TEXAS MATH SOLUTION

Grade 8

Teacher's Implementation Guide

Skills Program Edition

SY 2022-2023

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with Kelly Edenfield and Josh Fisher

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Our Manifesto

WE BELIEVE that quality math education is important for all students, to help them develop into creative problem solvers, critical thinkers, life-long learners, and more capable adults.

WE BELIEVE that math education is about more than memorizing equations or performing on tests—it's about delivering the deep conceptual learning that supports ongoing growth and future development.

WE BELIEVE all students learn math best when teachers believe in them, expect them to participate, and encourage them to own their learning.

WE BELIEVE teachers are fundamental to student success and need powerful, flexible resources and support to build dynamic cultures of collaborative learning.

WE BELIEVE our learning solutions and services can help accomplish this, and that by working together with educators and communities we serve, we guide the way to better math learning.

LONG + LIVE + MATH

At Carnegie Learning, we choose the path that has been proven most effective by research and classroom experience. We call that path the Carnegie Learning Way. Follow this code to take a look inside.

Acknowledgments

Middle School Math Solution Authors

- Sandy Bartle Finocchi, Chief Mathematics Officer
- Amy Jones Lewis, Senior Director of Instructional Design
- Kelly Edenfield, Instructional Designer
- Josh Fisher, Instructional Designer

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"Mathematics is so much more than rules and algorithms. It is learning to reason, to make connections, and to make sense of the world. We believe in Learning by Doing™—students need to actively engage with the content if they are to benefit from it. Your classroom environment will determine what type of discourse, questioning, and sharing will take place. Students deserve a safe place to talk, to make mistakes, and to build deep understanding of mathematics. My hope is that these instructional materials help you shift the mathematical authority in your class to your students. Be mindful to facilitate conversations that enhance trust and reduce fear."

Sandy Bartle Finocchi, Chief Mathematics Officer

"My hope is that you know that your students are capable of thinking like mathematicians. This book is designed to give them the opportunity to struggle with challenging tasks, to talk about math with their classmates, and to make and fix mistakes. I hope that you use this book to build this capacity in your students—to ask the necessary questions to uncover what students already know and connect it to what they are learning, to encourage creative thinking, and to give just enough support to keep students on the right path."

Amy Jones Lewis, Senior Director of Instructional Design

"At Carnegie Learning, we have created an organization whose mission and culture is defined by student success. Our passion is creating products that make sense of the world of mathematics and ignite a passion in students. Our hope is that students will enjoy our resources as much as we enjoyed creating them."

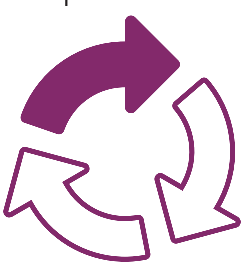
Barry Malkin, CEO

The Carnegie Learning Way

At Carnegie Learning, we choose the path that has been proven most effective by research and classroom experience. We call that path the **Carnegie Learning Way.**

Our Instructional Approach

Carnegie Learning's instructional approach is a culmination of the collective knowledge of our researchers, instructional designers, cognitive learning scientists, and master practitioners. It is based on a scientific understanding of how people learn, as well as an understanding of how to apply the science to the classroom.
At its core, our instructional approach is based on three simple, key components:



ENGAGE

Activate student thinking by tapping into prior knowledge and real-world experiences. Provide an introduction that generates curiosity and plants the seeds for deeper learning.

DEVELOP

Build a deep understanding of mathematics through a variety of activities. Students encounter real-world problems, sorting activities, Worked Examples, and peer analysis—in an environment where collaboration, conversations, and questioning are routine practices.

DEMONSTRATE

Reflect on and evaluate what was learned. Ongoing formative assessment underlies the entire learning experience, driving real-time adjustments, next steps, insights, and measurements.

Our Research

Carnegie Learning has been deeply immersed in research ever since it was founded by cognitive and computer scientists from Carnegie Mellon University. Our research extends far beyond our own walls, playing an active role in the constantly evolving field of cognitive and learning science. Our internal researchers collaborate with a variety of independent research organizations, tirelessly working to understand more about how people learn, and how learning is best facilitated. We supplement this information with feedback and data from our own products, teachers, and students, to continuously evaluate and elevate our instructional approach and its delivery.

Our Support

We're all in. In addition to our books and software, implementing Carnegie Learning in your classroom means you get access to an entire ecosystem of ongoing classroom support, including:

Professional Learning: Our team of Master Math Practitioners is always there for you, from implementation to math academies to a variety of other options to help you hone your teaching practice.

Texas Support Center: We've customized a Support Center just for you and your students. The Texas Support Center provides articles and videos to help you implement the Texas Math Solution, from the basics to get you started to more targeted support to guide you as you scaffold instruction for all learners in your classroom. Visit **www.CarnegieLearning.com/texas-help** to explore online and to access content that you can also share with your students and their caregivers.

MyCL: This is the central hub that gives you access to all of the products and resources that you and your students will need. Visit MyCL at **www.CarnegieLearning.com/login**.

LONG + LIVE + MATH: When you join this community of like-minded math educators, suddenly you're not alone. You're part of a collective, with access to special content, events, meetups, book clubs, and more. Because it's a community, it's constantly evolving! Visit **www.longlivemath.com** to get started.

Scan this code to visit the Texas Support Center and look for references throughout the Front Matter to learn more about the robust resources you will find in the Support Center.

Our Blend of Learning

The Texas Math Solution delivers instructional resources that make learning math attainable for all students. Learning Together and Learning Individually resources work in parallel to engage students with various learning experiences they need to understand the mathematics at each grade level.

For **Learning Together**, the student textbook is a consumable resource that empowers students to become creators of their mathematical knowledge. This resource is designed to support teachers in facilitating active learning so that students feel confident in sharing ideas, listening to each other, and learning together.

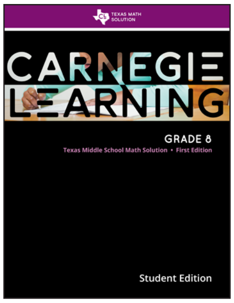
Over the course of a year, based on the recommended pacing, teachers will spend approximately 60% of their instructional time teaching whole-class activities as students learn together.

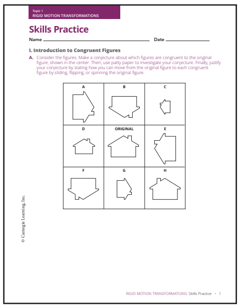
For **Learning Individually**, the Skills Practice provides students the opportunity to engage with problems that target each lesson's skills, concepts, and applications. This resource is designed to target discrete skills for development and mastery, therefore, scaffolding and extension opportunities are provided in the problem sets.

An additional Learning Individually resource is MATHia®, an intelligent software that provides just-in-time support and tracks student progress against fine-grained skills to deliver the right content they need to become proficient with the mathematics.

Over the course of the year, based on the recommended pacing, teachers will spend approximately 40% of their instructional time monitoring students as they work and learn individually.

Learning Together | Learning Individually





TEXTBOOK

I am a record of student thinking, reasoning, and problem solving. My lessons allow students to build new knowledge based upon prior knowledge and experiences, apply math to real-world situations, and learn together in a collaborative classroom.

My purpose is to create mathematical thinkers who are active learners that participate in class.

SKILLS PRACTICE

I am targeted practice of each lesson's skills, mathematical concepts, and applications for each topic in the student textbook.

My purpose is to provide additional problem sets for teachers to assign as needed for additional practice or remediation.

MATHia

I am designed to empower students to learn individually at their own pace with sophisticated AI technology that personalizes their learning experiences, while giving teachers real-time insights to monitor student progress.

My purpose is to coach students alongside teachers as students learn, practice, do, and look forward.

Visit the Texas Support Center for additional information on the Learning Individually resources.

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Module 1: Transforming Geometric Objects

Module 2: Developing Function Foundations

Module 3: Data Data Everywhere

Topic 1: Patterns in Bivariate Data

Topic 2: Variablility and Sampling

Module 4: Modeling Linear Equations

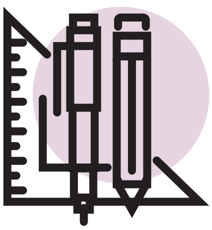
Topic 1: Solving Linear Equations

Module 5: Applying Powers

End of Course Topic

Instructional Design

In a word, every single piece of Carnegie Learning's Texas Math Solution is **intentional.** Our instructional designers work alongside our master math practitioners, cognitive scientists, and researchers to intentionally design, draft, debate, test, and revise every piece, incorporating the latest in learning science.



Intentional Mathematics Design

Carnegie Learning's Texas Math Solution is thoroughly and thoughtfully designed to ensure students build the foundation they'll need to experience ongoing growth in mathematics.

Mathematical Coherence: The MSMS arc of mathematics develops coherently, building understanding by linking together within and across grades, so students can learn concepts more deeply and apply what they've learned to more complex problems going forward.

Mathematical Process Standards: Carnegie Learning is organized around the Mathematical Process Standards to encourage experimentation, creativity, and false starts, which is critical if we expect students to tackle difficult problems in the real world, and persevere when they struggle.

Multiple Representations: Carnegie Learning recognizes the importance of connecting multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

Transfer: Carnegie Learning focuses on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.

Texas Math Solution Overview

The instructional materials in the Carnegie Learning Texas Math Solution emphasize active learning and making sense of the mathematics. We ask deep questions that require students to thoroughly understand the mathematical concepts they are learning. We think about how to guide students to connect interrelated ideas in a holistic way to integrate students' understanding with their developing habits of mind.

What are the Carnegie Learning Texas Math Solution guiding principles?

The Texas Math Solution has been strongly influenced by scientific research into the learning process and student motivations for academic success. Its guiding principles are active learning, discourse through collaboration, and personalized learning.

Active Learning: The research makes it clear that students need to actively engage with content in order to benefit from it. Studies show that as instruction moves up the scale from entirely passive to fully interactive, learning becomes more robust. All of the activities we provide for the classroom encourage students to be thoughtful about their work, to consider hypotheses and conclusions from different perspectives, and to build a deep understanding of mathematics. The format of the student text, as a consumable workbook, supports active instruction.

Discourse through Collaborative Learning: Effective collaboration encourages students to articulate their thinking, resulting in self-explanation. Reviewing other students' approaches and receiving feedback on their own provides further metacognitive feedback. Collaborative problem-solving encourages an interactive instructional model, and we have looked to research to provide practical guidance for making collaboration work. The collaborative activities within our lessons are designed to promote active dialogue centered on structured activities.

Personalized Learning: One of the ways to build intrinsic motivation is to relate activities to students' existing interests. Research has proven that problems that capture student interests are more likely to be taken seriously. In the textbook, problems often begin with the students' intuitive understanding of the world and build to an abstract concept, rather than the other way around.

How is the mathematical content delivered to promote productive mathematical processes?

Students deserve math learning that develops them into creative problem solvers, critical thinkers, life-long learners, and more capable adults, while teachers deserve instructional resources that will support them in bringing learning to life. There are three organizing principles that guide these instructional resources.

Seeing Connections: Activities make use of models—e.g., real-world situations, graphs, diagrams, and worked examples—to help students see and make connections between different topics. In each lesson, learning is linked to prior knowledge and experiences so that students build their new understanding on the firm foundation of what they already know. We help students move from concrete representations and an intuitive understanding of the world to more abstract representations and procedures. Activities thus focus on real-world situations to demonstrate the usefulness of mathematics.

Exploring Structure: Questions are phrased in a way that promotes analysis, develops higher-order-thinking skills, and encourages the seeking of mathematical relationships. Students inspect a given figure, equation, or data set, and in each case, they are asked to discern a pattern or structure. We want students to become fluent in seeing how the structure of each representation—verbal, graphic, numerical, and algebraic—reveals properties of the relationship it defines. We want students to become fluent at composing and decomposing expressions, equations, and data sets. As students gain proficiency in manipulating structure, they become capable of comparing, contrasting, composing, decomposing, transforming, solving, representing, clarifying, and defining the characteristics of figures, equations leading to functions, and data sets.

Reflecting and Communicating: A student-centered approach focuses on students thinking about and discussing mathematics as active participants in their own learning. Through articulating their thinking in conversations with a partner, in a group, or as a class, students integrate each piece of new knowledge into their existing cognitive structure. They use new insights to build new connections. Through collaborative activities and the examination of peer work—both within their groups and from examples provided in the lessons—students give and receive feedback, which leads to verifying, clarifying, and/or improving the strategy.

Texas Math Solution Year at a Glance

This Year at a Glance highlights the sequence of topics and the number of blended instructional days (1 day is a 45-minute instructional session) allocated for Grade 8 in the Texas Math Solution. The pacing information also includes time for assessments, providing you with an instructional map that covers 180 days of the school year. As you set out at the beginning of the year, we encourage you to still modify this plan as necessary.

Want More Support Designing Your Long-Term Plan?

You can find this Year at a Glance and additional guidance on planning intentionally and flexibly on the Texas Support Center at **www.CarnegieLearning.com/texas-help**.

