided in the lessons—students give and receive feedback, which leads to verifying, clarifying, and/or improving the strategy.

Texas Math Solution Year at a Glance

This Year at a Glance highlights the sequence of topics and the number of blended instructional days (1 day is a 45-minute instructional session) allocated for Grade 7 in the Texas Math Solution. The pacing information also includes time for assessments, providing you with an instructional map that covers 180 days of the school year. As you set out at the beginning of the year, we encourage you to still modify this plan as necessary.

Want More Support Designing Your Long-Term Plan?

You can find this Year at a Glance and additional guidance on planning intentionally and flexibly on the Texas Support Center at www.CarnegieLearning.com/texas-help.



Texas Grade 7: Year at a Glance

*1 Day Pacing = 45 min. Session

Module	Topic	Pacing	TEKS
Process Standards are embedded in every module: 7.1A, 7.1B, 7.1C, 7.1D, 7.1E, 7.1F, 7.1G			
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		12	
Total Days:		180	

Connecting Content and Practice

Lesson Structure

Each lesson of the Texas Math Solution has the same structure. This consistency allows both you and your students to track your progress through each lesson. Key features of each lesson are noted.

ENGAGE

Establishing Mathematical Goals to Focus Learning

Create a classroom climate of collaboration and establish the learning process as a partnership between you and students.

Communicate continuously with students about the learning goals of the lesson to encourage self-monitoring of their learning.

Visit the Texas Support Center for additional guidance on how to foster a classroom environment that promotes collaboration and communication.



Lesson Structure

1. Learning Goals

Learning goals are stated for each lesson to help you take ownership of the learning objectives.

2. Connection

Each lesson begins with a statement connecting what you have learned with a question to ponder.

Return to this question at the end of this lesson to gauge your understanding.

WARM UP
Scale up or down to determine an equivalent ratio.

- $\frac{18 \text{ miles}}{3 \text{ hours}} = \frac{?}{1 \text{ hour}}$
- $\frac{\$750}{4 \text{ days}} = \frac{?}{1 \text{ day}}$
- $\frac{12 \text{ in.}}{1 \text{ ft}} = \frac{?}{5 \text{ ft}}$
- $\frac{48 \text{ oz}}{3 \text{ lb}} = \frac{?}{1 \text{ lb}}$

LEARNING GOALS ①

- Identify pi (π) as the ratio of the circumference of a circle to its diameter.
- Construct circles using a compass and identify various parts of circles.
- Know and write the formula for the circumference of a circle, and use the formula to solve problems.

KEY TERMS

- congruent
- circle
- radius
- diameter
- circumference
- pi

② You have learned about ratios. How can you use ratios to analyze the properties of geometric figures such as circles?

LESSON 1: Pi: The Ultimate Ratio • 1

“ Mathematics is the science of patterns. So, we encourage students throughout this course to notice, test, and interpret patterns in a variety of ways—to put their “mental tentacles” to work in every lesson, every activity. Our hope is that this book encourages you to do the same for your students, and create an environment in your math classroom where productive and persistent learners develop and thrive.

Josh Fisher, Instructional Designer

” Activating Student Thinking

Your students enter each class with varying degrees of experience and mathematical success. The focus of the Getting Started is to tap into prior knowledge and real-world experiences, to generate curiosity, and to plant seeds for deeper learning. Pay particular attention to the strategies students use, for these strategies reveal underlying thought processes and present opportunities for connections as students proceed through the lesson.

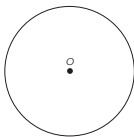
Supporting Emergent Bilingual Students

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they complete the Getting Started activities in each lesson.


3
Getting Started

Across and Around

A circle is shown with a point drawn at the center of the circle. The name of the point is O, so let's call this Circle O.



“ Be sure to include units when you record your measurements. ”



1. Analyze the distance around the circle.
 - a. Use a string and a centimeter ruler to determine the distance around the circle.
 - b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.
2. Draw a line from a point on the circle to the center of the circle, point O.
 - a. Measure your line using your centimeter ruler.
 - b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.

2 • TOPIC 1: Circles and Ratio

Lesson Structure • FM-13



Aligning Teaching to Learning

Students learn when they are actively engaged in a task: reasoning about the math, writing their solutions, justifying their strategies, and sharing their knowledge with peers.

Support productive struggle by allowing students time to engage with and persevere through the mathematics.

Support student-to-student discourse as well as whole-class conversations that elicit and use evidence of student thinking.

Supporting Emergent Bilingual Students

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they engage in mathematical discourse throughout each lesson.



4. Activities
You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about answer-getting. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, Worked Examples, or analyzing simple student work.

Be prepared to share your solutions and methods with your classmates.

4

ACTIVITY 1.1 Analyzing the Parts of a Circle

Everyone can identify a circle when they see it, but defining a circle is a wordy math task. **Step 1:** In the space

ACTIVITY 1.2 Measuring the Distance Around a Circle

Let's explore circles. Use circles A, B, D, E, and O provided at the end of the lesson.

ACTIVITY 1.3 The Circumference Formula

The number pi (π) is the ratio of the circumference of a circle to its diameter. That is $\pi = \frac{\text{circumference of a circle}}{\text{diameter of a circle}}$ or $\pi = \frac{C}{d}$, where C is the circumference of the circle, and d is the diameter of the circle. The number π has an infinite number of decimal digits that never repeat. Some approximations used for the value π are 3.14 and $\frac{22}{7}$.

- Use this information to write a formula for the circumference of a circle, where d represents the diameter of a circle and C represents the circumference of a circle.
- Rewrite the formula for the circumference of a circle, where r represents the radius of a circle and C represents the circumference of a circle.
- Use different representations for π to calculate the circumference of a circle.
 - Calculate the circumference of a circle with a diameter of 4.5 centimeters and a circle with a radius of 6 inches. Round your answer to the nearest ten-thousandths, if necessary.

Value for π	$d = 4.5$ centimeters	$r = 6$ inches
π		
Use the π key on a calculator		
Use 3.14 for π		
Use $\frac{22}{7}$ for π		

6 • TOPIC 1: Circles and Ratio

Ongoing Formative Assessment Drives Instruction

For students to take responsibility for their own learning, they need to be encouraged to self-assess. Students can use the Talk the Talk to monitor their own progress towards mastering the learning goals. Listen and review their answers and explanations and provide feedback to help them improve their understanding.

As you plan the next lesson, consider the connections you can make to build off the strengths or fill any gaps identified from this formative assessment.



NOTES

TALK the TALK 5

Twice

Use what you have learned to compare circles by their characteristics.

1. Using your compass, draw each circle.

a. radius length of 3 centimeters
b. diameter length of 3 centimeters

Measuring the Distance Around a Circle

LESSON 1: Pi: The Ultimate Ratio • 9

5. Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

Lesson Structure • FM-15

Student Lesson Overview Videos

Each lesson has a corresponding lesson overview video(s) for students to utilize and reference to support their learning. The videos provide an overview of key concepts, strategies, and/or worked examples from the lessons.

Assignment

An intentionally designed Assignment follows each lesson.

There is one Assignment per lesson. Lessons often span multiple days. Be thoughtful about which portion of the Assignment students can complete based on that day's progress.

The **Stretch** section is not necessarily appropriate for all learners. Assign this to students who are ready for more advanced concepts.

The **Review** section provides spaced practice of concepts from the previous lesson and topic and of the fluency skills important for the course.

