## CL <br> TEXAS MATH SOLUTION

## Geometry

## Student Textbook Skills Program Edition SY 2022-2023

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## MANIFESTO

## Our Manifesto

WE BELIEVE that quality math education is important for all students, to help them develop into creative problem solvers, critical thinkers, life-long learners, and more capable adults.

WE BELIEVE that math education is about more than memorizing equations or performing on tests-it's about delivering the deep conceptual learning that supports ongoing growth and future development.

WE BELIEVE all students learn math best when teachers believe in them, expect them to participate, and encourage them to own their learning.

WE BELIEVE teachers are fundamental to student success and need powerful, flexible resources and support to build dynamic cultures of collaborative learning.

WE BELIEVE our learning solutions and services can help accomplish this, and that by working together with educators and communities we serve, we guide the way to better math learning.

## LONG + LIVE + MATH

## ACKNOWLEDGMENTS

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- Lumina Datamatics, Ltd.
- Mathematical Expressions, LLC


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www.pixabay.com

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Mathematics is so much more than memorizing rules. It is learning to reason, to make connections, and to make sense of the world. We believe in Learning by Doing ${ }^{\text {TM }}$-you need to actively engage with the content if you are to benefit from it. The lessons were designed to take you from your intuitive understanding of the world and build on your prior experiences to then learn new concepts. My hope is that these instructional materials help you build a deep understanding of math.

Sandy Bartle Finocchi, Chief Mathematics Officer

You have been learning math for a very long time-both in school and in your interactions in the world. You know a lot of math! In this course, there's nothing brand new. It all builds on what you already know. So, as you approach each activity, use all of your knowledge to solve problems, to ask questions, to fix mistakes, and to think creatively.

Amy Jones Lewis, Senior Director of Instructional Design

At Carnegie Learning, we have created an organization whose mission and culture is defined by your success. Our passion is creating products that make sense of the world of mathematics and ignite a passion in you. Our hope is that you will enjoy our resources as much as we enjoyed creating them.

Barry Malkin, CEO

## Module 1: Reasoning with Shapes

## Topic 1: Using a Rectangular Coordinate System

1 The Squariest Square
2 Hip to Be Square
3 Ts and Train Tracks
Parallel and Perpendicular Lines
4 Where Has Polly Gone?
Classifying Shapes on the Coordinate Plane
5 In and Out and All About $\begin{aligned} & \text { Area and Perimeter on the Coordinate Plane }\end{aligned}$
Topic 2: Rigid Motions on a Plane
1 Put Your Input In, Take Your Output Out
Geometric Components of Rigid Motions
2 Bow Thai
Translations as Functions
3 Staring Back at Me
Reflections as Functions
4 Turn Yourself Around
Rotations as Functions
5 Slide, Flip, Turn: The Latest Dance Craze?
Translations, Rotations, and Reflections on the Coordinate Plane
6 OKEECHOBEE
Reflectional and Rotational Symmetry

## Topic 3: Congruence Through Transformations

1 Elemental

2 ASA, SAS, and SSS
Proving Triangle Congruence Theorems
3 I Never Forget a Face
Using Triangle Congruence to Solve Problems

## Module 2: Establishing Congruence

## Topic 1: Composing and Decomposing Shapes

1 Running Circles Around Geometry
Using Circles to Make Conjectures
2 The Quad Squad
Conjectures About Quadrilaterals
3 Into the Ring
Constructing an Inscribed Regular Polygon
4 Tri- Tri- Tri- and Separate Them
Conjectures About Triangles
5 What's the Point?
Points of Concurrency

## Topic 2: Justifying Line and Angle Relationships

1 Proof Positive
Forms of Proof
2 A Parallel Universe
Proving Parallel Line Theorems
3 Ins and Outs
Interior and Exterior Angles of Polygons
4 Identical Twins
Perpendicular Bisector and Isosceles Triangle Theorems
5 Corners in a Round Room
Angle Relationships Inside and Outside Circles

## Topic 3: Using Congruence Theorems

1 SSS, SAS, AAS, ... S.O.S!

2 Props to You
Properties of Quadrilaterals
3 Three-Chord Song
Relationships Between Chords

## Module 3: Investigating Proportionality

## Topic 1: Similarity

1 Big, Little, Big Little
2 Similar Triangles or Not?
Establishing Triangle Similarity Criteria
3 Keep It in Proportion
Theorems About Proportionality
4 This Isn't Your Average Mean More Similar Triangles