Texas Grade 8: Year at a Glance

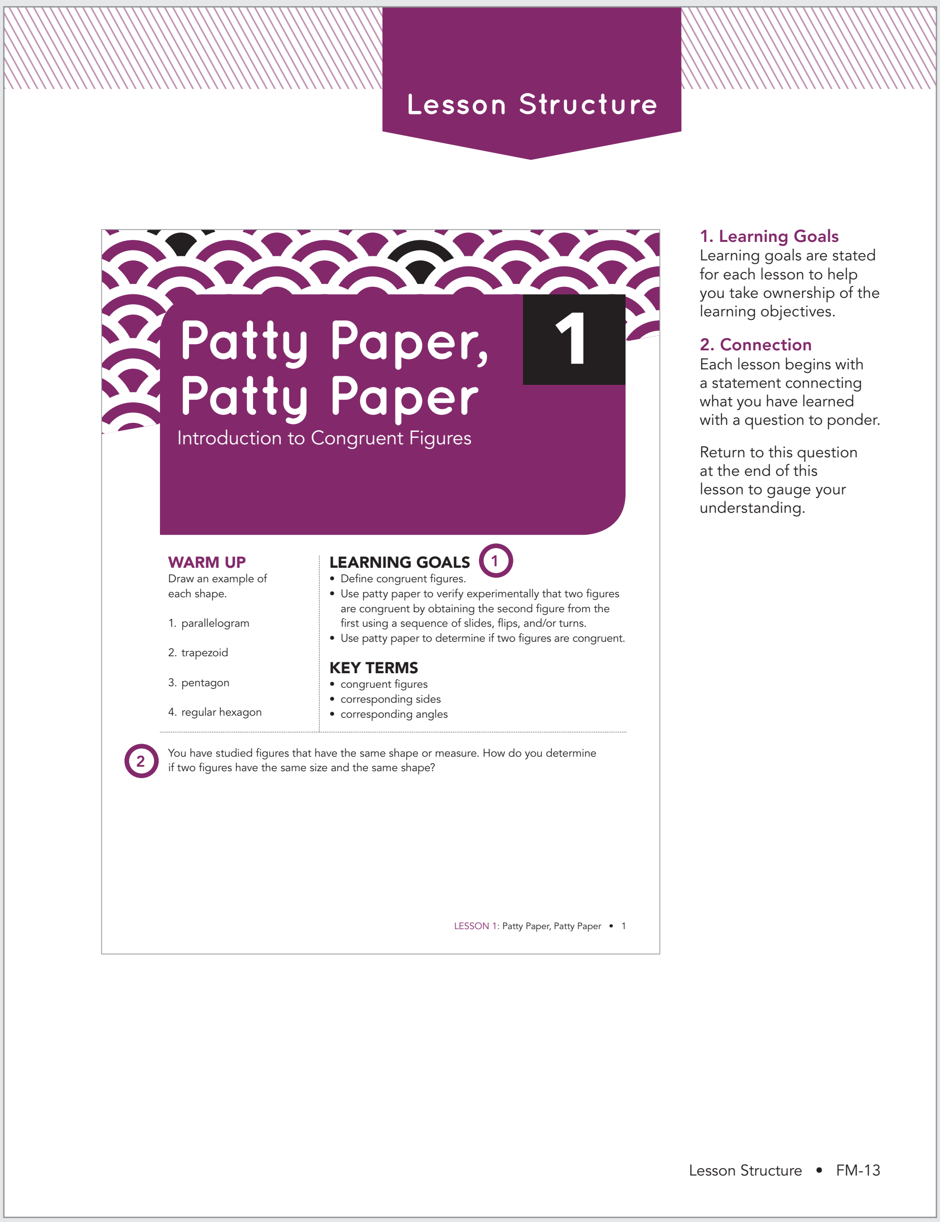
*1 Day Pacing = 45 min. Session

Module	Topic	Pacing	TEKS
Process Standards are embedded in every module: 8.1A, 8.1B, 8.1C, 8.1D, 8.1E, 8.1F, 8.1G			
1 Transforming Geometric Objects	1: Rigid Motion Transformations	20	8.10A, 8.10B, 8.10C
	2: Similarity	10	8.3A, 8.3B, 8.3C, 8.10A, 8.10B, 8.10C, 8.10D
	3: Line and Angle Relationships	9	8.8D
		39	
2 Developing Function Foundations	1: From Proportions to Linear Relationships	17	8.3C, 8.4A, 8.4B, 8.4C, 8.5A, 8.5E, 8.5F, 8.5H, 8.10C, 8.10D
	2: Linear Relationships	14	8.4A, 8.4C, 8.5B, 8.5F, 8.5I
	3: Introduction to Functions	16	8.4C, 8.5B, 8.5F, 8.5G, 8.5I
		47	
3 Data Data Everywhere	1: Patterns in Bivariate Data	10	8.5C, 8.5D, 8.5I, 8.11A
	2: Variability and Sampling	11	8.11B, 8.11C
		21	
4 Modeling LInear Equations	1: Solving Linear Equations	8	8.8A, 8.8B, 8.8C
	2: Systems of Linear Equations	10	8.5B, 8.9A
		18	
5 Applying Powers	1: Real Numbers	11	8.2A, 8.2B, 8.2C, 8.2D
	2: The Pythagorean Theorem	14	8.6C, 8.7C, 8.7D
	3: Financial Literacy: Your Financial Future	8	8.12A, 8.12B, 8.12C, 8.12D, 8.12E, 8.12F, 8.12G
	4: Volume of Curved Figures	14	8.6A, 8.6B, 8.7A, 8.7B
		47	
End of Course Formative Assessment	Performance Tasks	8	8.4A, 8.4B, 8.4C, 8.5A, 8.5B, 8.5F, 8.5I, 8.8A, 8.8C
		8	
	Total Days:	180	

Connecting Content and Practice

Lesson Structure

Each lesson of the Texas Math Solution has the same structure. This consistency allows both you and your students to track your progress through each lesson. Key features of each lesson are noted.



Lesson Structure

Patty Paper, Patty Paper 1

Introduction to Congruent Figures

WARM UP
Draw an example of each shape.

1. parallelogram
2. trapezoid
3. pentagon
4. regular hexagon

LEARNING GOALS (1)
- Define congruent figures.
- Use patty paper to verify experimentally that two figures are congruent by obtaining the second figure from the first using a sequence of slides, flips, and/or turns.
- Use patty paper to determine if two figures are congruent.

KEY TERMS
- congruent figures
- corresponding sides
- corresponding angles

(2) You have studied figures that have the same shape or measure. How do you determine if two figures have the same size and the same shape?

LESSON 1: Patty Paper, Patty Paper • 1

1. Learning Goals
Learning goals are stated for each lesson to help you take ownership of the learning objectives.

2. Connection
Each lesson begins with a statement connecting what you have learned with a question to ponder.

Return to this question at the end of this lesson to gauge your understanding.

Lesson Structure • FM-13

ENGAGE

Establishing Mathematical Goals to Focus Learning

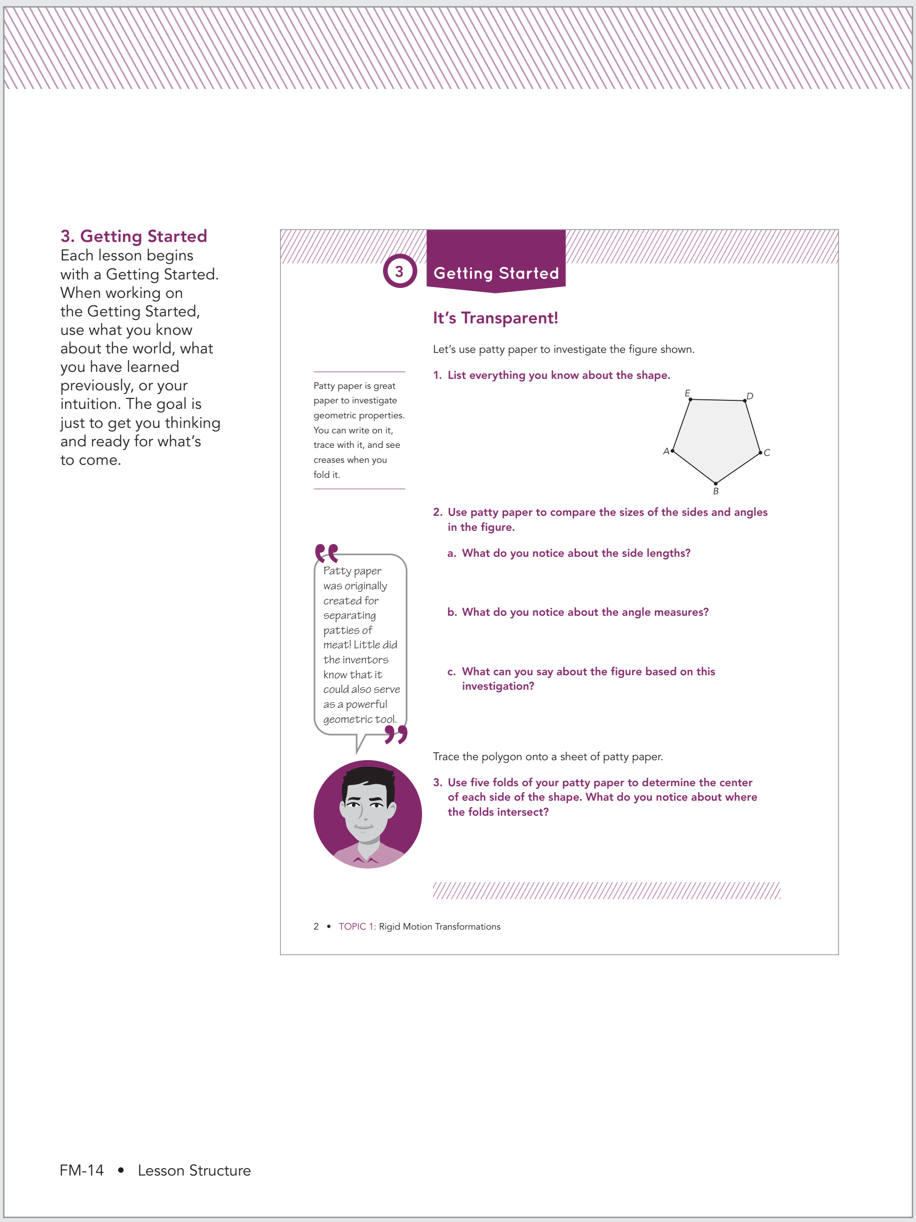
Create a classroom climate of collaboration and establish the learning process as a partnership between you and students.

Communicate continuously with students about the learning goals of the lesson to encourage self-monitoring of their learning.

Visit the Texas Support Center for additional guidance on how to foster a classroom environment that promotes collaboration and communication.

> Mathematics is the science of patterns. So, we encourage students throughout this course to notice, test, and interpret patterns in a variety of ways—to put their "mental tentacles" to work in every lesson, every activity. Our hope is that this book encourages you to do the same for your students, and create an environment in your math classroom where productive and persistent learners develop and thrive.

Josh Fisher, Instructional Designer



3. Getting Started
Each lesson begins with a Getting Started. When working on the Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.

3 Getting Started

It's Transparent!

Let's use patty paper to investigate the figure shown.

Patty paper is great paper to investigate geometric properties. You can write on it, trace with it, and see creases when you fold it.

1. List everything you know about the shape.

2. Use patty paper to compare the sizes of the sides and angles in the figure.
   a. What do you notice about the side lengths?
   b. What do you notice about the angle measures?
   c. What can you say about the figure based on this investigation?

"Patty paper was originally created for separating patties of meat! Little did the inventors know that it could also serve as a powerful geometric tool."

Trace the polygon onto a sheet of patty paper.

3. Use five folds of your patty paper to determine the center of each side of the shape. What do you notice about where the folds intersect?

2 • TOPIC 1: Rigid Motion Transformations

FM-14 • Lesson Structure

Activating Student Thinking

Your students enter each class with varying degrees of experience and mathematical success. The focus of the Getting Started is to tap into prior knowledge and real-world experiences, to generate curiosity, and to plant seeds for deeper learning. Pay particular attention to the strategies students use, for these strategies reveal underlying thought processes and present opportunities for connections as students proceed through the lesson.

Supporting Emergent Bilingual Students

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they complete the Getting Started activities in each lesson.

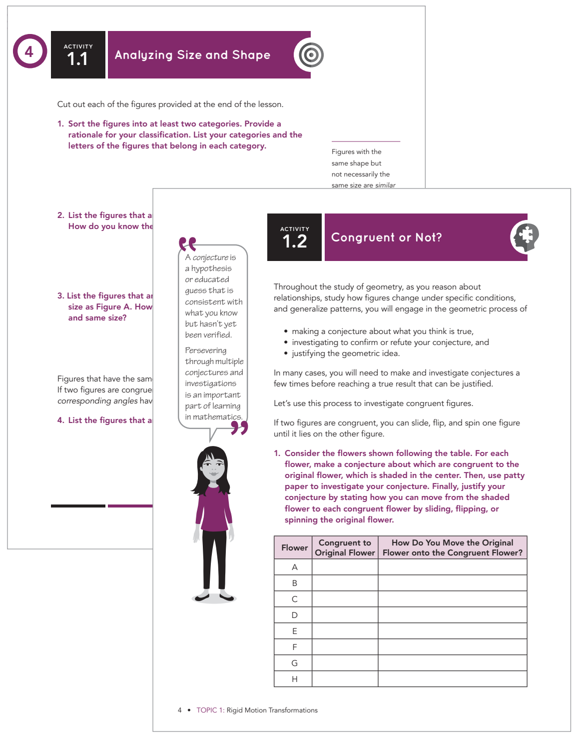
DEVELOP

Aligning Teaching to Learning

Students learn when they are actively engaged in a task: reasoning about the math, writing their solutions, justifying their strategies, and sharing their knowledge with peers.

Support productive struggle by allowing students time to engage with, and persevere through, the mathematics.

Support student-to-student discourse as well as whole-class conversations that elicit and use evidence of student thinking.



4 ACTIVITY 1.1 Analyzing Size and Shape

Cut out each of the figures provided at the end of the lesson.

1. Sort the figures into at least two categories. Provide a rationale for your classification. List your categories and the letters of the figures that belong in each category.

Figures with the same shape but not necessarily the same size are *similar*

2. List the figures that a
How do you know the

3. List the figures that a
size as Figure A. How
and same size?

Figures that have the sam
If two figures are congrue
corresponding angles hav

4. List the figures that a

ACTIVITY 1.2 Congruent or Not?

A *conjecture* is a hypothesis or educated guess that is consistent with what you know but hasn't yet been verified.

Persevering through multiple conjectures and investigations is an important part of learning in mathematics.

Throughout the study of geometry, as you reason about relationships, study how figures change under specific conditions, and generalize patterns, you will engage in the geometric process of

- making a conjecture about what you think is true,
- investigating to confirm or refute your conjecture, and
- justifying the geometric idea.

In many cases, you will need to make and investigate conjectures a few times before reaching a true result that can be justified.

Let's use this process to investigate congruent figures.

If two figures are congruent, you can slide, flip, and spin one figure until it lies on the other figure.

1. Consider the flowers shown following the table. For each flower, make a conjecture about which are congruent to the original flower, which is shaded in the center. Then, use patty paper to investigate your conjecture. Finally, justify your conjecture by stating how you can move from the shaded flower to each congruent flower by sliding, flipping, or spinning the original flower.

Flower	Congruent to Original Flower	How Do You Move the Original Flower onto the Congruent Flower?
A		
B		
C		
D		
E		
F		
G		
H		

4 • TOPIC 1: Rigid Motion Transformations

4. Activities
You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about answer-getting. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, Worked Examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

Lesson Structure • FM-15

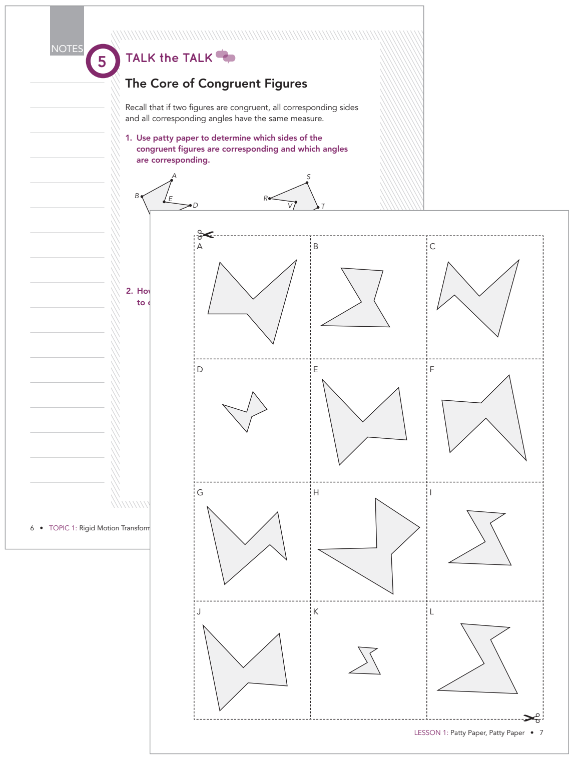
Supporting Emergent Bilingual Students

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they engage in mathematical discourse throughout each lesson.