Assignment

6. Write
Reflect on your work and clarify your thinking.

7. Remember
Take note of the key concepts from the lesson.

8. Practice
Use the concepts learned in the lesson to solve problems.

9. Stretch
Ready for a challenge?

10. Review
Remember what you've learned by practicing concepts from previous lessons and topics.

Assignment LESSON 1: Pi: The Ultimate Ratio

6 Write
Define each term in your own words.
1. circle
2. radius
3. diameter
4. pi

8 Practice
Answer each question. Use 3.14 for π . Round your answer to the nearest hundredth, if necessary.
1. Although she's only in middle school, her favorite place to drive is at the track. Track 1 has a radius of 60 feet and a radius of 110 feet.
a. Compute the circumference.
b. Compute the circumference.
c. Compute the circumference.
d. Driver's Delight is considering 150 feet. Compute the circumference.
2. Tamaka wants to build a circular track.
a. If she wants the track to have a radius of 100 feet, how long is the track?
b. If she wants the track to have a diameter of 200 feet, how long is the track?
c. If she wants the track to have a circumference of 1,000 feet, how long is the track?

9 Stretch
A rope is arranged using three circles as shown. The distance between the centers of the two outer circles is 3 in. What is the radius of each circle?

7 Remember
The circumference of a circle is the distance around the circle. The formulas to determine the circumference of a circle are $C = \pi d$ or $C = 2\pi r$, where d represents the diameter, r represents the radius, and π is a constant value equal to approximately 3.14 or $\frac{22}{7}$. The constant π represents the ratio of the circumference of a circle to its diameter.

10 Review
1. Ethan and Connor are training for a marathon.
a. Connor runs 13.5 miles in 2 hours. What is her rate?
b. Ethan wants to run the 26.2 miles of the marathon in 4.5 hours. At what rate will he have to run to reach this goal? Round to the nearest tenth.
2. Fifteen seventh graders were randomly selected to see how many pushups in a row they could do. Their data are shown:
45, 40, 36, 38, 42, 48, 40, 40, 70, 45, 42, 43, 48, 36
a. Determine the mean of this data set.
b. Determine the median of this data set.
3. Convert each measurement.
a. $4\frac{1}{2}$ pounds = _____ oz
b. 22.88 cm = _____ in.

2 • TOPIC 1: Circles and Ratio

FM-16 • Assignment

Topic Summary

A Topic Summary is provided for students at the end of each topic. The Topic Summary lists all key terms of the topic and provides a summary of each lesson. Each lesson summary defines key terms and reviews key concepts, strategies, and/or worked examples.

Circles and Ratio Summary

KEY TERMS

- congruent
- circle
- radius
- diameter
- circumference
- pi
- unit rate

LESSON 1

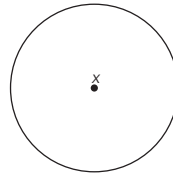
A **circle** is a shape with one curved surface. The center of the circle is a point.

A **radius** of a circle is a line segment from the center of the circle to the edge. A diameter is a line segment connecting two points on the circle that passes through the center.

Circles are measured by their circumference. The diameter of a circle is the distance across the circle through the center.

The distance around the circle is the circumference. The formula for the circumference of a circle is $C = \pi d$, where d is the diameter of the circle. Some approximate values for π are 3.14 and $\frac{22}{7}$.

Congruent means that it has the same shape and size. For example, Circle X is congruent to Circle B. If line segment AH on Circle B has a length of 10 centimeters, then the circumference of Circle X is $C = \pi(10)$ centimeters, or approximately 31.4 centimeters.

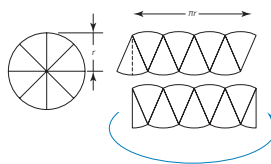


LESSON 2

That's a Spicy Pizza!

The circumference of a circle is the distance around the circle, while the area of a circle is the amount of space contained inside the circle. The formula for the area of a circle is $A = \pi r^2$.

The area formula for a circle can be derived by dividing a circle into a large number of equal-sized wedges. Laying these wedges as shown, you can see that they will form an approximate rectangle with a length of πr and a height of r .



A **unit rate** is a ratio of two different measures in which either the numerator or denominator is 1.

For example, a large pizza with a diameter of 18 inches costs \$14.99. The rate of area to cost is $\frac{\pi \cdot 9^2}{14.99} = \frac{81\pi}{14.99}$. Using 3.14 for π , the unit rate is approximately 16.97 square inches per dollar. The unit rate of cost to area is $\frac{1}{16.97}$, or approximately \$0.06 per square inch.

LESSON
3

Circular Reasoning

Given a specific length to form a perimeter or circumference, arranging that length into the shape of a circle provides the maximum area.

For example, suppose you have 176 feet of fencing to use to fence off a portion of your backyard for planting vegetables. You want to maximize the amount of fenced land. Calculate the maximum fenced area you will have.

The length of the
Use the formula for
diameter of a circle

If the diameter is
28 feet. Use the formula

The maximum area is

Many geometric shapes
known as circles. It is
necessary to know the

For example, the area of
semi-circles is half the area of

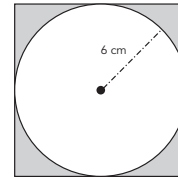
Calculate the area of the square.
 $A = l \times w$
 $A = (11) \times (11)$
 $A = 121$

The area of the circle is
or 84.25 square centimeters.

When determining the area of a shaded region of a figure, it is often necessary to calculate the area of a figure and subtract it from the area of a second figure.

For example, this figure shows a circle inscribed in a square. Determine the area of the shaded region.

When a circle is inscribed in a square, the diameter of the circle is equal to the side length of the square.



Calculate the area of the square.

$$A = s^2$$

$$A = 12^2$$

$$A = 144 \text{ square centimeters}$$

Calculate the area of the circle.

$$A = \pi r^2$$

$$A = \pi(6)^2$$

$$A = 36\pi \approx 113.04 \text{ square centimeters}$$

The area of the shaded region is approximately 144 square centimeters minus 113.04 square centimeters, or 30.96 square centimeters.

Problem Types You Will See

Lessons include a variety of problem types to engage students in reasoning about the math.

Problem Types You Will See

WORKED EXAMPLE

	$\frac{11}{3}x + 5 = \frac{17}{3}$	$\frac{1}{2}x + \frac{3}{4} = 2$
Step 1:	$3\left(\frac{11}{3}x + 5\right) = 3\left(\frac{17}{3}\right)$	$4\left(\frac{1}{2}x + \frac{3}{4}\right) = 4(2)$
Step 2:	$11x + 15 = 17$	$2x + 3 = 8$
Step 3:	$x = \frac{17 - 15}{11}$ $= \frac{2}{11}$	$x = \frac{8 - 3}{2}$ $= \frac{5}{2}$

Worked Example

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

Ask Yourself:

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

Thumbs Up

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.

Ask Yourself:

- Why is this method correct?
- Have I used this method before?

Thumbs Down

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

Ask Yourself:

- Where is the error?
- Why is it an error?
- How can I correct it?

Analyze the solution strategy and solution for each inequality.

Ella



$$\begin{aligned} -\frac{1}{2}x + \frac{3}{4} &< 2 \\ -4\left(-\frac{1}{2}x + \frac{3}{4} < 2\right) \\ 2x - 3 &> -8 \\ 2x &> -5 \\ x &> -\frac{5}{2} \\ x &> -2.5 \end{aligned}$$

Describe the strategy that Ella used correctly.

Jeff



$$\begin{aligned} -12x + 20 &< 32 \\ \frac{-12x + 20}{-4} &< \frac{32}{-4} \\ 3x - 5 &< -8 \\ 3x &< -3 \\ x &< -1 \end{aligned}$$

Identify the error in Jeff's strategy and determine the correct solution.