5 Run It Up the Flagpole
Application of Similar Triangles
6 Jack's Spare Key Partitioning Segments in Given Ratios

## Topic 2: Trigonometry

1 Three Angle Measure Introduction to Trigonometry
2 Going on a Tangent Tangent Ratio and Inverse Tangent

3 Show Me a Sine Sine Ratio and Inverse Sine

4 Can I Get a Cosine?
Cosine Ratio and Inverse Cosine
5 Fishing for Complements Complement Angle Relationships

Module 4: Connecting Geometric and Algebraic Descriptions

## Topic 1: Circles and Volume

1 All Circles Great and Small
Similarity Relationships in Circles
2 A Piece of Pi
Sectors and Segments of a Circle
3 Do Me a Solid
Building Three-Dimensional Figures
4 Get to the Point
Building Volume and Surface Area Formulas for Pyramids, Cones, and Spheres

## Topic 2: Circles and Cross Sections

1 Give Me a Slice
Cross-Sections
$2 X^{2}$ Plus $Y^{2}$ Equals Radius ${ }^{2}$
Deriving the Equation for a Circle
3 A Blip on the Radar
Determining Points on a Circle

## Module 5: Making Informed Decisions

## Topic 1: Independence and Conditional Probability

1 What Are the Chances?
Compound Sample Spaces
2 And?
Compound Probability with And
3 Or?
Compound Probability with Or
4 And, Or, and More!
Calculating Compound Probability

## Topic 2: Computing Probabilities

1 Table Talk
Compound Probability for Data Displayed in Two-Way Tables
2 It All Depends
Conditional Probability
3 Give Me 5!
Permutations and Combinations
4 A Different Kind of Court Trial
Independent Trials
5 What Do You Expect?
Expected Value

## End of Course Topic

## Formative Assessment

1 Shape Up!
Performance Task
2 Map My Route
Performance Task
3 It's a Bird! It's a Plane! It's ... a Drone?
Performance Task

## Glossary

Each lesson has the same structure. Key features are noted.


## 1. Learning Goals

 Learning goals are stated for each lesson to help you take ownership of the learning objectives.
## 2. Connection

 Each lesson begins with a statement connecting what you have learned with a question to ponder.Return to this question at the end of this lesson to gauge your understanding.

## 3. Getting Started

 Each lesson begins with Getting Started. When working on Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.


## 4. Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about answer-getting. The process is important.
- Making mistakes are a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, worked examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.
5. Talk the Talk Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.


## ASSIGNMENT



## 6. Write

 Reflect on your work and clarify your thinking.7. Remember Take note of the key concepts from the lesson.

## 8. Practice

 Use the concepts learned in the lesson to solve problems.
## 9. Stretch

 Ready for a challenge?
## 10. Review

 Remember what you've learned by practicing concepts from previous lessons and topics.
## PROBLEM TYPES YOU WILL SEE

## Worked Example

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.


## Ask Yourself:

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?


## Worked Example

Consider $\triangle A B C$ and $\triangle A D E$ shown. They are both $45^{\circ}-45^{\circ}-90^{\circ}$ triangles.
leg length of $\triangle A D E$
hypotenuse length of $\triangle A B C$
Triangle $A B C$ is similar to $\triangle A D E$ by the AA Similarity Theorem.
Therefore, the lengths of the corresponding sides are proportional.

$$
\frac{A E}{A C}=\frac{A D}{A B}
$$

You can rewrite the proportion.
side length adjacent to $\angle A$

length of hypotenuse
So, given the same reference angle measure, the ratio $\frac{\text { side length adjacent to reference angle }}{\text { length of hypotenuse }}$ is constant in similar right triangles.

## Who's Correct?

## When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or not correct.


## Ask Yourself:

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

5. Jun says that the sine and cosecant value of every acute angle is less than 1. Todd says that the sine value of every acute angle is less than 1 , but the cosecant value is greater than 1 . Who is correct? Explain your reasoning.

## Thumbs Up

## When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.


## Ask Yourself:

- Why is this method correct?
- Have I used this method before?


## Gabriel



The side length ratios of the opposite side to the hypotenuse or the adjacent side to the hypotenuse is a percent. If the ratio is approximately 0.70 , that means the length of the side is about $70 \%$ the length of the hypotenuse.

## Thumbs Down

## When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.


## Ask Yourself:

- Where is the error?
- Why is it an error?
- How can I correct it?


## Alicia

The ratio $\frac{B C}{A B}$ is equal to the ratio $\frac{D C}{A D}$, because the ratio $\frac{\text { side opposite } \angle A}{\text { hypotenuse }}$ is the same for both $\triangle A B C$ and $\triangle A D C$, given the reference angle $A$, which is $45^{\circ}$.