5. Talk the Talk
Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.



FM-16 • Lesson Structure

DEMONSTRATE

Ongoing Formative Assessment Drives Instruction

For students to take responsibility for their own learning, they need to be encouraged to self-assess. Students can use the Talk the Talk to monitor their own progress towards mastering the learning goals. Listen and review their answers and explanations and provide feedback to help them improve their understanding.

As you plan the next lesson, consider the connections you can make to build off the strengths or fill any gaps identified from this formative assessment.

Student Lesson Overview Videos

Each lesson has a corresponding lesson overview video(s) for students to utilize and reference to support their learning. The videos provide an overview of key concepts, strategies, and/or worked examples from the lessons.

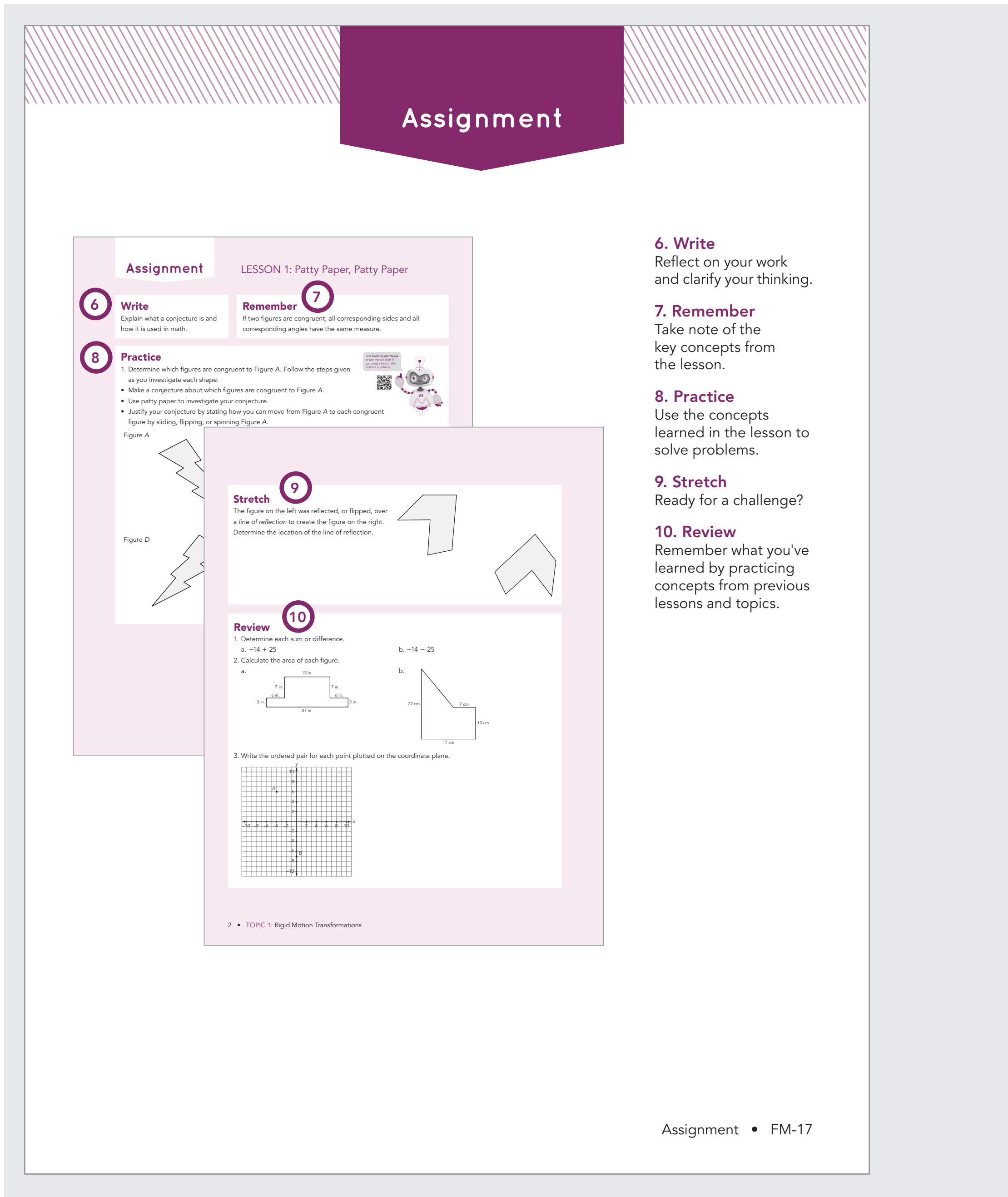
Assignment

An intentionally designed Assignment follows each lesson.

There is one Assignment per lesson. Lessons often span multiple days. Be thoughtful about which portion of the Assignment students can complete based on that day's progress.

The **Stretch** section is not necessarily appropriate for all learners. Assign this to students who are ready for more advanced concepts.

The **Review** section provides spaced practice of concepts from the previous lesson and topic and of the fluency skills important for the course.



Assignment

Assignment LESSON 1: Patty Paper, Patty Paper

6 Write
Explain what a conjecture is and how it is used in math.

7 Remember
If two figures are congruent, all corresponding sides and all corresponding angles have the same measure.

8 Practice
1. Determine which figures are congruent to Figure A. Follow the steps given as you investigate each shape.
- Make a conjecture about which figures are congruent to Figure A.
- Use patty paper to investigate your conjecture.
- Justify your conjecture by stating how you can move from Figure A to each congruent figure by sliding, flipping, or spinning Figure A.

Figure A

Figure D

9 Stretch
The figure on the left was reflected, or flipped, over a *line of reflection* to create the figure on the right. Determine the location of the line of reflection.

10 Review
1. Determine each sum or difference.
a. −14 + 25 b. −14 − 25
2. Calculate the area of each figure.
a. b.
3. Write the ordered pair for each point plotted on the coordinate plane.

2 • TOPIC 1: Rigid Motion Transformations

6. Write
Reflect on your work and clarify your thinking.

7. Remember
Take note of the key concepts from the lesson.

8. Practice
Use the concepts learned in the lesson to solve problems.

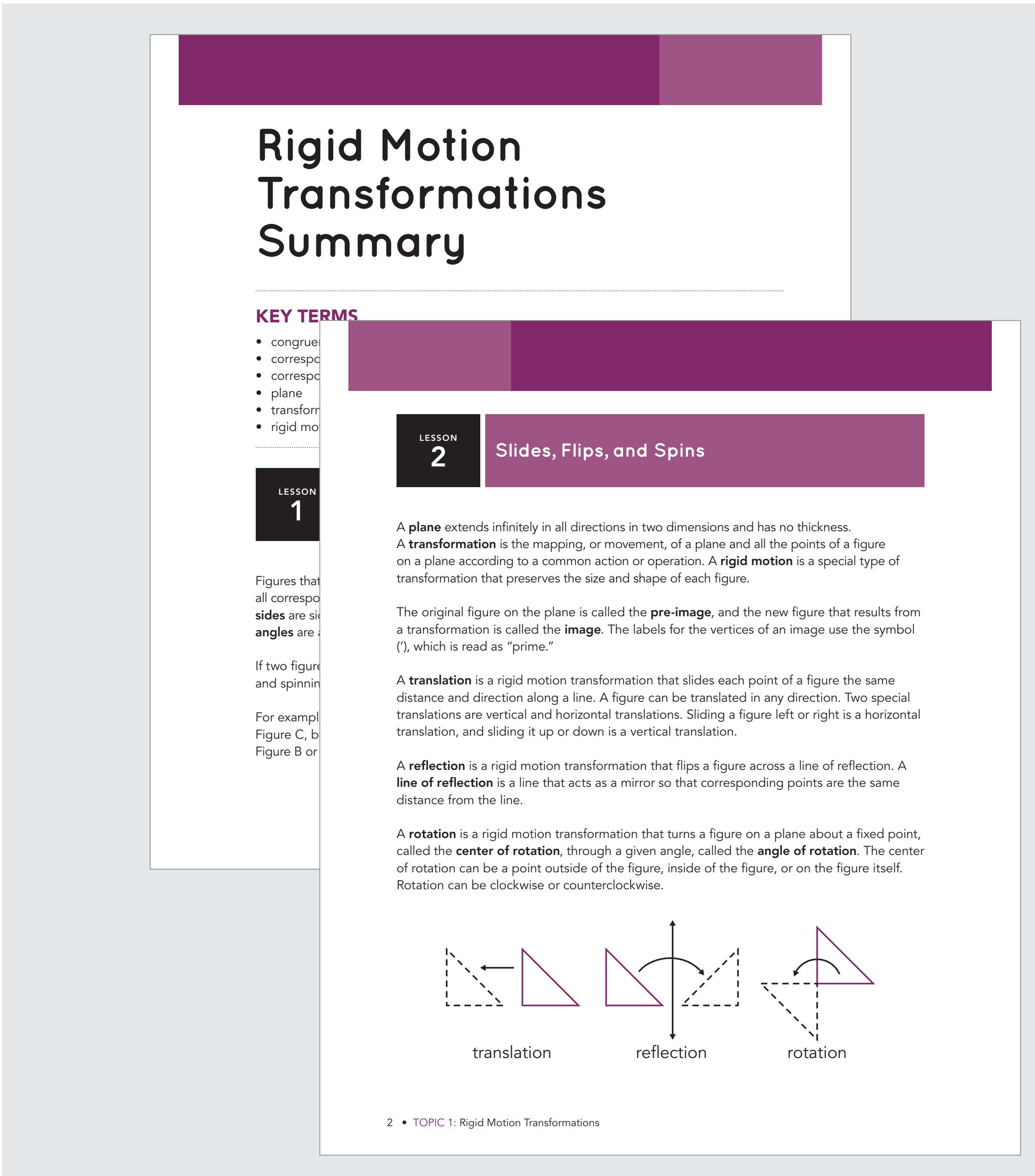
9. Stretch
Ready for a challenge?

10. Review
Remember what you've learned by practicing concepts from previous lessons and topics.

Assignment • FM-17

Topic Summary

A Topic Summary is provided for students at the end of each topic. The Topic Summary lists all key terms of the topic and provides a summary of each lesson. Each lesson summary defines key terms and reviews key concepts, strategies, and/or worked examples.



Rigid Motion Transformations Summary

KEY TERMS

- congrue
- correspo
- correspo
- plane
- transforr
- rigid mo

LESSON 1

Figures that
all correspo
sides are si
angles are

If two figure
and spinnin

For exampl
Figure C, b
Figure B or

LESSON 2 Slides, Flips, and Spins

A **plane** extends infinitely in all directions in two dimensions and has no thickness. A **transformation** is the mapping, or movement, of a plane and all the points of a figure on a plane according to a common action or operation. A **rigid motion** is a special type of transformation that preserves the size and shape of each figure.

The original figure on the plane is called the **pre-image**, and the new figure that results from a transformation is called the **image**. The labels for the vertices of an image use the symbol ('), which is read as "prime."

A **translation** is a rigid motion transformation that slides each point of a figure the same distance and direction along a line. A figure can be translated in any direction. Two special translations are vertical and horizontal translations. Sliding a figure left or right is a horizontal translation, and sliding it up or down is a vertical translation.

A **reflection** is a rigid motion transformation that flips a figure across a line of reflection. A **line of reflection** is a line that acts as a mirror so that corresponding points are the same distance from the line.

A **rotation** is a rigid motion transformation that turns a figure on a plane about a fixed point, called the **center of rotation**, through a given angle, called the **angle of rotation**. The center of rotation can be a point outside of the figure, inside of the figure, or on the figure itself. Rotation can be clockwise or counterclockwise.

translation reflection rotation

2 • TOPIC 1: Rigid Motion Transformations

LESSON 3

Lateral Moves

A translation slides an image on the coordinate plane. When an image is horizontally translated *c* units on the coordinate plane, the value of the *x*-coordinates change by *c* units. When an image is vertically translated *c* units on the coordinate plane, the value of the *y*-coo summarized

Original F
(*x*, *y*)

For examp *A* (0, 2), *B* (2

When ∆*AB* the coordin *A*′ (0, −6), *B*

When ∆*AB* the coordin *A*″ (6, 2), *B*″

LESSON 4

Mirror, Mirror

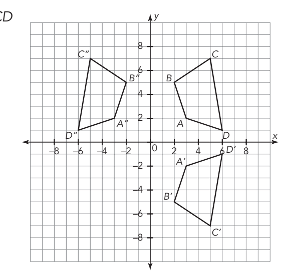
A reflection flips an image across a line of reflection. When an image on the coordinate plane is reflected across the *y*-axis, the value of the *x*-coordinate of the image is opposite the *x*-coordinate of the pre-image. When an image on the coordinate plane is reflected across the *x*-axis, the value of the *y*-coordinate of the image is opposite the *y*-coordinate of the pre-image. The coordinates of an image after a reflection on the coordinate plane are summarized in the table.

Original Point	Reflection Over *x*-Axis	Reflection Over *y*-Axis
(*x*, *y*)	(*x*, −*y*)	(−*x*, *y*)

For example, the coordinates of Quadrilateral *ABCD* are *A* (3, 2), *B* (2, 5), *C* (5, 7), and *D* (6, 1).

When Quadrilateral *ABCD* is reflected across the *x*-axis, the coordinates of the image are *A*′ (3, −2), *B*′ (2, −5), *C*′ (5, −7), and *D*′ (6, −1).

When Quadrilateral *ABCD* is reflected across the *y*-axis, the coordinates of the image are *A*″ (−3, 2), *B*″ (−2, 5), *C*″ (−5, 7), and *D*″ (−6, 1).



4 • TOPIC 1: Rigid Motion Transformations

Problem Types You Will See

Lessons include a variety of problem types to engage students in reasoning about the math.