Problem Types • FM-17

Worked Examples

Worked Examples help students develop their skills as they question their understanding, make connections with the steps, and ultimately explain the progression of the steps towards the final outcome. They represent and mimic an internal dialogue about the mathematics and the strategies, and the questions that follow them are designed to serve as a model for self-questioning and self-explanations, while making sure that students don't skip over a Worked Example without interacting with it, thinking about it, and responding to its accompanying questions. This approach aids students as they develop their desired habits of mind for being conscientious about the importance of steps and their order.

Thumbs Up / Thumbs Down

Thumbs Up problems give students the opportunity to analyze viable methods and problem-solving strategies. Questions are presented to help students consider the various strategies in depth and to focus on an analysis of correct responses. Because research shows that providing only positive examples is less effective for eliminating common student misconceptions than also showing negative examples, incorrect responses are provided alongside the correct responses. From the incorrect responses, students learn to determine where the error in calculation is, why the method is wrong or is being used wrong, and also how to correct the method to calculate the solution properly.

Who's Correct?

"Who's Correct?" problems are an advanced form of correct vs. incorrect responses. In this problem type, students are not told who is correct. Students have to think more deeply about what the strategies really mean and whether each of the solutions makes sense. Students will determine what is correct and what is incorrect, and then explain their reasoning. These types of problems will help students analyze their own work for errors and correctness.

Who's Correct?



When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or not correct.

Ask Yourself:

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

Vanessa was given a math problem to determine how many different rectangles can be constructed with an area of 12 square inches.

Vanessa thinks that there are only two: one with a width of 2 inches and a length of 6 inches, and another with a width of 3 inches and a length of 4 inches. Is she correct? Explain your reasoning.

Promoting Self-Reflection

The Crew

The Crew is here to help you on your journey. Sometimes they will remind you about things you already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are members of your group—someone you can rely on!



Teacher aides will guide you along your journey. They will help you make connections and remind you to think about the details.



The Crew • FM-19

The Crew

Characters are embedded throughout the Texas Math Solution to remind students to stop and think in order to promote productive reflection. The characters are used in a variety of ways: they may remind students to recall a previous mathematical concept, help students develop expertise to think through problems, and occasionally, present a fun fact.

Mathematical Process Standards

Note

Each lesson provides opportunities for students to think, reason, and communicate their mathematical understanding. However, it is your responsibility as a teacher to recognize these opportunities and incorporate these practices into your daily rituals. Expertise is a long-term goal, and students must be encouraged to apply these practices to new content throughout their school career.

Mathematical Process Standards

Texas Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The “I can” expectations listed below align with the TEKS Mathematical Process Standards and encourage students to develop their mathematical learning and understanding.

► Apply mathematics to problems arising in everyday life, society, and the workplace.

I can:

- use the mathematics that I learn to solve real world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

► Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

I can:

- explain what a problem “means” in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

FM-20 • Mathematical Process Standards



Supporting Students to Use Mathematical Tools

Visit the Texas Support Center for strategies to support students as they use mathematical tools, including formula charts and reference sheets.

- ▶ **Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.**

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

- ▶ **Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.**

I can:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

Mathematical Process Standards • FM-21

Note

When you are facilitating each lesson, listen carefully and value diversity of thought, redirect students' questions with guiding questions, provide additional support with those struggling with a task, and hold students accountable for an end product. When students share their work, make your expectations clear, require that students defend and talk about their solutions, and monitor student progress by checking for understanding.

Consider having students create "I can" statements to promote their self-reflection.

There is one more page of mathematical process standards that is not provided here, but is available in the Student Textbook Front Matter.

Supporting ALL Learners

Visit the Texas Support Center for facilitation strategies to support ALL students as they engage in the Mathematical Process Standards.



Academic Glossary

Language Expectations

It is critical for students to possess an understanding of the language of their text. Students must learn to read for different purposes and write about what they are learning. Encourage students to become familiar with the key words and the questions they can ask themselves when they encounter these words.

It is our recommendation to be explicit about your expectations of language use and the way students write responses throughout the text. Encourage students to answer questions with complete sentences. Complete sentences help students reflect on how they arrived at a solution, make connections between topics, and consider what a solution means both mathematically as well as in context.

Academic Glossary

There are important terms you will encounter throughout this book. It is important that you have an understanding of these words as you get started on your journey through the mathematical concepts. Knowing what is meant by these terms and using these terms will help you think, reason, and communicate your ideas.

ANALYZE

Definition
To study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?


EXPLAIN YOUR REASONING

Definition
To give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

Visit the Students & Caregivers Portal on the Texas Support Center at www.CarnegieLearning.com/texas-help to access the Mathematics Glossary for this course anytime, anywhere.



Related Phrases

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

Related Phrases

- Show your work
- Explain your calculation
- Justify
- Why or why not?

Academic Glossary • FM-23



Supporting Students at Varying Levels of Language Proficiency

Visit the Texas Support Center for guidance on how to leverage the Academic Glossary to support students at varying levels of language proficiency.

Related Phrases

REPRESENT

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

Definition

To display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

Related Phrases

ESTIMATE

- Predict
- Approximate
- Expect
- About how much?

Definition

To make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

Related Phrases

DESCRIBE

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

Definition

To represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

FM-24 • Academic Glossary

Ask Yourself

The Ask Yourself questions help students develop the proficiency to explain to themselves the meaning of problems.

Real-World Context

Real-world contexts confirm concrete examples of mathematics. The scenarios in the lessons help students recognize and understand that quantitative relationships seen in the real world are no different than quantitative relationships in mathematics. Some problems begin with a real-world context to remind students that the quantitative relationships they already use can be formalized mathematically. Other problems will use real-world situations as an application of mathematical concepts.

Mathematics Glossary

A course-specific mathematics glossary is available for students to utilize and reference during their learning. Definitions and examples of key terms are provided in the glossary.

Facilitating Student Learning

Visit the Texas Support Center at www.CarnegieLearning.com/texas-help for additional resources to support you anytime, anywhere.



Teacher's Implementation Guide

The Teacher's Implementation Guide (TIG) is designed to fully support a wide range of teachers implementing our materials: from first-year teachers to 30-year veterans and from first-time Carnegie Learning users to master practitioners.

One goal in developing the TIG was to make our instructional design apparent to the users.

The lessons of each topic were written to be accessible to the full range of learners. With every instructional decision you make, keep in mind your mathematical objectives for the topic and module and the course. Plan each lesson by thinking about how you will create access for your particular group of students, maintain access and pace throughout the lesson, and assess their understanding along the way. We recommend that you do the math in each topic before implementing the activities with your specific group of students.

What makes this TIG useful?

Effective Lesson Design: Each lesson has a consistent structure for teachers and students to follow. The learning experiences are engaging and effective for students.

Pacing: Each course is designed to be taught in a 180-day school year. Pacing suggestions are provided for each lesson. Each day in the pacing guide is an equivalent to about a 45-minute instructional session.

Instructional Supports: Guiding questions are provided for teachers to use as they're circulating the room, as well as differentiation strategies, common student misconceptions, and student look-fors.

Clearly Defined Mathematics: The content and instructional goals are clearly described at the module, topic, lesson, and activity levels.

The TIG is critical to understanding how the mathematics that students encounter should be realized in the classroom. The TIG describes the depth of understanding that students need to develop for each standard and a pathway for all learners to be successful. It provides differentiation strategies to support students who struggle, to extend certain activities for students who are advanced in their understanding of the content, and to support emergent bilingual students.

Module and Topic Overviews

You are responsible for teaching the essential concepts associated with a particular course. You need to understand how activities within lessons build to achieve understanding within topics, and how topics build to achieve understanding throughout the course. In the Texas Math Solution, Carnegie Learning seeks to establish a shared curriculum vision with you.