Problem Types You Will See

Worked Example

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

Ask Yourself:

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

WORKED EXAMPLE

The first right triangle has sides of length 3 units, 4 units, and 5 units, where the sides of length 3 units and 4 units are the legs and the side with length 5 units is the hypotenuse.

The sum of the squares of the lengths of the legs: $3^2 + 4^2 = 9 + 16$

$= 25$

The square of the hypotenuse: $5^2 = 25$

Therefore $3^2 + 4^2 = 5^2$, which verifies the Pythagorean Theorem, holds true.

The Pythagorean Theorem can be used to determine unknown side lengths in a right triangle. Evan and Sophi are using the theorem to determine the length of the hypotenuse, c, with leg lengths of 2 and 4. Examine their work.

Sophi

$c^2 = 2^2 + 4^2$

$c^2 = 4 + 16 = 20$

$c = \sqrt{20} \approx 4.5$

The length of the hypotenuse is approximately 4.5 units.

Evan

$c^2 = 2^2 + 4^2$

$c^2 = 6^2$

$c = 6$

The length of the hypotenuse is 6 units.

Thumbs Up

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.

Ask Yourself:

- Why is this method correct?
- Have I used this method before?

Thumbs Down

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

Ask Yourself:

- Where is the error?
- Why is it an error?
- How can I correct it?

FM-18 • Problem Types

Worked Examples

Worked Examples help students develop their skills as they question their understanding, make connections with the steps, and ultimately explain the progression of the steps towards the final outcome. They represent and mimic an internal dialogue about the mathematics and the strategies, and the questions that follow them are designed to serve as a model for self-questioning and self-explanations, while making sure that students don't skip over a Worked Example without interacting with it, thinking about it, and responding to its accompanying questions. This approach aids students as they develop their desired habits of mind for being conscientious about the importance of steps and their order.

Thumbs Up / Thumbs Down

Thumbs Up problems give students the opportunity to analyze viable methods and problem-solving strategies. Questions are presented to help students consider the various strategies in depth and to focus on an analysis of correct responses. Because research shows that providing only positive examples is less effective for eliminating common student misconceptions than also showing negative examples, incorrect responses are provided alongside the correct responses. From the incorrect responses, students learn to determine where the error in calculation is, why the method is wrong or is being used wrong, and also how to correct the method to calculate the solution properly.

Who's Correct?

"Who's Correct?" problems are an advanced form of correct vs. incorrect responses. In this problem type, students are not told who is correct. Students have to think more deeply about what the strategies really mean and whether each of the solutions makes sense. Students will determine what is correct and what is incorrect, and then explain their reasoning. These types of problems will help students analyze their own work for errors and correctness.

Isabel says that $2^2 + 2^3 = 2^5$, and Elizabeth says that $2^2 + 2^3 \neq 2^5$. Who is correct? Explain your reasoning.

Who's Correct?

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or not correct.

Ask Yourself:

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

Problem Types • FM-19

Promoting Self-Reflection


The Crew

The Crew is here to help you on your journey. Sometimes they will remind you about things you already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are members of your group—someone you can rely on!

Teacher aides will guide you along your journey. They will help you make connections and remind you to think about the details.

FM-20 • The Crew

The Crew

Characters are embedded throughout the Texas Math Solution to remind students to stop and think in order to promote productive reflection. The characters are used in a variety of ways: they may remind students to recall a previous mathematical concept, help students develop expertise to think through problems, and occasionally, present a fun fact.

Mathematical Process Standards

Note

Each lesson provides opportunities for students to think, reason, and communicate their mathematical understanding. However, it is your responsibility as a teacher to recognize these opportunities and incorporate these practices into your daily rituals. Expertise is a long-term goal, and students must be encouraged to apply these practices to new content throughout their school career.

Mathematical Process Standards

Texas Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The "I can" expectations listed below align with the TEKS Mathematical Process Standards and encourage students to develop their mathematical learning and understanding.

▶ Apply mathematics to problems arising in everyday life, society, and the workplace.

I can:

- use the mathematics that I learn to solve real world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

▶ Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

I can:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Mathematical Process Standards • FM-21

Supporting Students to Use Mathematical Tools

Visit the Texas Support Center for strategies to support students as they use mathematical tools, including formula charts and reference sheets.

► **Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.**

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

► **Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.**

I can:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

FM-22 • Mathematical Process Standards

Note

When you are facilitating each lesson, listen carefully and value diversity of thought, redirect students' questions with guiding questions, provide additional support with those struggling with a task, and hold students accountable for an end product. When students share their work, make your expectations clear, require that students defend and talk about their solutions, and monitor student progress by checking for understanding.

There is one more page of mathematical process standards that is not provided here, but is available in the Student Textbook Front Matter.

Supporting ALL Learners

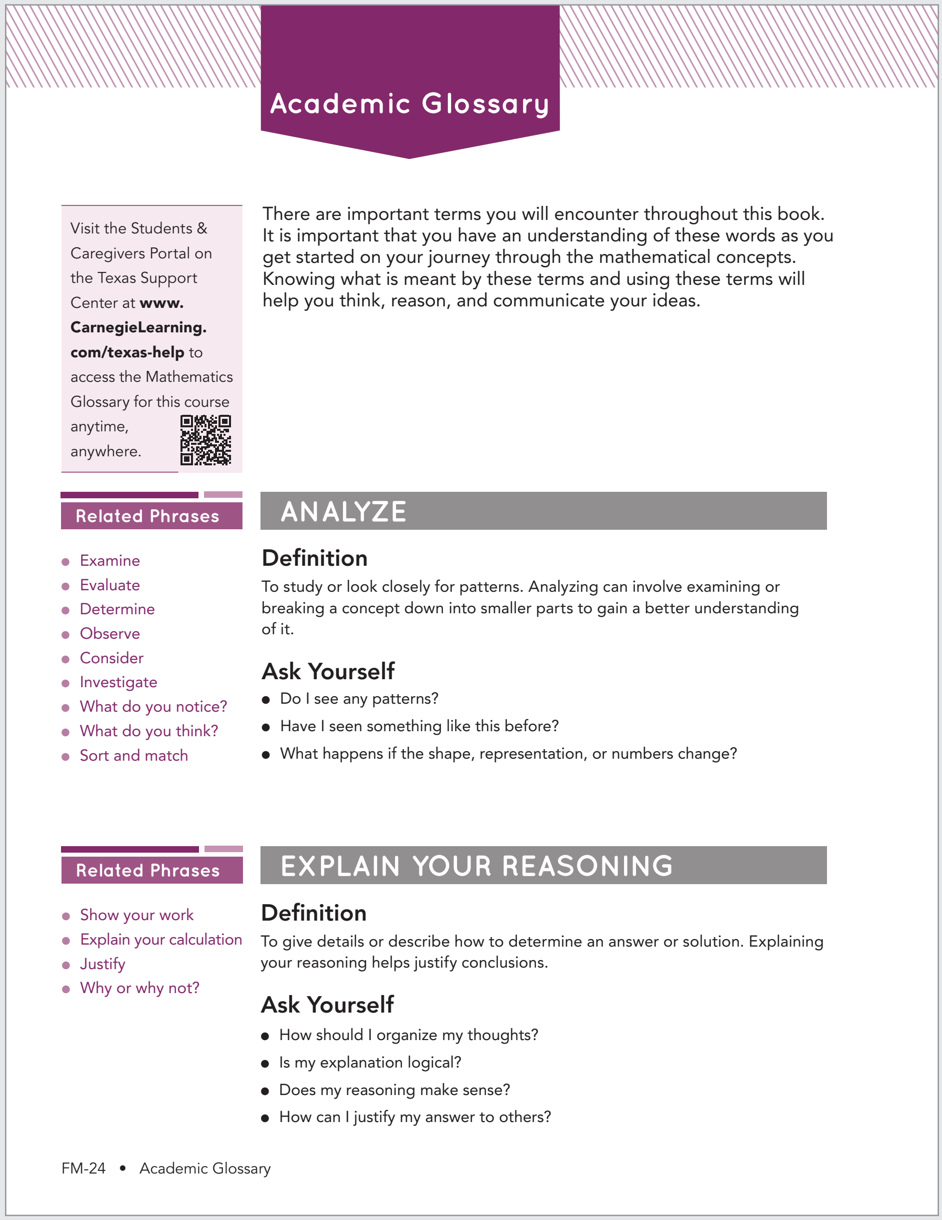
Visit the Texas Support Center for facilitation strategies to support ALL students as the engage in the Mathematical Process Standards.

Academic Glossary

Language Expectations

It is critical for students to possess an understanding of the language of their text. Students must learn to read for different purposes and write about what they are learning. Encourage students to become familiar with the key words and the questions they can ask themselves when they encounter these words.

It is our recommendation to be explicit about your expectations of language use and the way students write responses throughout the text. Encourage students to answer questions with complete sentences. Complete sentences help students reflect on how they arrived at a solution, make connections between topics, and consider what a solution means both mathematically as well as in context.



Academic Glossary

Visit the Students & Caregivers Portal on the Texas Support Center at **www.CarnegieLearning.com/texas-help** to access the Mathematics Glossary for this course anytime, anywhere.

There are important terms you will encounter throughout this book. It is important that you have an understanding of these words as you get started on your journey through the mathematical concepts. Knowing what is meant by these terms and using these terms will help you think, reason, and communicate your ideas.

Related Phrases

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

ANALYZE

Definition

To study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

Related Phrases

- Show your work
- Explain your calculation
- Justify
- Why or why not?

EXPLAIN YOUR REASONING

Definition

To give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

FM-24 • Academic Glossary

Supporting Students at Varying Levels of Language Proficiency

Visit the Texas Support Center for guidance on how to leverage the Academic Glossary to support students at varying levels of language proficiency.

REPRESENT

Definition

To display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

Related Phrases

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

ESTIMATE

Definition

To make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

Related Phrases

- Predict
- Approximate
- Expect
- About how much?

DESCRIBE

Definition

To represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

Related Phrases

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

Academic Glossary • FM-25

Ask Yourself

The Ask Yourself questions help students develop the proficiency to explain to themselves the meaning of problems.

Real-World Context

Real-world contexts confirm concrete examples of mathematics. The scenarios in the lessons help students recognize and understand that quantitative relationships seen in the real world are no different than quantitative relationships in mathematics. Some problems begin with a real-world context to remind students that the quantitative relationships they already use can be formalized mathematically. Other problems will use real-world situations as an application of mathematical concepts.

Mathematics Glossary

A course-specific mathematics glossary is available for students to utilize and reference during their learning. Definitions and examples of key terms are provided in the glossary.

Visit the Texas Support Center at **www.CarnegieLearning.com/texas-help** for additional resources to support you anytime, anywhere.

Teacher's Implementation Guide

The Teacher's Implementation Guide (TIG) is designed to fully support a wide range of teachers implementing our materials: from first-year teachers to 30-year veterans and from first-time Carnegie Learning users to master practitioners.

One goal in developing the TIG was to make our instructional design apparent to the users.

The lessons of each topic were written to be accessible to the full range of learners. With every instructional decision you make, keep in mind your mathematical objectives for the topic and module and the course. Plan each lesson by thinking about how you will create access for your particular group of students, maintain access and pace throughout the lesson, and assess their understanding along the way. We recommend that you do the math in each topic before implementing the activities with your specific group of students.

What makes this TIG useful?

Effective Lesson Design: Each lesson has a consistent structure for teachers and students to follow. The learning experiences are engaging and effective for students.

Pacing: Each course is designed to be taught in a 180-day school year. Pacing suggestions are provided for each lesson. Each day in the pacing guide is an equivalent to about a 45-minute instructional session.

Instructional Supports: Guiding questions are provided for teachers to use as they're circulating the room, as well as differentiation strategies, common student misconceptions, and student look-fors.

Clearly Defined Mathematics: The content and instructional goals are clearly described at the module, topic, lesson, and activity levels.

The TIG is critical to understanding how the mathematics that students encounter should be realized in the classroom. The TIG describes the depth of understanding that students need to develop for each standard and a pathway for all learners to be successful. It provides differentiation strategies to support students who struggle, to extend certain activities for students who are advanced in their understanding of the content, and to support emergent bilingual students.

Module and Topic Overviews

You are responsible for teaching the essential concepts associated with a particular course. You need to understand how activities within lessons build to achieve understanding within topics, and how topics build to achieve understanding throughout the course. In the Texas Math Solution, Carnegie Learning seeks to establish a shared curriculum vision with you.

Module Overview

Each module begins with an overview that describes the reasoning behind the name, the mathematics being developed, the connections to prior learning, and the connections to future learning.

Topic Overview

A Topic Overview describes how the topic is organized, the entry point for students, how a student will demonstrate understanding, why the mathematics is important, how the activities promote expertise in the mathematical process standards, materials needed for the topic, examples of visual representations or strategies used, and more detailed information to help with pacing.

"Teachers must first develop their ideas about where the curriculum program is going mathematically (curriculum vision) before deciding whether the curriculum materials will help them reach that mathematical goal (curriculum trust)" (Drake & Sherin, 2009, p. 325).

Facilitation Notes

For each lesson, you are provided with detailed facilitation notes to fully support your planning process. This valuable resource provides point-of-use support that serves as your primary resource for planning, guiding, and facilitating student learning.

1. Materials

Materials required for the lesson are identified.

2. Lesson Overview

The Lesson Overview sets the purpose and describes the overarching mathematics of the lesson, explaining how the activities build and how the concepts are developed.

3. TEKS Addressed

The focus standards for each lesson are listed. Carnegie Learning recognizes that some lessons could list several TEKS based on the skills needed to complete the activities, however, the TEKS listed are what the lesson is focused on developing or mastering.

Jack and Jill Went Up the Hill 2

Using Similar Triangles to Describe the Steepness of a Line

MATERIALS (1)

Patty paper
Straightedge
Scissors

(2) **Lesson Overview**

Students connect the previously learned concepts of unit rate, constant of proportionality, and scale factor with the concept of slope, which is introduced here as the rate of change of the dependent quantity compared to the independent quantity. In this lesson, *slope* is defined as the steepness and direction of a line. The formula to calculate slope is introduced in the next topic. Students derive the equation for a proportional relationship, $y = mx$. By translating the line b units, they derive the equation for a non-proportional linear relationship, $y = mx + b$. They practice writing equations from graphs. Students begin with incomplete tables and graphs to create their own proportional and non-proportional linear relationships. They also investigate the slope of a horizontal line.

Grade 8

Proportionality

(3) **(4) The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope. The student is expected to:**

(A) use similar right triangles to develop an understanding that slope, m, given as the rate comparing the change in y-values to the change in x-values, $\frac{(y_2 - y_1)}{(x_2 - x_1)}$, is the same for any two points (x_1, y_1) and (x_2, y_2) on the same line.