Module 3 Overview
Reasoning Algebraically

"Effective algebraic thinking sometimes involves reversibility (i.e., being able to undo mathematical processes as well as do them). In effect, it is the capacity not only to use a process to get to a goal, but also to understand the process well enough to work backward from the answer to the starting point." (*Fostering Algebraic Thinking: A Guide for Teachers Grades 6–10*, pp. 1–2)

Why is this Module named Reasoning Algebraically?
Reasoning Algebraically continues to build students' facility with... Students need to view a patterns and sense-making procedures and rules to the primary focus of this sense of and reasoning and equations. The expressions and inequalities student this module are more commonly studied in grade 6, involving rational numbers and rather than one. Through students are expected to quantities, interpret quantities and inequalities, and real connections across representing equations and inequalities students should also build fluency in operating with equations, and inequalities negative rational coefficients reasoning about the algorithm should remain at the form

What is the mathematics of Reasoning Algebraically?
Reasoning Algebraically contains

Two-Step Equations and Inequalities
Topic 3 Overview

How is Two-Step Equations and Inequalities organized?
Two-Step Equations and Inequalities continues to develop students' understanding of a solution to an equation or the solution set of an inequality. Just as they did with one-step equations in grade 6, students begin this topic by reasoning about expressions and equations. They use bar models to write and solve equations from problem situations. Next, they use double number lines, similar to those used to determine equivalent ratios, but with variable expressions. Throughout these reasoning exercises, the meaning of a solution to an equation is reinforced: students check their solutions with substitution and write equations from solutions.

with a variety of rational coefficients and to generalize strategies to solve equations of the form $ax + b = c$ and $a(x + b) = c$. Additionally, students write and solve literal equations for geometry concepts.

Students extend their understanding of solving equations to solving two-step inequalities and graphing the solution sets on number lines. They use numeric examples to build simple solution sets to investigate and develop properties of inequalities. Students add, subtract, multiply, and divide by positive and negative rational numbers and recognize that dividing or multiplying each side of an inequality by a negative rational number reverses the sign of the inequality.

What is the entry point for students?
In grade 6, students first encountered variable equations and used models to solve one-step equations and inequalities. *Two-Step Equations and Inequalities* builds on students' knowledge of expressions and equations to introduce two-step equations. Students write and interpret the meanings of parts of expressions and then set expressions equal to each other. They reason about solutions using their bar models and number sense. As students

TOPIC 3: Two-Step Equations and Inequalities • 1

Module Overview

Each module begins with an overview that describes the reasoning behind the name, the mathematics being developed, the connections to prior learning, and the connections to future learning.

Topic Overview

A Topic Overview describes how the topic is organized, the entry point for students, how a student will demonstrate understanding, why the mathematics is important, how the activities promote expertise in the mathematical process standards, materials needed for the topic, examples of visual representations or strategies used, and more detailed information to help with pacing.

"Teachers must first develop their ideas about where the curriculum program is going mathematically (curriculum vision) before deciding whether the curriculum materials will help them reach that mathematical goal (curriculum trust)" (Drake & Sherin, 2009, p. 325).

Facilitation Notes

For each lesson, you are provided with detailed facilitation notes to fully support your planning process. This valuable resource provides point-of-use support that serves as your primary resource for planning, guiding, and facilitating student learning.

1. Materials

Materials required for the lesson are identified.

2. Lesson Overview

The Lesson Overview sets the purpose and describes the overarching mathematics of the lesson, explaining how the activities build and how the concepts are developed.

3. TEKS Addressed

The focus TEKS for each lesson are listed. Carnegie Learning recognizes that some lessons could list several TEKS based on the skills needed to complete the activities, however, the TEKS listed are what the lesson is focused on developing or mastering.

4. ELPS Addressed

The English Language Proficiency Standards for each lesson are listed. As you plan, consider these ELPS and determine the instructional strategies that you will use to meet these ELPS.

Pi: The Ultimate Ratio

Exploring the Ratio of Circle Circumference to Diameter

MATERIALS

- Centimeter ruler
- String
- Compass
- Calculator with π key

2 Lesson Overview

Students explore the relationship between the distance around a circle and the distance across a circle. They learn the terms *circumference*, *diameter*, and *radius*. Students use hands-on tools to measure the distances and compare the ratio of the circumference to the length of the diameter. They then use a compass to create their own circles and realize that for every circle the ratio of circumference to diameter is pi. Students practice solving for the diameter or the circumference in problems.

Grade 7 Proportionality

(5) The student applies mathematical process standards to use geometry to describe or solve problems involving proportional relationships. The student is expected to:

(B) describe π as the ratio of the circumference of a circle to its diameter.

Expressions, Equations, and Relationships

(8) The student applies mathematical process standards to develop geometric relationships with volume. The student is expected to:

(C) use models to determine the approximate formulas for the circumference and area of a circle and connect the models to the actual formulas.

(9) The student applies mathematical process standards to solve geometric problems. The student is expected to:

(B) determine the circumference and area of circles.

4 ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

5 Essential Ideas

- The circumference of a circle is the distance around the circle.
- The ratio of the circumference of a circle to the diameter of a circle is approximately 3.14 or pi.

LESSON 1: Pi: The Ultimate Ratio • 1

5. Essential Ideas

These statements are derived from the standards and state the concepts students will develop.

6 Lesson Structure and Pacing: 2 Days 7

Day 1

Engage

Getting Started: . . . Stay Together

Students analyze double number lines and write equations to represent the information presented in double number lines. Students then use what they know to extend each of the number lines to the left and right of the values presented. They do this for equal expressions involving addition, subtraction, negative numbers, and fractions.

Develop

Activity 2.1: Solving a Two-Step Equation

Students recall a scenario from the previous lesson and analyze how to solve the equation by representing the situation using a double number line. Students describe the operations used in each step and interpret the solution. Students then solve a two-step equation on their own using a double number line.

Day 2

Activity 2.2: Practice Solving Two-Step Equations

Students model and solve a variety of equations using double number lines. In each case, students describe the steps and reasoning they use to solve, including the operations at each step. Students investigate alternative ways of solving equations using a double number line.

Activity 2.3: Reasoning with Negatives to Solve Equations

An equation with a negative coefficient is solved for students, and students are asked to analyze and interpret the steps used to solve. A number of equations involving negative coefficients are given. Students are asked to solve them using a double number line and explain their process.

Demonstrate

Talk the Talk: Keeping It Together

Students start with an unknown solution and generate an equation. They describe the steps they use to compose the equation. Students then record the steps their partner uses to solve the equation, noting that these steps can be the opposite of the steps used to compose the equation. Finally, students reflect on what it means to maintain equality when solving equations.

6. Lesson Structure

This section highlights how the parts of the lesson fit within the instructional design: Engage, Develop, and Demonstrate. A summary of each activity is included.

7. Pacing

Lessons often span more than one 45-minute class period. Suggested pacing is provided for each lesson so that the entire course can be completed in a school year.



Position yourself to take full advantage of the richness of the mathematics addressed in the textbook. The Facilitation Notes provide guidance to reach each student from their current level of understanding to advance to the next stage. Place yourself in the position of the student by experiencing the textbook activities prior to class. Realize your role in the classroom—empower your students! Step back and let them do the math with confidence in their role as learner and your role as facilitator of learning.



Janet Sinopoli, Instructional Designer

8. Facilitation Notes by Activity

A detailed set of guidelines walks the teacher through implementing the Getting Started, Activities, and Talk the Talk portions of the lesson. These guidelines include an activity overview, grouping strategies, guiding questions, possible student misconceptions, differentiation strategies, student look-fors, and an activity summary.