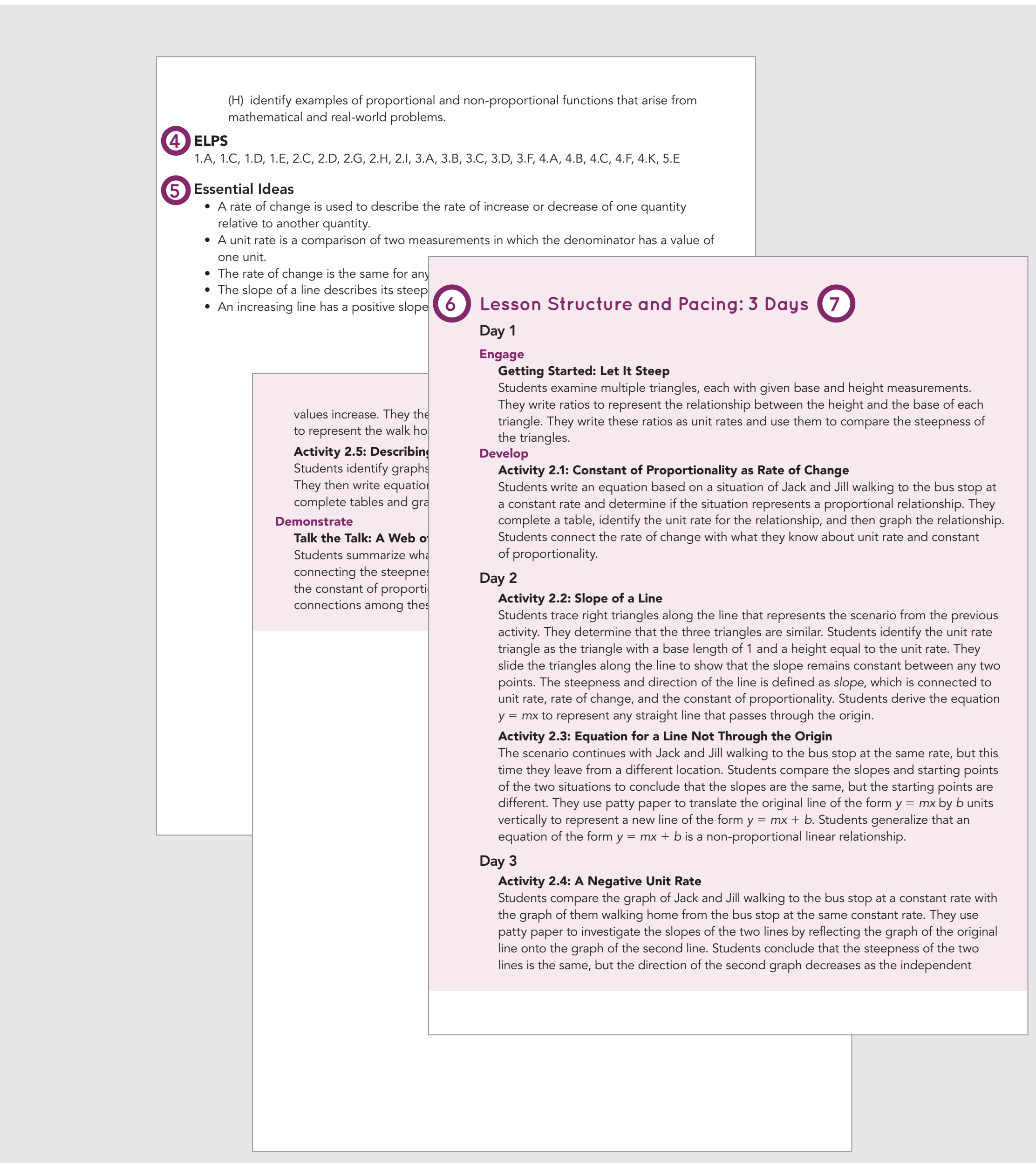
(B) graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship.

(C) use data from a table or graph to determine the rate of change or slope and y-intercept in mathematical and real-world problems.

Proportionality

(5) The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to:

(F) distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form $y = kx$ or $y = mx + b$, where $b \neq 0$.



(H) identify examples of proportional and non-proportional functions that arise from mathematical and real-world problems.

4 **ELPS**
1.A, 1.C, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.F, 4.K, 5.E

5 **Essential Ideas**
- A rate of change is used to describe the rate of increase or decrease of one quantity relative to another quantity.
- A unit rate is a comparison of two measurements in which the denominator has a value of one unit.
- The rate of change is the same for any
- The slope of a line describes its steep
- An increasing line has a positive slope

values increase. They the
to represent the walk ho
Activity 2.5: Describing
Students identify graphs
They then write equation
complete tables and gra
Demonstrate
Talk the Talk: A Web o
Students summarize wha
connecting the steepnes
the constant of proporti
connections among thes

6 **Lesson Structure and Pacing: 3 Days** 7

Day 1

Engage

Getting Started: Let It Steep
Students examine multiple triangles, each with given base and height measurements. They write ratios to represent the relationship between the height and the base of each triangle. They write these ratios as unit rates and use them to compare the steepness of the triangles.

Develop

Activity 2.1: Constant of Proportionality as Rate of Change
Students write an equation based on a situation of Jack and Jill walking to the bus stop at a constant rate and determine if the situation represents a proportional relationship. They complete a table, identify the unit rate for the relationship, and then graph the relationship. Students connect the rate of change with what they know about unit rate and constant of proportionality.

Day 2

Activity 2.2: Slope of a Line
Students trace right triangles along the line that represents the scenario from the previous activity. They determine that the three triangles are similar. Students identify the unit rate triangle as the triangle with a base length of 1 and a height equal to the unit rate. They slide the triangles along the line to show that the slope remains constant between any two points. The steepness and direction of the line is defined as *slope*, which is connected to unit rate, rate of change, and the constant of proportionality. Students derive the equation $y = mx$ to represent any straight line that passes through the origin.

Activity 2.3: Equation for a Line Not Through the Origin
The scenario continues with Jack and Jill walking to the bus stop at the same rate, but this time they leave from a different location. Students compare the slopes and starting points of the two situations to conclude that the slopes are the same, but the starting points are different. They use patty paper to translate the original line of the form $y = mx$ by b units vertically to represent a new line of the form $y = mx + b$. Students generalize that an equation of the form $y = mx + b$ is a non-proportional linear relationship.

Day 3

Activity 2.4: A Negative Unit Rate
Students compare the graph of Jack and Jill walking to the bus stop at a constant rate with the graph of them walking home from the bus stop at the same constant rate. They use patty paper to investigate the slopes of the two lines by reflecting the graph of the original line onto the graph of the second line. Students conclude that the steepness of the two lines is the same, but the direction of the second graph decreases as the independent

4. ELPS Addressed

The English Language Proficiency Standards for each lesson are listed. As you plan, consider these ELPS and determine the instructional strategies that you will use to meet these ELPS.

5. Essential Ideas

These statements are derived from the standards and state the concepts students will develop.

6. Lesson Structure

This section highlights how the parts of the lesson fit within the instructional design: Engage, Develop, and Demonstrate. A summary of each activity is included.

7. Pacing

Lessons often span more than one 45-minute class period. Suggested pacing is provided for each lesson so that the entire course can be completed in a school year.

8. Facilitation Notes by Activity

A detailed set of guidelines walks the teacher through implementing the Getting Started, Activities, and Talk the Talk portions of the lesson. These guidelines include an activity overview, grouping strategies, guiding questions, possible student misconceptions, differentiation strategies, student look-fors, and an activity summary.

9. Activity Overview

Each set of Facilitation Notes begins with an overview that highlights how students will actively engage with the task to achieve the learning goals.

8

9

Getting Started: Let It Steep

ENGAGE

Facilitation Notes

In this activity, students write ratios to represent the relationship between the height and the base of different triangles and use unit rates for comparison purposes.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- Is the height of the triangle written in the numerator or denominator of the ratio?
- Is the base of the triangle written in the numerator or denominator of the ratio?
- Is the ratio $\frac{10}{10}$ equal to the ratio $\frac{15}{15}$?
- How did you determine the unit rate?
- Does the smallest rate identify with the least steep or most steep line on the graph?
- Does the largest rate identify with the least steep or most steep line on the graph?

Misconception

If students calculate $\frac{x}{y}$, take the time to interpret the meaning of their calculations.

Summary

Unit rates can be used to compare different ratios.

Activity 2.1

Constant of Proportionality as Rate of Change

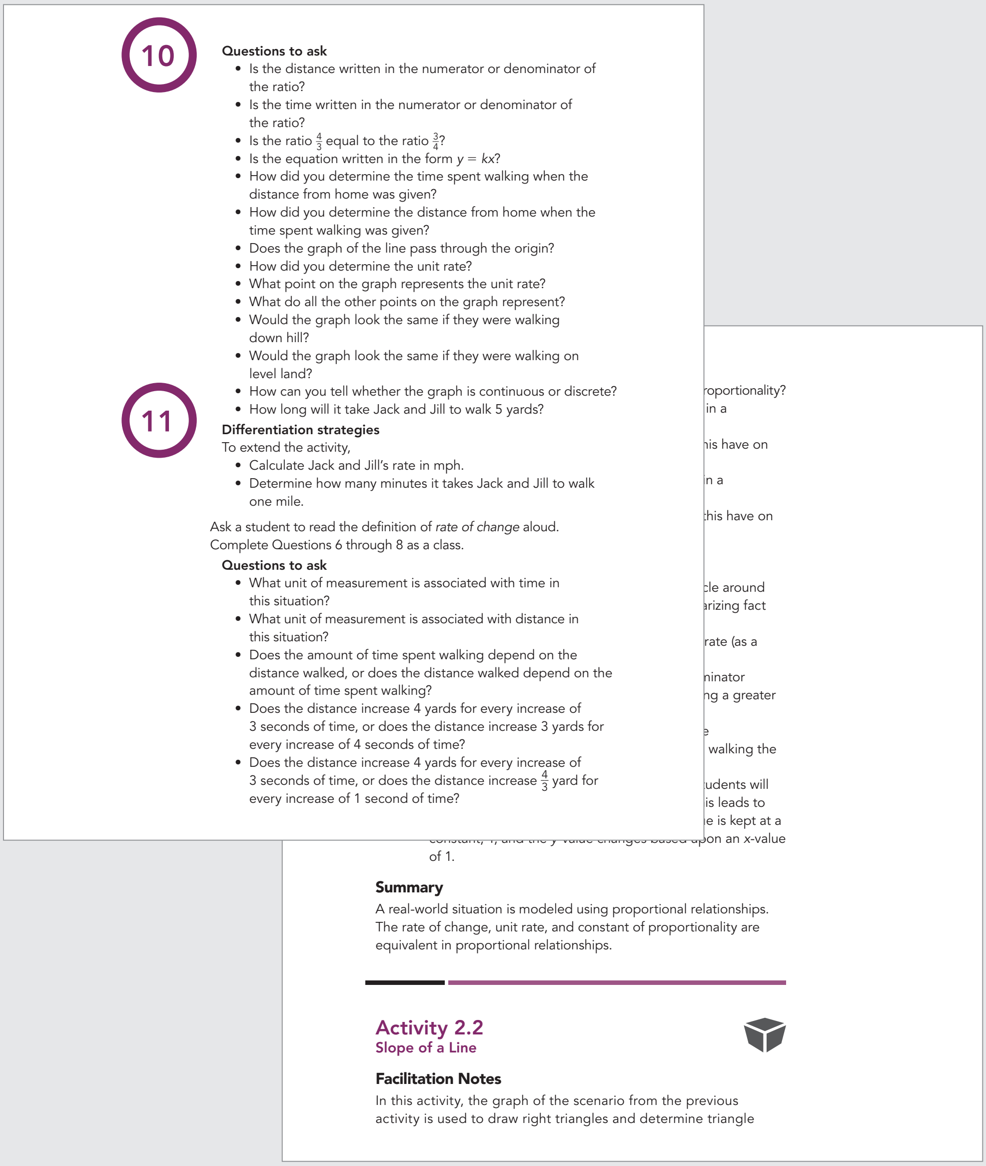
DEVELOP

Facilitation Notes

In this activity, students are given a scenario and asked to write an equation, complete a table, identify the unit rate, and graph the relationship. Connections are made between the rate of change, the unit rate, and the constant of proportionality.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

> Position yourself to take full advantage of the richness of the mathematics addressed in the textbook. The Facilitation Notes provide guidance to reach each student from their current level of understanding to advance to the next stage. Place yourself in the position of the student by experiencing the textbook activities prior to class. Realize your role in the classroom—empower your students! Step back and let them do the math with confidence in their role as learner and your role as facilitator of learning.
>
> Janet Sinopoli, Instructional Designer



10

Questions to ask

- Is the distance written in the numerator or denominator of the ratio?
- Is the time written in the numerator or denominator of the ratio?
- Is the ratio $\frac{4}{3}$ equal to the ratio $\frac{3}{4}$?
- Is the equation written in the form $y = kx$?
- How did you determine the time spent walking when the distance from home was given?
- How did you determine the distance from home when the time spent walking was given?
- Does the graph of the line pass through the origin?
- How did you determine the unit rate?
- What point on the graph represents the unit rate?
- What do all the other points on the graph represent?
- Would the graph look the same if they were walking down hill?
- Would the graph look the same if they were walking on level land?
- How can you tell whether the graph is continuous or discrete?
- How long will it take Jack and Jill to walk 5 yards?

11

Differentiation strategies

To extend the activity,

- Calculate Jack and Jill's rate in mph.
- Determine how many minutes it takes Jack and Jill to walk one mile.

Ask a student to read the definition of *rate of change* aloud. Complete Questions 6 through 8 as a class.

Questions to ask

- What unit of measurement is associated with time in this situation?
- What unit of measurement is associated with distance in this situation?
- Does the amount of time spent walking depend on the distance walked, or does the distance walked depend on the amount of time spent walking?
- Does the distance increase 4 yards for every increase of 3 seconds of time, or does the distance increase 3 yards for every increase of 4 seconds of time?
- Does the distance increase 4 yards for every increase of 3 seconds of time, or does the distance increase $\frac{4}{3}$ yard for every increase of 1 second of time?

Summary

A real-world situation is modeled using proportional relationships. The rate of change, unit rate, and constant of proportionality are equivalent in proportional relationships.

Activity 2.2
Slope of a Line

Facilitation Notes

In this activity, the graph of the scenario from the previous activity is used to draw right triangles and determine triangle

10. Questions to Ask

The overarching questioning strategies throughout each lesson promote analysis and higher-order thinking skills beyond simple yes or no responses. These questions can be used to gather information, probe thinking, make the mathematics explicit, and encourage reflection and justification as students are working together or when they are sharing responses as a class. These questions are an embedded formative assessment strategy to provide feedback as students are actively engaged in learning.

11. Differentiation Strategies

To extend an activity for students who are ready to advance beyond the scope of the activity, additional challenges are provided.

Note

Differentiation strategies are provided that will ensure all students acquire the knowledge of the activity. These strategies provide flexibility within the lesson to allow for varying student acquisition and demonstration of learning. These strategies provide suggestions for all students, including those with learning strengths or learning gaps.

12. Differentiation Strategies

To assist all students, instructional strategies are provided that benefit the full range of learners.

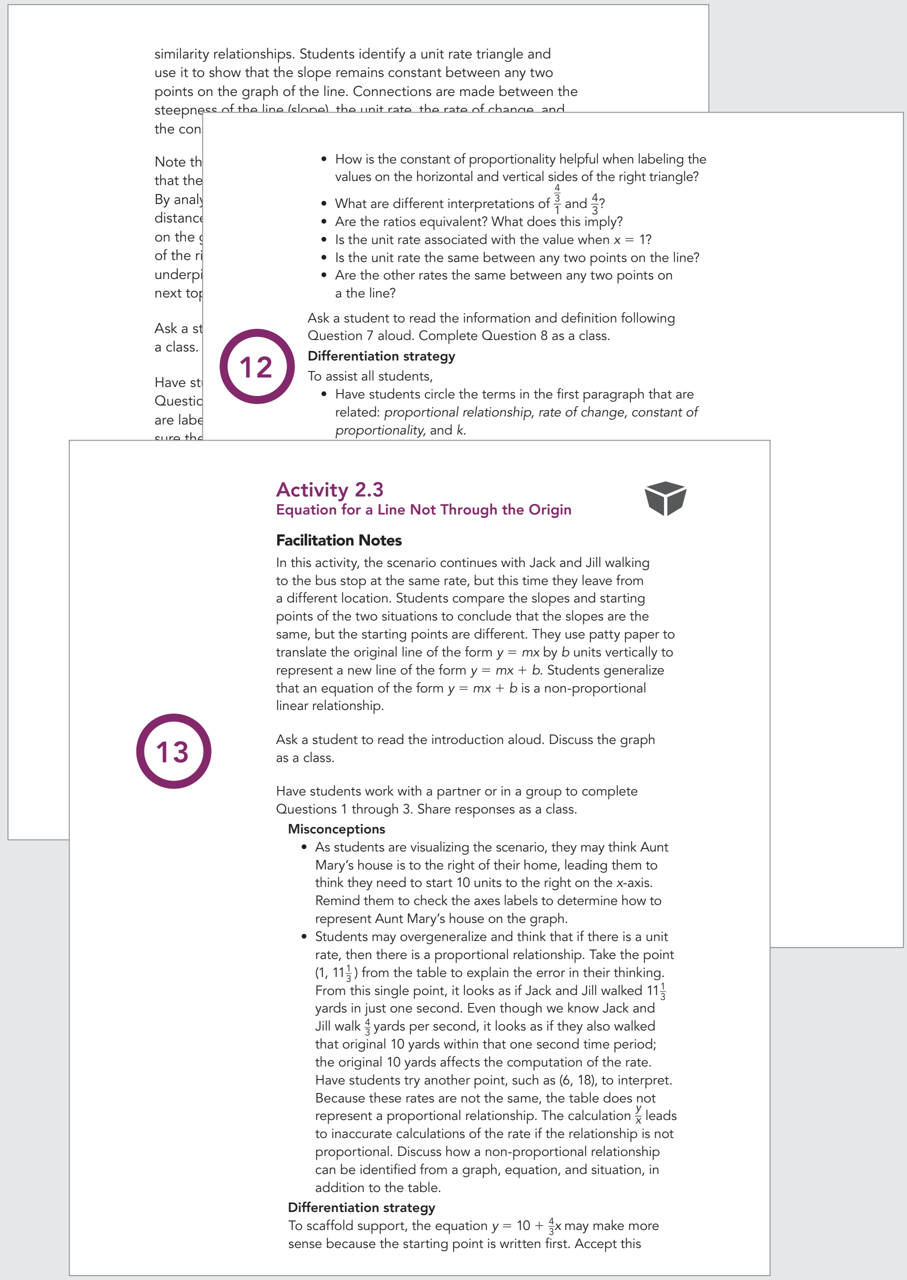
13. Grouping Strategies

Suggestions appear to help chunk each activity into manageable pieces and establish the cadence of the lesson.

Learning is social. Whether students work in pairs or in groups, the critical element is that they are engaged in discussion. Carnegie Learning believes, and research supports, that student-to-student discourse is a motivating factor; it increases student learning and supports ongoing formative assessment. Additionally, it provides students with opportunities to have mathematical authority.

Working collaboratively can, when done well, encourage students to articulate their thinking (resulting in self-explanation) and also provides metacognitive feedback (by reviewing other students' approaches and receiving feedback on their own).

The student discussion is then transported to a classroom discussion facilitated by the teacher



to guarantee all necessary mathematics is addressed, once again, with the same benefits of discussion.

Alternative Grouping Strategies

Differentiation strategies that provide other grouping strategies, such as whole class participation and the jigsaw method, are sometimes recommended for specific activities. These are listed as Differentiation Strategies.

More information about grouping strategies is available online in the Texas Support Center at **www.CarnegieLearning.com/texas-help**.

response; it is mathematically correct and reflects students' understanding of rate and starting position. Discuss what property demonstrates that $y = 10 + \frac{4}{3}x$ and $y = \frac{4}{3}x + 10$ are equivalent. Allow for flexibility in the formats of student responses.

Questions to ask

- What location or point on the graph represents Aunt Mary's house?
- Is the point (0, 10) or the poi Mary's house?
- Is Aunt Mary's house located
- Where on the graph did the
- Where on the graph does thi
- If the graph of a line does no the line represent a proportio
- Does the translation add 10 to each coordinate?
- Do translations preserve unit
- Why can't the equation $y = \frac{4}{3}$ this situation?
- Can this equation be written why not?

Ask a student to read the informatio Complete Question 4 as a class.

15 **Summary**

When a proportional relationship de $y = mx$ is translated vertically b unit equation is $y = mx + b$, where m re represents the y-intercept.

Activity 2.4
A Negative Unit Rate

Facilitation Notes

In this activity, students compare th to the bus stop at a constant rate w home from the bus stop at the sam paper to investigate the slopes of th graph of the original line onto the g conclude that the steepness of the

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direction of the second graph decreases as the independent values increase. They then write an equation of the form $y = mx + b$, where m is negative, to represent the walk home from the bus stop.

Note that this activity is designed for students to understand the concept of slope as the direction (either increasing or decreasing from left to right) and steepness (the ratio of the change in vertical distance to the change in horizontal distance between any two points on the line) of a line.

Ask a student to read the introduction aloud. Discuss the graphs as a class.

Have students work with a partner or in a group to complete Questions 1 through 4. Students may try a variety of transformations. Remind them of the units that define each axis. To maintain that relationship, they should reflect the triangle across a horizontal line of reflection. Share responses as a class.

Questions to ask

- What location or point on the graph represents the bus stop?
- Is the point (0, 30) or the point (30, 0) the location of the bus stop?
- Is the bus stop located on the x- or y-axis?
- Where on the graph did the original line begin?
- Where on the graph does this line begin?
- Where on the graph did the line begin when they started from Aunt Mary's house?
- If the graph of a line does not pass through the origin, does the line represent a proportional relationship?
- Is the constant of proportionality the same for every set of points on the line?
- Is the line increasing or decreasing as you read the graph from left to right?
- Does the unit rate remain constant between any two points on the line?
- Does the translation add 30 to the x-value or y-value for each coordinate?
- Do translations preserve unit rates?

14 **Misconception**

Some students have difficulty understanding how you can tell whether a line is increasing or decreasing because it looks to them

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14. Misconceptions

Common student misconceptions are provided in places where students may overgeneralize mathematical relationships or have confusion over the vocabulary used. Suggestions are provided to address the given misconception.

15. Summary

The summary brings the activity to closure. This statement encapsulates the big mathematical ideas of the particular activity.

16. As Students Work, Look For

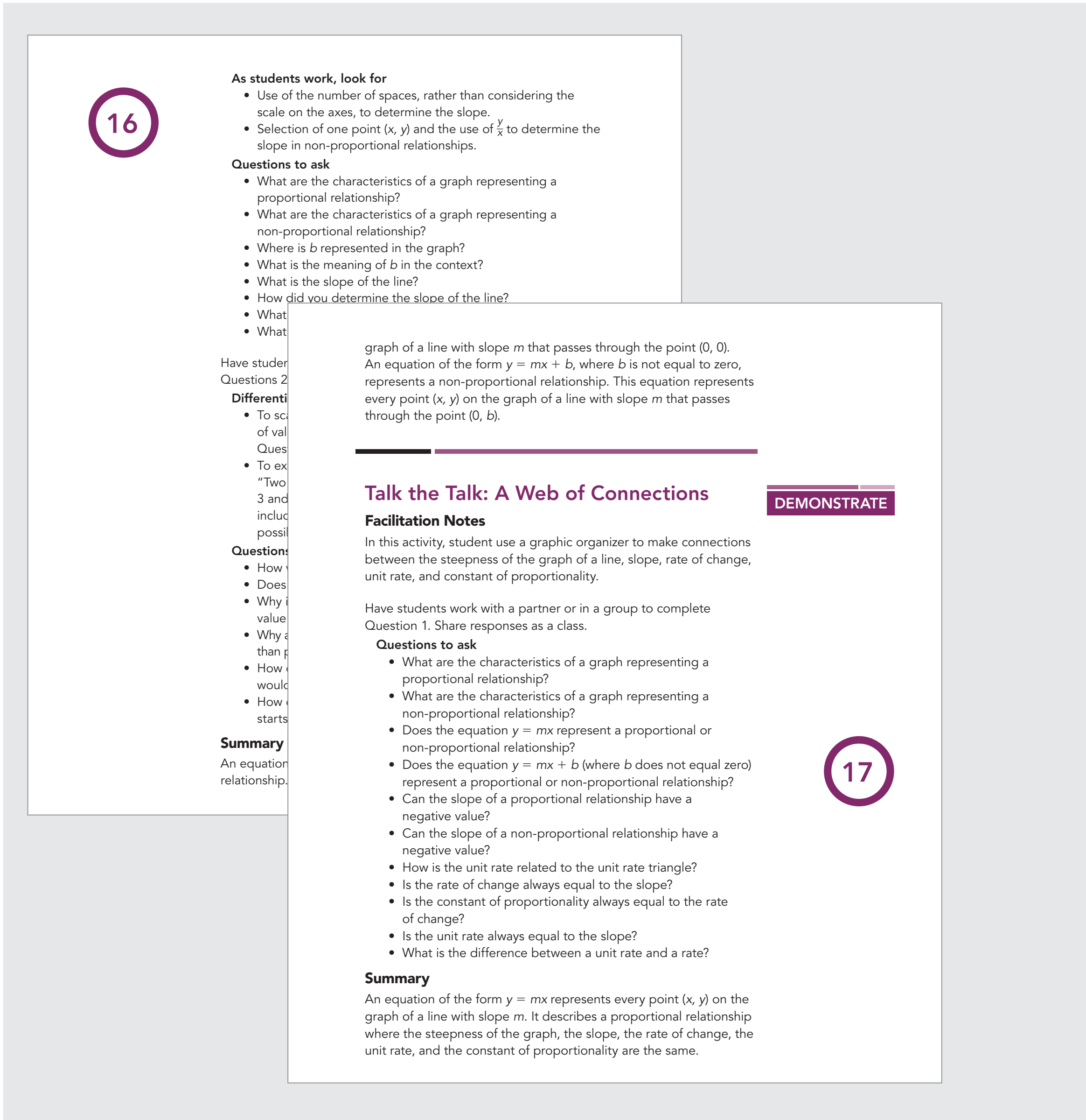
These notes provide specific language, strategies, and/or errors to look and listen for you as you circulate and monitor students working in pairs or groups. You can incorporate these ideas when students share their responses with the class.

Note

Talk the Talk helps you to assess student learning and to make decisions about helpful connections you need to make in future lessons.

17. White Space

The white space in each margin is intentional. Use this space to make additional planning notes or to reflect on the implementation of the lesson.



16

As students work, look for
- Use of the number of spaces, rather than considering the scale on the axes, to determine the slope.
- Selection of one point (*x*, *y*) and the use of $\frac{y}{x}$ to determine the slope in non-proportional relationships.

Questions to ask
- What are the characteristics of a graph representing a proportional relationship?
- What are the characteristics of a graph representing a non-proportional relationship?
- Where is *b* represented in the graph?
- What is the meaning of *b* in the context?
- What is the slope of the line?
- How did you determine the slope of the line?
- What
- What

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Summary
An equation
relationship.

graph of a line with slope *m* that passes through the point (0, 0). An equation of the form $y = mx + b$, where *b* is not equal to zero, represents a non-proportional relationship. This equation represents every point (*x*, *y*) on the graph of a line with slope *m* that passes through the point (0, *b*).