9. Activity Overview

Each set of Facilitation Notes begins with an overview that highlights how students will actively engage with the task to achieve the learning goals.

10. Differentiation Strategies

To extend an activity for students who are ready to advance beyond the scope of the activity, additional challenges are provided.

8

Getting Started: A Winning Formula

ENGAGE

9

Facilitation Notes

In this activity, given the circumference of a circle, students develop a strategy and use it to solve for the area.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

10

Differentiation strategies

For students who struggle to get started students because there is not a direct solution path, ask start up questions, such as,

- What do you know?
- Can you draw a diagram to represent that?
- What numbers can you label in your diagram?
- Are there other measurements you could figure out?
- Are there any formulas that would be helpful?

Questions to ask

- What is the formula for calculating the area of a circle?
- What information is needed to calculate the area of a circle?
- What is the formula for calculating the circumference of a circle?
- Knowing the circumference, how can the length of the radius be determined?
- Knowing the circumference, how can the length of the diameter be determined?
- What is the length of the radius in this situation?
- What is the length of the diameter in this situation?

Summary

Given the circumference of a circle, the radius or diameter can be determined and used to calculate the area.

Activity 3.1 A Maximum Area Problem



DEVELOP

Facilitation Notes

In this activity, students are given 120 feet of fencing and asked to construct a freestanding dog pen in such a way that the maximum amount of area is fenced in.

Have students work with a partner or in a group to complete this activity. Share responses as a class.

LESSON 3: Circular Reasoning • 3

Note

Differentiation strategies are provided that will ensure all students acquire the knowledge of the activity. These strategies provide flexibility within the lesson to allow for varying student acquisition and demonstration of learning. These strategies provide suggestions for all students, including those with learning strengths or learning gaps.

- What words give you clues to which formula you should use?
- What unit was used to describe circumference in this situation?
- What unit was used to describe area in this situation?
- Why are different units used for circumference and area?

Summary

Circumference ($C = 2\pi r$) and area of a circle ($A = \pi r^2$) formulas are applied to solve real-world problem situations.

Activity 2.3 Unit Rates and Circle Area



Facilitation Notes

In this activity, students use unit rates and the area of a circle formula to determine the best buy in a real-world problem situation.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

12

As students work, look for

- Confusion between square inches per dollar and cents per square inch.
- Decimal errors in calculations.
- Rounding errors in decimals.
- Reversing numerator and denominator when determining square inches per dollar.
- Reversing numerator and denominator when determining cents per square inch.
- Misinterpreting unit rates to determine the better buy.
- Errors due to not using parentheses correctly on the calculator.

Questions to ask

- Who determined the rate of the number of square inches for every dollar?
- Who determined the rate of the amount, in dollars, for every square inch?
- Is the greater or smaller value for square inches per dollar the better buy?
- Is the smaller or greater value for dollars per square inch the better buy?

11. Mathematical Process Standards

Each activity is denoted with an icon that represents a mathematical practice or pair of practices that are intentionally being developed. For example, this icon indicates students are expected to look for and make use of structure and look for and express regularity in repeated reasoning.

12. As Students Work, Look For

These notes provide specific language, strategies, and/or errors to look and listen for as you circulate and monitor students working in pairs or groups. You can incorporate these ideas when students share their responses with the class.

Note: Alternative Grouping Strategies

Differentiation strategies that provide other grouping strategies, such as whole class participation and the jigsaw method, are sometimes recommended for specific activities. These are listed as Differentiation Strategies.

More information about grouping strategies is available online in the Texas Support Center at www.CarnegieLearning.com/texas-help

13. Grouping Strategies

Suggestions appear to help chunk each activity into manageable pieces and establish the cadence of the lesson.

Learning is social. Whether students work in pairs or in groups, the critical element is that they are engaged in discussion. Carnegie Learning believes, and research supports, that student-to-student discourse is a motivating factor; it increases student learning and supports ongoing formative assessment. Additionally, it provides students with opportunities to have mathematical authority.

Working collaboratively can, when done well, encourage students to articulate their thinking (resulting in self-explanation) and also provides metacognitive feedback (by reviewing other students' approaches and receiving feedback on their own).

The student discussion is then transported to a classroom discussion facilitated by the teacher to guarantee all necessary mathematics is addressed, once again, with the same benefits of discussion.

Activity 2.1 Solving a Two-Step Equation



DEVELOP

Facilitation Notes

In this activity, students recall a scenario from the previous lesson and analyze how to solve the equation representing the situation using a double number line. Students describe the operations used in each step and interpret the solution. Students then solve a two-step equation on their own using a double number line.

Ask a student to read the introductory paragraphs aloud. Discuss as a class.

13

Have students work with a partner or in groups to read through the Worked Example and answer Questions 1 through 3. Share responses as a class.

14

Differentiation strategies

- Have students interact with the Worked Example. Have them letter the steps so that they can recall the process used; this also coincides with the notation used in Activity 2.3. It may also be helpful for students to write the operation used above each shaded portion of the number line.
- To support students who struggle, have them write a list of generalized steps to solve future problems.

Questions to ask

- What are the first steps used in any double number line problem?
- Does it matter if the 0 or 0x is labeled in the top number line?
- Where do you get the expressions to place on the number line?
- When placing the equivalent relationship on the double number line, why does it make sense to place the constant, rather the algebraic expression first?
- Why can't you tell by looking only at the algebraic expression if the expression lies to the left or right of 0?
- What is the goal of using double number lines?
- How do you know when you have solved the equation?

Differentiation strategy

To extend the activity, ask students to solve the problem another way, basically to divide first and then subtract. Check that students divide both terms in the expression by 2. Compare answers and methods.

Have students work with a partner or in groups to complete Question 4. Share responses as a class.

Questions to ask

- How do you know when you have solved the equation?

LESSON 2: Expressions That Play Together . . . • 5

ould have given you the

equivalent expressions on
lines can then be used to



variety of equations using
describe the steps and
operations at each step.
olving equations using a

roups to complete
class.

urage students to set
ven equation and not to
es. Throughout this lesson
d peer analysis problems
s to identify equations such

division first, then subtraction. Later in the module, students will be asked to generalize the solution of the equation $a(x + b) = c$ as $x = \frac{c}{a} - b$. While it may be worthwhile to address both solution paths, please do not address this type of question exclusively through distribution.

Questions to ask

- Where do you get the expressions to place on the number line?
- How did you know whether to place the expressions to the left or right of 0?
- How did you decide what operations to use?
- Is there another set of equalities you could have used to solve this equation?
- Why is each resulting pair of expressions equivalent?

6 • TOPIC 3: Two-Step Equations and Inequalities

14. Differentiation Strategies

To assist all students, instructional strategies are provided that benefit the full range of learners.

- How do you know when you have solved the equation?
- What method is most efficient?
- How can you check your solution?

Summary

Double number lines can be used to solve equations. The operations to solve the equations will vary depending upon the expressions in the equations.

Activity 2.3

Reasoning with Negatives to Solve Equations



Facilitation Notes

In this activity, an equation is given to students, and students are used to solve. A number line is given. Students are asked to solve the equation on the number line and explain their process.

Have students work with a partner or in groups to complete Questions 1 through 3. Share responses as a class.

Differentiation strategies

- Provide each student with a number line so that they can “act out” the process and write the solution.
- To scaffold support from A to B, provide equivalent expressions for division by 2, then by 3, and use addition of opposites. Division is more challenging, especially when the denominator is a fraction.