Talk the Talk: A Web of Connections

DEMONSTRATE

Facilitation Notes

In this activity, student use a graphic organizer to make connections between the steepness of the graph of a line, slope, rate of change, unit rate, and constant of proportionality.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

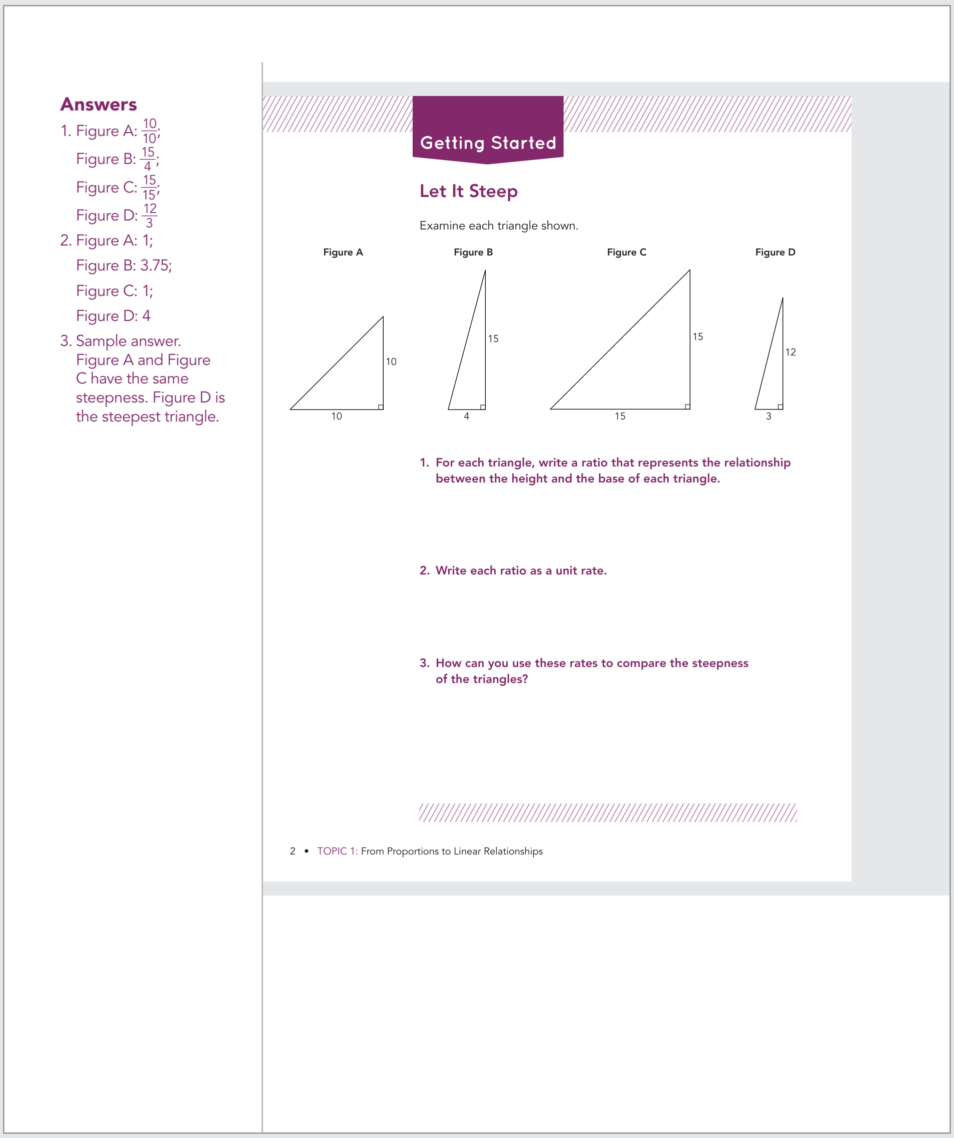
Questions to ask
- What are the characteristics of a graph representing a proportional relationship?
- What are the characteristics of a graph representing a non-proportional relationship?
- Does the equation $y = mx$ represent a proportional or non-proportional relationship?
- Does the equation $y = mx + b$ (where *b* does not equal zero) represent a proportional or non-proportional relationship?
- Can the slope of a proportional relationship have a negative value?
- Can the slope of a non-proportional relationship have a negative value?
- How is the unit rate related to the unit rate triangle?
- Is the rate of change always equal to the slope?
- Is the constant of proportionality always equal to the rate of change?
- Is the unit rate always equal to the slope?
- What is the difference between a unit rate and a rate?

17

Summary
An equation of the form $y = mx$ represents every point (*x*, *y*) on the graph of a line with slope *m*. It describes a proportional relationship where the steepness of the graph, the slope, the rate of change, the unit rate, and the constant of proportionality are the same.

Supporting Emergent Bilingual Students

Emergent bilingual students often face multiple challenges in the mathematics classroom beyond language development skills, including a lack of confidence, peer-to-peer understanding, and building solid conceptual mastery. The Carnegie Learning Texas Math Solution seeks to support emergent bilingual students as they develop skills in both mathematics and language.



Answers

1. Figure A: $\frac{10}{10}$; Figure B: $\frac{15}{4}$; Figure C: $\frac{15}{15}$; Figure D: $\frac{12}{3}$
2. Figure A: 1; Figure B: 3.75; Figure C: 1; Figure D: 4
3. Sample answer. Figure A and Figure C have the same steepness. Figure D is the steepest triangle.

Getting Started

Let It Steep

Examine each triangle shown.

1. For each triangle, write a ratio that represents the relationship between the height and the base of each triangle.

2. Write each ratio as a unit rate.

3. How can you use these rates to compare the steepness of the triangles?

2 • TOPIC 1: From Proportions to Linear Relationships

Throughout instruction, ELL tips are placed for teachers at point-of-use on the mini-lesson page in the TIG. They provide additional modifications to support this special population.

These tips:

- Inform teachers of potential learning obstacles specific to the lesson.
- Provide engaging activities for learning and assessment.
- Reinforce newly acquired mathematical language to gain an increasing level of comprehension of English.
- Introduce students to language needed to understand a specific context.

Students internalize new content language by using and reusing it in meaningful ways in a variety of different speaking activities that build concept and language attainment.

For More Support

Visit the Texas Support Center for many more resources to support you and your students who are emergent bilingual students.

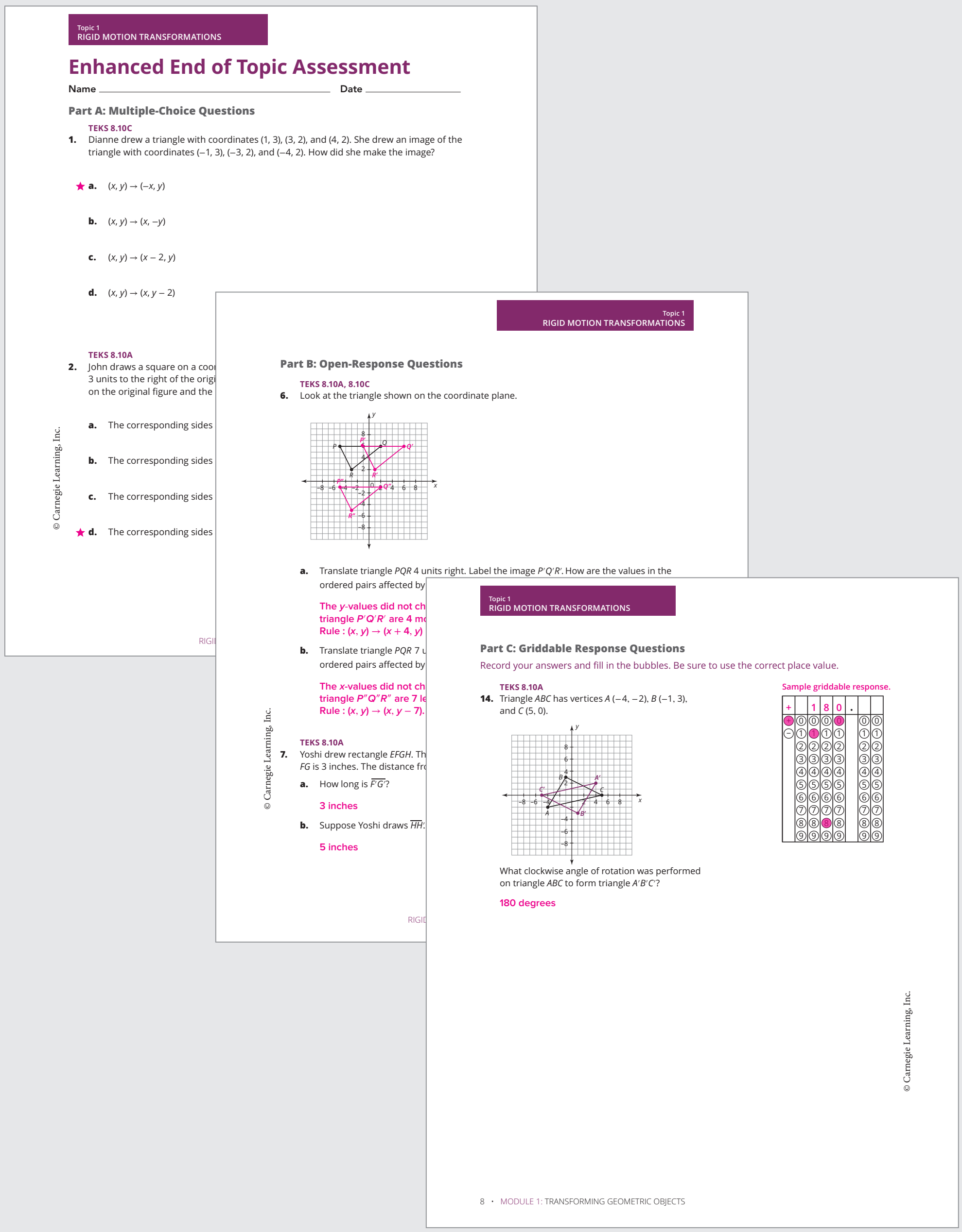
Assessments

Formative assessment tools are provided throughout each lesson, providing you with ongoing feedback of student performance and encouraging students to monitor their own progress. End of topic summative assessments are provided to measure student performance on a clearly denoted set of standards. For certain topics that extend longer than four instructional weeks, a mid-topic summative assessment is also provided.

Enhanced End of Topic Assessment

There are three problem type sections per assessment. Multiple-choice questions, open-response questions, and griddable response questions prepare students for enhanced standardized tests.

The answer key provides teachers with the TEKS aligned to each question, as well as sample answers for open-response and griddable response questions.



Supporting Students to Use Mathematical Tools

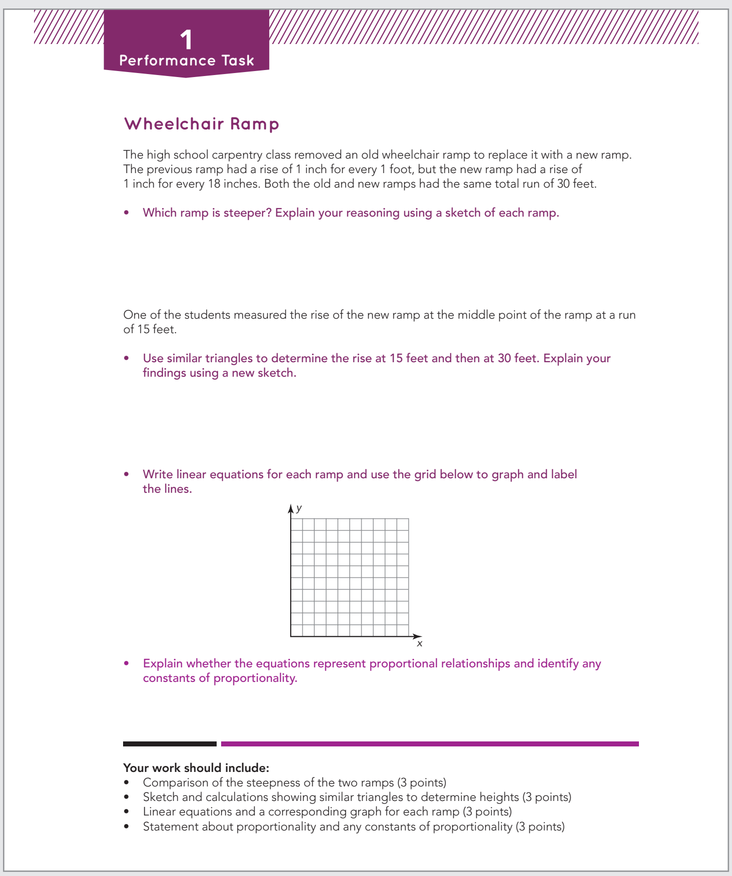
Visit the Texas Support Center for strategies to support students as they use mathematical tools, including formula charts and reference sheets.

End of Course Topic

The End of Course Topic is the final topic of the course which includes a collection of problem-based performance tasks that are aligned with selected priority math standards of the course. This final topic provides students an additional opportunity to demonstrate their ability to make sense of multi-step, real-world problems, communicate their thinking, represent solutions, and justify their reasoning on content aligned with these selected math standards.

Performance Tasks

Each performance task is a formative assessment tool that allows students to demonstrate their learning of the selected course content. At the end of each task, a section titled "Your Work Should Include" lists the categories and the corresponding maximum scoring points from the grading rubric.



1
Performance Task

Wheelchair Ramp

The high school carpentry class removed an old wheelchair ramp to replace it with a new ramp. The previous ramp had a rise of 1 inch for every 1 foot, but the new ramp had a rise of 1 inch for every 18 inches. Both the old and new ramps had the same total run of 30 feet.

- Which ramp is steeper? Explain your reasoning using a sketch of each ramp.

One of the students measured the rise of the new ramp at the middle point of the ramp at a run of 15 feet.

- Use similar triangles to determine the rise at 15 feet and then at 30 feet. Explain your findings using a new sketch.

- Write linear equations for each ramp and use the grid below to graph and label the lines.

- Explain whether the equations represent proportional relationships and identify any constants of proportionality.

Your work should include:
- Comparison of the steepness of the two ramps (3 points)
- Sketch and calculations showing similar triangles to determine heights (3 points)
- Linear equations and a corresponding graph for each ramp (3 points)
- Statement about proportionality and any constants of proportionality (3 points)

Grading Rubric

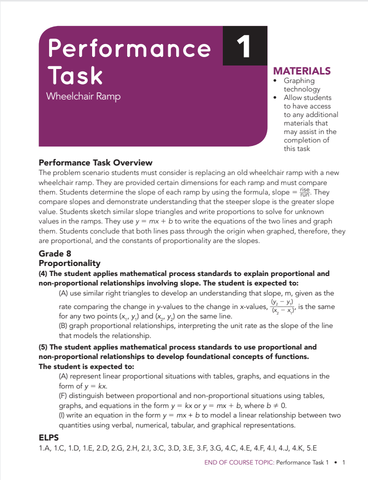
The grading rubric is for students and teachers to set clear expectations for how each completed performance task will be evaluated. Students should use the rubric to guide their work and self-monitor their progress. Teachers should use the rubric to evaluate and provide feedback for the completed performance task.

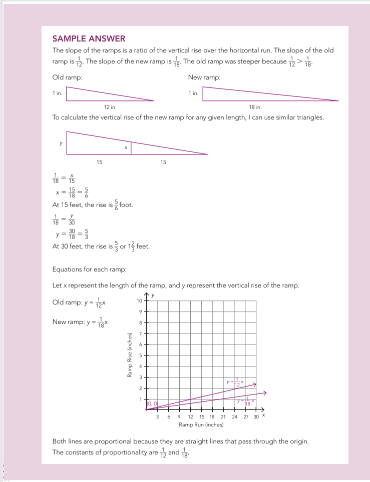
RUBRIC: 12 Total Points

	0 points	1 point	2 points	3 points
Comparison of Steepness of Ramps	No comparison given.	The comparison is incorrect.	The comparison is correct, but the explanation is missing.	The comparison is correct and has a complete explanation.
Sketches and Calculations	No sketches or calculations given.	The sketch is given, but the calculations are missing.	The sketch is correct, but the calculations include errors.	The sketch and the calculations are complete and correct.
Equations	No equations given.	One or both equations are incorrect.	Both equations are correct, but the variables aren't defined.	Both equations are complete and correct with variables defined.
Statement	No statement given.	The statement is given but contains errors. No mention of the constants of proportionality.	The statement is correct, but the constants of proportionality are incorrect for at least one ramp.	The statements of proportionality and the constants of proportionality are both correct.

Teacher's Implementation Guide

The Teacher's Implementation Guide for the End of Course Topic contains a performance task overview, list of aligned TEKS and ELPS, essential ideas, facilitation notes which describe how to pace the two-day performance task, sample answer, and grading rubric.





Similar to the other topics in this course, the End of Course Topic also has a Topic Family Guide for students and caregivers, and a Topic Overview for teachers. The End of Course Topic does not include an end of topic assessment since each performance task is a formative assessment.

Carnegie Learning recognizes that it is the classroom teachers who make the material come alive for students, transforming the way math is taught. Implementation requires integrating learning together and learning individually.

PREPARE YOURSELF

Prepare for Learning Together

The most important first step you can take in preparing to teach with these instructional materials is to become comfortable with the mathematics.

- Read through the Module 1 Overview and the Topic 1 Overview.
- Do the math of the first Topic, and consider the facilitation notes.
- Prepare team-building activities to intentionally create a student-centered environment.

Prepare for Learning Individually

Plan how you will utilize Skills Practice as a Learning Individually resource. Then, determine how you will introduce Skills Practice to students. Explain to them the benefits of of working individually and why practice is important.

- Read through Module 1 Topic 1 Skills Practice.
- Determine which problem sets align with the activities in the corresponding student lessons.
- Based on student performance in the lesson, be prepared to assign the class, small groups of students, or individual students different problem sets to practice skills to develop mastery.

Plan how you will introduce students to MATHia. Explain to them the benefits of working individually and why practice is important.

- Test out the computers or tablets that your students will be using.
- Verify your classes have been set up in Teacher's Toolkit with correct MATHia content assigned. Or manually set up your classes in Teacher's Toolkit if applicable.
- Use the Content Browser in Teacher's Toolkit to explore the content students are assigned.
- Be prepared to demonstrate how students will access and log into MATHia.

PREPARE YOUR CLASSROOM

Prepare the Environment

The classroom is often considered the third teacher. Consider how to create a learning environment that engages students and fosters a sense of ownership. The use of space in your classroom should be flexible and encourage open sharing of ideas.

- Consider how your students are going to use the consumable book. It is the student's record of their learning. Many teachers have students move an entire topic to a three-ring binder as opposed to carrying the entire book.
- Arrange your desks so students can talk and collaborate with each other.
- Prepare a toolkit for groups to use as they work together and share their reasoning (read the materials list in each Topic Overview).
- Consider where you will display student work, both complete and in-progress.
- Create a word wall of key terms used in the textbook.

PREPARE YOUR STUDENTS

Prepare the Learners

If you expect students to work well together, they need to understand what it means to collaborate and how it will benefit them. It is important to establish classroom guidelines and structure groups to create a community of learners.

- Facilitate team-building activities and encourage students to learn each others' names.
- Set clear expectations for how the class will interact:
    - Their text is a record of their learning and is to be used as a reference for any assignments or tests you give.
    - They will be doing the thinking, talking, and writing in your classroom.
    - They will be working and sharing their strategies and reasoning with their peers.
    - Mistakes and struggles are normal and necessary.

PREPARE FAMILIES AND CAREGIVERS

Prepare the Support

- Prepare a letter to send home on the first day. Visit the Texas Support Center for a sample letter.
- Encourage families and caregivers to read the introduction of the student book.
- Ensure that families and caregivers receive the module Family and Caregiver guide at the start of each module. They should also receive the topic Family Guide at the start of the first topic and each subsequent topic.
- Consider a Family Math Night some time within the first few weeks of the school year.
- Encourage families and caregivers to explore the Students & Caregivers Portal on the Texas Support Center at **www.CarnegieLearning.com/texas-help/students-caregivers**.

Students and Caregivers Portal

Research has proven time and again that family engagement greatly improves a student's likelihood of success in school.

The Students & Caregivers Portal on the Texas Support Center provides:

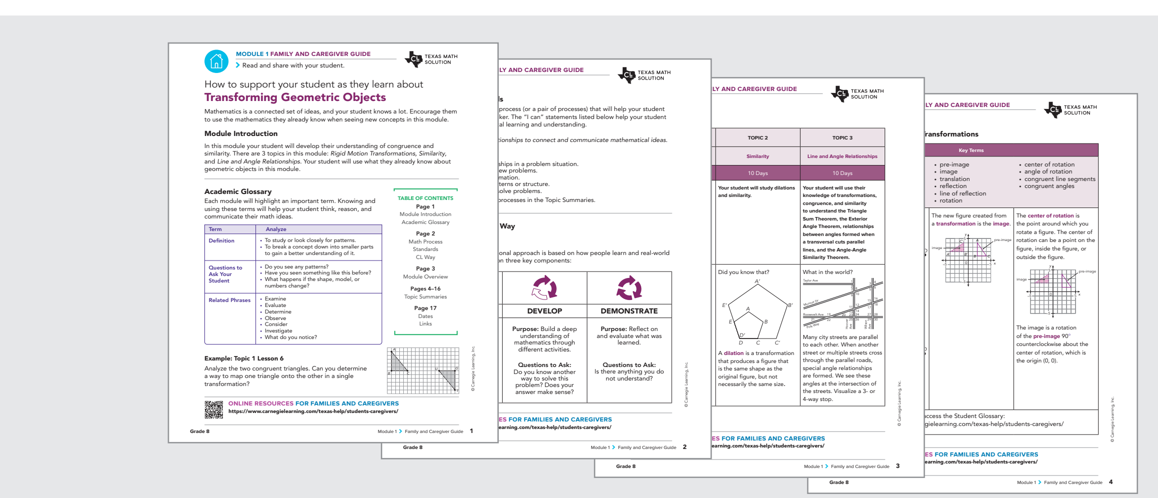
- Getting to Know Carnegie Learning video content to provide an introduction to the instructional materials and research.
- Articles and quick tip videos offering strategies for how families and caregivers can support student learning.

  Visit the Texas Support Center regularly to access new content and resources for students and caregivers as they learn mathematics in a variety of environments outside of the classroom.

Module Family and Caregiver Guides

Each module has a Family and Caregiver Guide available through the Students & Caregivers Portal on the Texas Support Center. Each module guide will provide a different highlight of the academic glossary, description and examples of TEKS Mathematical Process Standards, and an overview of a different component of our instructional approach known as The Carnegie Learning Way. Also included is a module overview of content, specific key terms, visual representations, and strategies students are learning in each topic of the module.

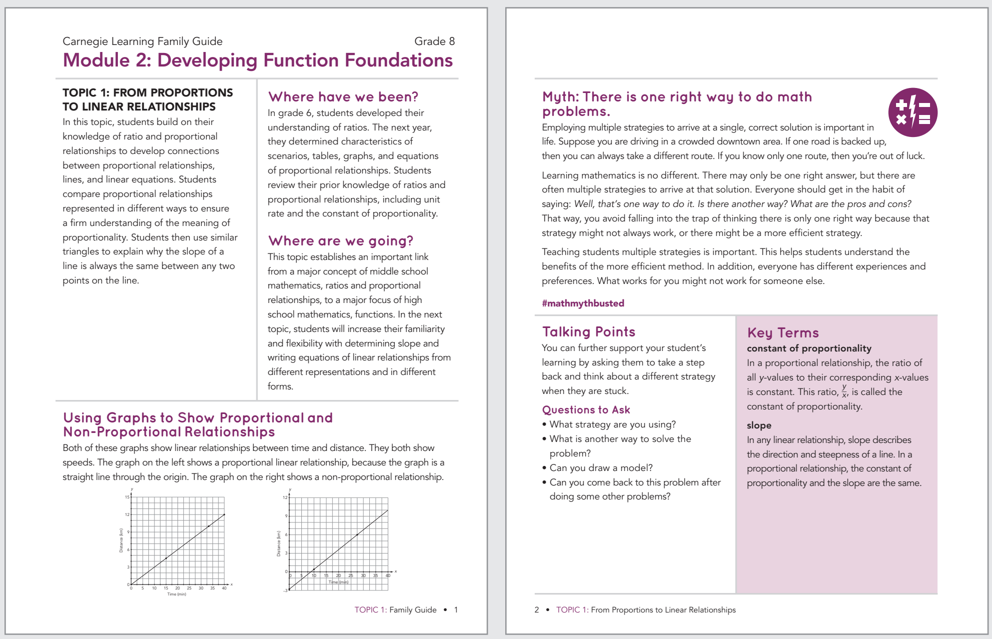
The purpose of the Family and Caregiver Guide is to bridge student learning in the classroom to student learning at home. The goal is to empower families and caregivers to understand the concepts and skills learned in the classroom so that families and caregivers can review, discuss, and solidify the understanding of these key concepts together. Videos will also be available on the Students & Caregivers Portal to provide added support.



Topic Family Guides

Each topic contains a Family Guide that provides an overview of the mathematics of the topic, how that math is connected to what students already know, and how that knowledge will be used in future learning. It provides families and caregivers an example of a math model or strategy their student is learning in the topic, busting of a math myth, questions to ask their student to support their learning, and a few of the key terms their student will learn.

We recognize that learning outside of the classroom is crucial to students' success at school. While we don't expect families and caregivers to be math teachers, the Family Guides are designed to assist families and caregivers as they talk to their students about what they are learning. Our hope is that both the students and their caregivers will read and benefit from the guides.



Carnegie Learning Family Guide Grade 8

Module 2: Developing Function Foundations

TOPIC 1: FROM PROPORTIONS TO LINEAR RELATIONSHIPS

In this topic, students build on their knowledge of ratio and proportional relationships to develop connections between proportional relationships, lines, and linear equations. Students compare proportional relationships represented in different ways to ensure a firm understanding of the meaning of proportionality. Students then use similar triangles to explain why the slope of a line is always the same between any two points on the line.

Where have we been?

In grade 6, students developed their understanding of ratios. The next year, they determined characteristics of scenarios, tables, graphs, and equations of proportional relationships. Students review their prior knowledge of ratios and proportional relationships, including unit rate and the constant of proportionality.

Where are we going?

This topic establishes an important link from a major concept of middle school mathematics, ratios and proportional relationships, to a major focus of high school mathematics, functions. In the next topic, students will increase their familiarity and flexibility with determining slope and writing equations of linear relationships from different representations and in different forms.

Using Graphs to Show Proportional and Non-Proportional Relationships

Both of these graphs show linear relationships between time and distance. They both show speeds. The graph on the left shows a proportional linear relationship, because the graph is a straight line through the origin. The graph on the right shows a non-proportional relationship.

TOPIC 1: Family Guide • 1

Myth: There is one right way to do math problems.

Employing multiple strategies to arrive at a single, correct solution is important in life. Suppose you are driving in a crowded downtown area. If one road is backed up, then you can always take a different route. If you know only one route, then you're out of luck.

Learning mathematics is no different. There may only be one right answer, but there are often multiple strategies to arrive at that solution. Everyone should get in the habit of saying: *Well, that's one way to do it. Is there another way? What are the pros and cons?* That way, you avoid falling into the trap of thinking there is only one right way because that strategy might not always work, or there might be a more efficient strategy.

Teaching students multiple strategies is important. This helps students understand the benefits of the more efficient method. In addition, everyone has different experiences and preferences. What works for you might not work for someone else.

#mathmythbusted

Talking Points

You can further support your student's learning by asking them to take a step back and think about a different strategy when they are stuck.

Questions to Ask

- What strategy are you using?
- What is another way to solve the problem?
- Can you draw a model?
- Can you come back to this problem after doing some other problems?

Key Terms

constant of proportionality

In a proportional relationship, the ratio of all *y*-values to their corresponding *x*-values is constant. This ratio, $\frac{y}{x}$, is called the constant of proportionality.

slope

In any linear relationship, slope describes the direction and steepness of a line. In a proportional relationship, the constant of proportionality and the slope are the same.

2 • TOPIC 1: From Proportions to Linear Relationships

You Might Be Wondering ...

Why are the student books consumable?

The Student Textbook contains all of the resources students need to complete the Learning Together component of the course. Students are to actively engage in this textbook, topic by topic, creating a record of their learning as they go. There is room to record answers, take notes, draw diagrams, and fix mistakes.

Why do we believe in our brand of blended: Learning Together and Learning Individually?

There has been a lot of research on the benefits of learning collaboratively. Independent practice is necessary for students to become fluent and automatic in a skill. A balance of these two pieces provides students with the opportunity to develop a deep conceptual understanding through collaboration with their peers, while demonstrating their understanding independently.

Why don't we have a Worked Example at the start of every lesson?

Throughout the Texas Math Solution, we do provide Worked Examples. Sweller and Cooper (1985) argue that Worked Examples are educationally efficient because they reduce working memory load. Ward and Sweller (1990) found that alternating between problem solving and viewing Worked Examples led to the best learning. Students often read Worked Examples with the intent to confirm that they understand the individual steps. However, the educational value of the Worked Example often lies in thinking about how the steps connect to each other and how particular steps might be added, omitted, or changed, depending on context.

Where are the colorful graphics to get students' attention?

Color and visuals make for stronger student engagement, right? Not quite. Our instructional materials have little extraneous material. This approach follows from research showing that "seductive details" used to spice up the presentation of material often have a negative effect on student learning (Mayer et al., 2001; Harp & Meyer, 1998). Students may not know which elements of an instructional presentation are essential and which are intended simply to provide visual interest. So, we focus on the essential materials. While we strive to make our educational materials attractive and engaging to students, research shows that only engagement based on the mathematical content leads to learning.

We're here for you.

The Carnegie Learning Texas Support Team is available to help with any issue at help@carnegielearning.com.

Monday–Friday
8:00 am–8:00 pm CT
via email, phone, or live chat.

Our expert team provides support for installations, networking, and technical issues, and can also help with general questions related to pedagogy, classroom management, content, and curricula.

Why so many words?

For students to deeply learn the math, they need to work through it. They also need to develop their work and demonstrate that they really understand it. Math isn't just about solving equations or formulas—it's about thinking, working through ideas, and seeing how the math relates to the real world.

Notes:

> If you have questions, reach out to us for support. Our team of master practitioners have been where you are. We made mistakes and we learned from them. We want to help you. We have many professional development options. Whether we come to your school for a workshop, join you in your classroom for modeling or coaching, or you join us online for a webinar or an entire course, our goal is to make sure you feel supported and prepared to use the tasks you'll find in this book to their fullest!

Kasey Bratcher, Senior VP of Professional Learning