Questions to ask

- Explain how you moved the number line.
- Does dividing by -3 result in the same result? Explain.
- What is another set of numbers that solve this equation?
- Does it matter where you reflect across 0 on the number line?

15

Have students work with a partner or in groups to complete Question 4. Share responses as a class.

Questions to ask

- Can you reflect an expression other than $-x$ across 0 on the number line?
- What would $-x + 10$ be if you reflected it across 0 on the number line?
- How did you handle moving from $\frac{3}{4}x$ to $1x$?
- What do your final double number lines look like in these 3 problems. Why do you think that is the case?
- Does it matter how many steps it takes to solve the equation? Explain.
- Does the order of your steps to solve the equation matter? Explain.

16

Misconception

When dealing with the expression $\frac{3}{4}x = 15$, students may think a viable option is to subtract $\frac{3}{4}x$, resulting in $0x = 15 - \frac{3}{4}x$. Revisit the goal of getting to $1x$ on the number line and how this would not be a productive move.

Differentiation strategies

- To scaffold support, they may need support dealing with the fraction, $-\frac{3}{4}$, in Question 4 part (b).
 - Have them refer to Getting Started Question 5. In this case, they used scaling to get equivalent expressions for $\frac{3}{4}x = 21$, although they did not need to write an equivalent expression using $1x$.
 - Suggest a scaling up strategy for $-\frac{3}{4}x = 15$: Multiplying both expressions by 4 results in $-3x = 60$, then dividing by 3 results in $-1x = 20$, then reflecting across 0 results in $x = -20$.
 - Suggest a scaling down strategy for $-\frac{3}{4}x = 15$: Students might find it easier to reflect the expression across 0 on the number line first, resulting in $\frac{3}{4}x = -15$. Then, they could get the numerator to have a value of 1 by dividing by 3, resulting in $\frac{1}{4}x = -5$. Lastly, they could multiply by 4 resulting in $x = -20$.
 - Discuss what strategy makes most sense to students.
- To extend the activity, have students investigate a case where $x = 0$. For example, have students solve $4x + 8 = 8$.

15. Questions to Ask

The overarching questioning strategies throughout each lesson promote analysis and higher-order thinking skills beyond simple yes or no responses. These questions can be used to gather information, probe thinking, make the mathematics explicit, and encourage reflection and justification as students are working together or when they are sharing responses as a class. These questions are an embedded formative assessment strategy to provide feedback as students are actively engaged in learning.

16. Misconceptions

Common student misconceptions are provided in places where students may overgeneralize mathematical relationships or have confusion over the vocabulary used. Suggestions are provided to address the given misconception.

Note

Talk the Talk helps you to assess student learning and to make decisions about helpful connections you need to make in future lessons.

17. White Space

The white space in each margin is intentional. Use this space to make additional planning notes or to reflect on the implementation of the lesson.

18. Summary

The summary brings the activity to closure. This statement encapsulates the big mathematical ideas of the particular activity.

Summary

When double number lines are used to solve equations containing a negative coefficient of x , reflection across 0 on the number line is necessary.

Talk the Talk: Keeping It Together

DEMONSTRATE

Facilitation Notes

In this activity, students start with an unknown solution and generate an equation. They describe the steps they use to compose the equation. Students then record the steps their partner uses to solve the equation, noting that these steps can be the opposite of the steps used to compose the equation. Finally, students reflect on what it means to maintain equality when solving equations.

Have students work with a partner to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- When solving your equation, does it matter if you add/subtract first or divide first?
- How would the steps be different if your classmate multiplied by a fraction to create the equation?
- Why do you think this idea of reversing the process solves the equation?

17

18

Summary

When solving an equation, you reverse the operations used to create the equation and maintain equal expressions throughout the process.

Supporting Emergent Bilingual Students

Emergent bilingual students often face multiple challenges in the mathematics classroom beyond language development skills, including a lack of confidence, peer-to-peer understanding, and building solid conceptual mastery. The Carnegie Learning Texas Math Solution seeks to support emergent bilingual students as they develop skills in both mathematics and language.

Throughout instruction, ELL Tips are placed for teachers at point-of-use on the mini-lesson page in the TIG. They provide additional modifications to support this special population.

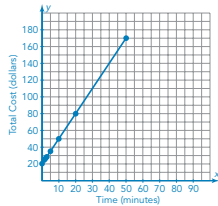
These tips:

- Inform teachers of potential learning obstacles specific to the lesson.
- Provide engaging activities for learning and assessment.
- Reinforce newly acquired mathematical language to gain an increasing level of comprehension of English.
- Introduce students to language needed to understand a specific context.

Students internalize new content language by using and reusing it in meaningful ways in a variety of different speaking activities that build concept and language attainment.

Answers

1. The unit rate of change is \$3 since the cost increases by \$3 for every minute.
2. Sample answer.
 $c = 20 + 3t$
3. The remaining three entries should be 50, 80, and 170.
- 4.



ACTIVITY
3.3

Starting with a Table to Solve a Problem



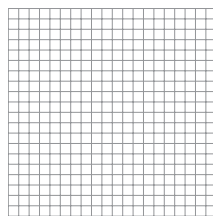
Herman and Melville found this table. The bottom three entries in the second column were smudged, and the boys couldn't read them.

Time (minutes)	Total Cost (dollars)
0	20
1	23
2	26
3	29
5	35
10	
20	
50	

Let's see if you can calculate the unknown values.

1. What is the unit rate of change shown in the table? Explain your reasoning.
2. Define variables for the quantities in the table, and write an equation that relates the two quantities.
3. Use your equation to complete the table. Show your work.

4. Use your completed table to construct a graph.



8 • TOPIC 4: Multiple Representations of Equations

ELL Tip

Support students in understanding unit rate by relating the meaning to common unit rates found in our everyday lives. Share and explain examples of unit rates, such as 60 miles per hour or \$0.99 per pound. Ask students to share other examples of unit rates.

18 • TOPIC 4: Multiple Representations of Equations

For More Support

Visit the Texas Support Center for many more resources to support you and your students who are emergent bilingual students.



Assessments

Formative assessment tools are provided throughout each lesson, providing you with ongoing feedback of student performance and encouraging students to monitor their own progress. End of topic summative assessments are provided to measure student performance on a clearly denoted set of standards. For certain topics that extend longer than four instructional weeks, a mid-topic summative assessment is also provided.

Enhanced End of Topic Assessment

There are three problem type sections per assessment. Multiple-choice questions, open-response questions, and griddable response questions prepare students for enhanced standardized tests.

The answer key provides teachers with the TEKS aligned to each question, as well as sample answers for open-response and griddable response questions.

Topic 1
CIRCLES AND RATIO

Enhanced End of Topic Assessment

Name _____ Date _____

Part A: Multiple-Choice Questions

TEKS 7.9B

1. Approximately how much fencing is needed to enclose a circular pond with a diameter of 12.5 feet?

- 122.66 feet
- 19.625 feet
- 78.5 feet
- ★ 39.25 feet

TEKS 7.4B, 7.9B

2. A circular marble tabletop has price per square inch?

- \$0.05 per square inch
- ★ \$0.20 per square inch
- \$4.25 per square inch
- \$4.95 per square inch

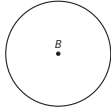
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Topic 1
CIRCLES AND RATIO

Part B: Open-Response Questions

TEKS 7.5B

6. Write an expression that is equivalent to x using the given information.



The circumference of Circle B is 94.2 square centimeters.

$$C = 2\pi r$$

$$94.2 = 2\pi r$$

$$r = \frac{94.2}{2\pi}$$

TEKS 7.9B

8. Compute the diameter of a circle with a circumference of 25 feet.

$$d = \frac{C}{\pi}$$

$$d = \frac{25}{3.14} \approx 7.96$$

The diameter is approximately 7.96 feet.

4 • MODULE 1: THINKING PROPORTIONALLY

Topic 1
CIRCLES AND RATIO

Part C: Griddable Response Questions

Record your answer and fill in the bubbles. Be sure to use correct place value.

TEKS 7.9B

13. Zach ran 5 laps around a circular track with a diameter of 150 feet. How far did he run? Use 3.14 for π .

2355 feet

Sample griddable response.

+	2	3	5	5	.	
⊙	⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙	⊙

TEKS 7.9C

14. Justin wants to cover the top of a circular swimming pool to keep insects out of the pool. He needs to know the area of the pool to order the correct size cover. If the diameter of the pool is 20 feet, what is the area of the pool? Use 3.14 for π .

314 square feet

Sample griddable response.

+	3	1	4	.	
⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙
⊙	⊙	⊙	⊙	⊙	⊙

6 • MODULE 1: THINKING PROPORTIONALLY

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Supporting Students to Use Mathematical Tools

Visit the Texas Support Center for strategies to support students as they use mathematical tools, including formula charts and reference sheets.

End of Course Topic

The End of Course Topic is the final topic of the course which includes a collection of problem-based performance tasks that are aligned with selected priority math standards of the course. This final topic provides students an additional opportunity to demonstrate their ability to make sense of multi-step, real-world problems, communicate their thinking, represent solutions, and justify their reasoning on content aligned with these selected math standards.

Performance Tasks

Each performance task is a formative assessment tool that allows students to demonstrate their learning of the selected course content. At the end of each task, a section titled “Your Work Should Include” lists the categories and the corresponding maximum scoring points from the grading rubric.

1
Performance Task

Lawn Boy

Greg has a job mowing lawns. He creates a table so that he can easily tell his parents how much he is earning.

Time (hours)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$
Amount Earned (\$)							64.75		

- Complete the table.
- How much does Greg earn per hour? Explain how to determine the unit rate from the table.

- Define your variables and write an equation to represent the relationship between Greg's earnings and the time worked.

- Greg's parents tell him if he works more than 60 hours in May, they will buy him a new lawn mower. In the month of May, Greg earns \$1,387.50. Use your equation to determine if Greg's parents will buy him a new lawn mower.

Your work should include:

- A completed table (3 points)
- An equation representing the situation with the variables defined (3 points)
- An explanation using math terms of how your equation relates to the table (3 points)
- Your calculations (3 points)
- An explanation about whether Greg's parents will buy him a new lawn mower (3 points)

Grading Rubric

The grading rubric is for students and teachers to set clear expectations for how each completed performance task will be evaluated. Students should use the rubric to guide their work and self-monitor their progress. Teachers should use the rubric to evaluate and provide feedback for the completed performance task.

RUBRIC: 15 Total Points

	0 points	1 point	2 points	3 points
Table	No table is shown.	Table is shown but is incomplete or incorrect.	Table is mostly complete and correct, but with minor errors.	Table is complete and correct.
Equation and Variables	There is no equation and the variables are not defined.	The equation is incorrect and/or variables are not correctly defined.	There is a minor error in the equation or the way the variables are defined.	The equation is correct and the variables are correctly defined.
Explanation	No explanation is given.	Explanation given uses no math terms.	Explanation includes math term(s), but does not correspond to the table.	Explanation corresponding to the table is complete and includes math term(s).
Calculations	No calculations are shown.	Calculations are shown, but include significant errors.	Calculations are shown, but include minor errors.	Calculations are shown and are complete and correct.
Statement	No statement is given.	Statement incorrectly gives Greg's unit rate.	Statement identifies correct unit rate, but does not explain whether Greg's parents will buy him a new lawn mower.	Statement correctly identifies unit rate and correctly explains why Greg's parents will buy him a new lawn mower.

Teacher's Implementation Guide

The Teacher's Implementation Guide for the End of Course Topic contains a performance task overview, list of aligned TEKS and ELPS, essential ideas, facilitation notes which describe how to pace the two-day performance task, sample answer, and grading rubric.

Performance Task 1
Lawn Boy

MATERIALS

- Graphing technology
- Allow students to have access to any additional materials that may assist in the completion of the task.

Performance Task Overview

Students are presented with a scenario about Greg, who has a job mowing lawns. They are given a rate table that includes the time worked and earnings. Using the values provided, students determine the unit rate for the earnings per hour and complete the table. Students define variables and write an equation in the form $y = kx$ to represent the relationship between Greg's earnings and the time worked. They use the equation to determine the number of hours worked when given a monthly dollar amount. Students then determine whether Greg worked enough hours to have his parents buy him a new lawn mower.

Grade 7
Number and Operations

(3) The student applies mathematical process standards to add, subtract, multiply, and divide while solving problems and justifying solutions. The student is expected to:

(B) apply and extend previous understandings of operations to solve problems using addition, subtraction, multiplication, and division of rational numbers.

Proportionality

(4) **The student applies mathematical process standards to represent and solve problems involving proportional relationships. The student is expected to:**

(A) represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including $d = rt$.

(B) calculate unit rates from rates in mathematical and real-world problems.

(C) solve problems involving ratios, rates, and percents, including multi-step problems involving percent increase and percent decrease, and financial literacy problems.

ELPS

1.A, 1.B, 1.C, 1.D, 1.F, 2.B, 2.C, 2.D, 2.I, 3.A, 3.B, 3.C, 3.D, 3.E, 4.D, 4.H, 4.I, 4.J, 4.K, 5.B, 5.C, 5.E, 5.F, 5.G

END-OF-COURSE TOPIC: Performance Task 1 • 1

SAMPLE ANSWER

According to the table, Greg earns \$64.75 for working $3\frac{1}{2}$ hours. To find the unit rate, I need to determine the amount of money that Greg earns for working 1 hour. I can take the money earned in $3\frac{1}{2}$ hours, and divide it by $3\frac{1}{2}$ hours to figure out how much money Greg earns per hour.

$$64.75 \div 3.5 = 18.50$$

So, Greg earns \$18.50 for each hour he mows lawns.

Let a represent the amount of money earned, and let t represent the number of hours worked. $a = 18.5t$

To complete the table, multiply the unit rate of \$18.50 by each time amount Greg worked.

Time (hours)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	4.5
Amount Earned (\$)	9.25	18.50	27.75	37.00	46.25	55.50	64.75	74.00	83.25

In May, Greg earned \$1,387.50. To determine how many hours of lawn mowing this equals, use your equation to determine the value of t when $a = 1,387.50$.

$$a = 18.5t$$

$$1,387.50 = 18.5t$$

$$\frac{1,387.50}{18.5} = \frac{18.5t}{18.5}$$

$$t = 75$$

Greg worked 75 hours in May. Since 75 hours is greater than 60 hours, Greg's parents will buy him a new lawn mower.

Similar to the other topics in this course, the End of Course Topic also has a Topic Family Guide for students and caregivers, and a Topic Overview for teachers. The End of Course Topic does not include an end of topic assessment since each performance task is a formative assessment.

Getting Ready

Carnegie Learning recognizes that it is the classroom teachers who make the material come alive for students, transforming the way math is taught. Implementation requires integrating learning together and learning individually.

Prepare for Learning Together

The most important first step you can take in preparing to teach with these instructional materials is to become comfortable with the mathematics.

- Read through the Module 1 Overview and the Topic 1 Overview.
- Do the math of the first Topic, and consider the facilitation notes.
- Prepare team-building activities to intentionally create a student-centered environment.

Prepare for Learning Individually

Plan how you will utilize Skills Practice as a Learning Individually resource. Then, determine how you will introduce Skills Practice to students. Explain to them the benefits of working individually and why practice is important.

- Read through Module 1 Topic 1 Skills Practice.
- Determine which problem sets align with the activities in the corresponding student lessons.
- Based on student performance in the lesson, be prepared to assign the class, small groups of students, or individual students different problem sets to practice skills to develop mastery.

Plan how you will introduce students to MATHia. Explain to them the benefits of working individually and why practice is important.

- Test out the computers or tablets that your students will be using.
- Verify your classes have been set up in Teacher's Toolkit with correct MATHia content assigned. Or manually set up your classes in Teacher's Toolkit if applicable.
- Use the Content Browser in Teacher's Toolkit to explore the content students are assigned.
- Be prepared to demonstrate how students will access and log into MATHia.

PREPARE YOURSELF

PREPARE YOUR CLASSROOM

Prepare the Environment

The classroom is often considered the third teacher. Consider how to create a learning environment that engages students and fosters a sense of ownership. The use of space in your classroom should be flexible and encourage open sharing of ideas.

- Consider how your students are going to use the consumable book. It is the student's record of their learning. Many teachers have students move an entire topic to a three-ring binder as opposed to carrying the entire book.
- Arrange your desks so students can talk and collaborate with each other.
- Prepare a toolkit for groups to use as they work together and share their reasoning (read the materials list in each Topic Overview).
- Consider where you will display student work, both complete and in-progress.
- Create a word wall of key terms used in the textbook.

PREPARE YOUR STUDENTS

Prepare the Learners

If you expect students to work well together, they need to understand what it means to collaborate and how it will benefit them. It is important to establish classroom guidelines and structure groups to create a community of learners.

- Facilitate team-building activities and encourage students to learn each others' names.
- Set clear expectations for how the class will interact:
 - Their text is a record of their learning and is to be used as a reference for any assignments or tests you give.
 - They will be doing the thinking, talking, and writing in your classroom.
 - They will be working and sharing their strategies and reasoning with their peers.
 - Mistakes and struggles are normal and necessary.

PREPARE FAMILIES AND CAREGIVERS

Prepare the Support

- Prepare a letter to send home on the first day. Visit the Texas Support Center for a sample letter.
- Encourage families and caregivers to read the introduction of the student book.
- Ensure that families and caregivers receive the module Family and Caregiver guide at the start of each module. They should also receive the topic Family Guide at the start of the first topic and each subsequent topic.
- Consider a Family Math Night some time within the first few weeks of the school year.
- Encourage families and caregivers to explore the Students & Caregivers Portal on the Texas Support Center at www.CarnegieLearning.com/texas-help/students-caregivers.

Topic Family Guides

Each topic contains a Family Guide that provides an overview of the mathematics of the topic, how that math is connected to what students already know, and how that knowledge will be used in future learning. It provides families and caregivers an example of a math model or strategy their student is learning in the topic, busting of a math myth, questions to ask their student to support their learning, and a few of the key terms their student will learn.

We recognize that learning outside of the classroom is crucial to students' success at school. While we don't expect families and caregivers to be math teachers, the Family Guides are designed to assist families and caregivers as they talk to their students about what they are learning. Our hope is that both the students and their caregivers will read and benefit from the guides.

Carnegie Learning Family Guide Grade 7
Module 3: Reasoning Algebraically

TOPIC 3: MULTIPLE REPRESENTATIONS OF EQUATIONS
This topic broadens students' perspective on solving and interpreting linear equations and inequalities through the use of tables and graphs. Students write and solve two-step equations using positive and negative numbers on four-quadrant graphs. Students then compare graphs of linear equations in different forms. Finally, students practice solving problems by writing equations and inequalities for problem situations, analyzing tables and graphs to solve the equations or inequalities, and interpreting the quantities in each problem situation.

Where have we been?
In grade 6, students used multiple representations to model and solve problems, primarily one-step equations. They learned that quantities can vary in relation to each other and are often classified as independent and dependent quantities.

Where are we going?
Students' ability to use symbolic algebra can be supported through the use of visual representations. Using and connecting symbolic and graphical representations of equations and inequalities occurs throughout the study of functions in grade 8 and in high school.

Interpreting Situations in More Than One Quadrant
This graph shows the relationship between the time someone has owned a car, t , and the value of the car, v . We only have information on the values to the right of the vertical axis, but if we assume that the relationship is linear, we can use an equation to determine car values for negative time values.

TOPIC 3: Family Guide • M3-123

Myth: Memory is like an audio or video recording.

Let's play a game. Memorize the following list of words: strawberry, grape, watermelon, banana, orange, peach, cherry, blueberry, raspberry. Got it? Good. Some believe that the brain stores memories in pristine form. Memories last for a long time and do not change—like a recording. Without looking back at the original list, was apple on it?

If you answered "yes," then go back and look at the list. You'll see that apple does not appear, even though it seems like it should. In other words, memory is an active, reconstructive process that takes additional information, like the category of words (e.g., fruit), and makes assumptions about the stored information.

This simple demonstration suggests memory is not like a recording. Instead, it is influenced by prior knowledge and decays over time. Therefore, students need to see and engage with the same information multiple times to minimize forgetting (and distortions).

#mathmythbusted

Talking Points
You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to represent relationships involving the equivalence of values in a variety of ways.

Questions to Ask

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

Key Term
unit rate of change
The unit rate of change is the amount that the dependent value changes for every one unit that the independent value changes.

M3-124 • TOPIC 3: Multiple Representations of Equations

You Might Be Wondering...

Why are the student books consumable?

The Student Textbook contains all of the resources students need to complete the Learning Together component of the course. Students are to actively engage in this textbook, topic by topic, creating a record of their learning as they go. There is room to record answers, take notes, draw diagrams, and fix mistakes.

Why do we believe in our brand of blended: Learning Together and Learning Individually?

There has been a lot of research on the benefits of learning collaboratively. Independent practice is necessary for students to become fluent and automatic in a skill. A balance of these two pieces provides students with the opportunity to develop a deep conceptual understanding through collaboration with their peers, while demonstrating their understanding independently.

Why don't we have a Worked Example at the start of every lesson?

Throughout the Texas Math Solution, we do provide Worked Examples. Sweller and Cooper (1985) argue that Worked Examples are educationally efficient because they reduce working memory load. Ward and Sweller (1990) found that alternating between problem solving and viewing Worked Examples led to the best learning. Students often read Worked Examples with the intent to confirm that they understand the individual steps. However, the educational value of the Worked Example often lies in thinking about how the steps connect to each other and how particular steps might be added, omitted, or changed, depending on context.

Where are the colorful graphics to get students' attention?

Color and visuals make for stronger student engagement, right? Not quite. Our instructional materials have little extraneous material. This approach follows from research showing that "seductive details" used to spice up the presentation of material often have a negative effect on student learning (Mayer et al., 2001; Harp & Meyer, 1998). Students may not know which elements of an instructional presentation are essential and which are intended simply to provide visual interest. So, we focus on the essential materials. While we strive to make our educational materials attractive and engaging to students, research shows that only engagement based on the mathematical content leads to learning.

We're here for you.

The Carnegie Learning Texas Support Team is available to help with any issue at help@carnegielearning.com.

**Monday–Friday
8:00 am–8:00 pm CT**
via email, phone, or live chat.

Our expert team provides support for installations, networking, and technical issues, and can also help with general questions related to pedagogy, classroom management, content, and curricula.