## MATHEMATICAL PROCESS STANDARDS

## Texas Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The "I can" expectations listed below align with the TEKS Mathematical Process Standards and encourage students to develop their mathematical learning and understanding.

- Apply mathematics to problems arising in everyday life, society, and the workplace.


## I can:

- use the mathematics that I learn to solve real world problems.
- interpret mathematical results in the contexts of a variety of problem situations.
- Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

I can:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.
- Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.
- Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

I can:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.
- Create and use representations to organize, record, and communicate mathematical ideas.

I can:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.


## Analyze mathematical relationships to connect and communicate mathematical ideas.

I can:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.
- Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I can:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.


## ACADEMIC GLOSSARY

There are important terms you will encounter throughout this book. It is important that you have an understanding of these words as you get started on your journey through the mathematical concepts. Knowing what is meant by these terms and using these terms will help you think, reason, and communicate your ideas.

## ANALYZE

## Definition

To study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

## Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?


## EXPLAIN YOUR REASONING

## Definition

To give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

## Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

Visit the Students \& Caregivers Portal on the Texas Support
Center at www.
CarnegieLearning. com/texas-help to access the Mathematics Glossary for this course anytime, anywhere.

## Related Phrases

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match


## Related Phrases

- Show your work
- Explain your calculation
- Justify
- Why or why not?


## Related Phrases

- Show
- Sketch
- Draw
- Construct
- Create
- Plot
- Graph
- Write an equation
- Complete the table


## Related Phrases

- Predict
- Approximate
- Expect
- About how much?


## Related Phrases

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?


## REPRESENT

## Definition

To display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

## Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?


## ESTIMATE

## Definition

To make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

## Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?


## DESCRIBE

## Definition

To represent or give an account of in words. Describing communicates mathematical ideas to others.

## Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?


## Thought Bubbles

Look for these icons as you journey through the textbook. Sometimes they will remind you about things you already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are here to help and guide your learning.
side notes are included to provide helpful insights as you work.

## Student Lesson Overview Videos

Each lesson has a corresponding lesson overview video(s) for you to use and reference as you are learning. The videos provide an overview of key concepts, strategies, and/or worked examples from the lessons.


## Topic Summary

A Topic Summary is provided at the end of each topic. The Topic Summary lists all key terms of the topic and provides a summary of each lesson. Each lesson summary defines key terms and reviews key concepts, strategies, and/or worked examples.


## Mathematics Glossary

A course-specific mathematics glossary is available to utilize and reference while you are learning. Use the glossary to locate definitions and examples of math key terms.


## Addition Property of Equality

The addition property of equality states: "If $a=b$, then $a+c=b+c$.'

## Example

If $x=2$, then $x+5=2+5$, or $x+5=7$ is an example of the Addition Property of Equality

## Addition Rule for Probability

The Addition Rule for Probability states: "The probability that Event $A$ occurs or Event $B$ occurs is the probability that Event A occurs plus the probability that Event $B$ occurs minus the probability that both $A$ and $B$ occur."

$$
P(A \text { or } B)=P(A)+P(B)=P(A \text { and } B)
$$

## Example

You flip a coin two times. Calculate the probability of flipping a heads on the first flip or flipping a heads on the second flip.

Let $A$ represent the event of flipping a heads on the first flip. Let $B$ represent the event of flipping a heads on the second flip.
$P(A$ or $B)=P(A)+P(B)-P(A$ and $B)$
$P(A$ or $B)=\frac{1}{2}+\frac{1}{2}-\frac{1}{4}$
$P(A$ or $B)=\frac{3}{4}$
So, the probability of flipping a heads on the first flip or flipping a heads on the second
flip is $\frac{3}{4}$.
adjacent arcs
Adjacent arcs are two arcs of the same circle sharing a common endpoint.

## Example

Arcs $Z A$ and $A B$ are adjacent arcs.

adjacent side
The adjacent side of a triangle is the side adjacent to the reference angle that is not the hypotenuse.

## Example



## Module Family and Caregiver Guides

Each module guide will provide a different highlight of the academic glossary, description and examples of TEKS Mathematical Process Standards, and an overview of a different component of our instructional approach known as The Carnegie Learning Way. Also included is a module overview of content, specific key terms, visual representations, and strategies you are learning in each topic of the module.

The purpose of the Family and Caregiver Guides is to bridge student learning in the classroom to student learning at home. Our goal is to empower you and your family to understand the concepts and skills learned in the classroom so that you can review, discuss, and solidify the understanding of these key concepts together. Videos will also be available on the Students \& Caregivers Portal on the Texas Support Center to provide added support.


## Topic Family Guides

Each topic contains a Family Guide that provides an overview of the math of the topic and answers the questions, "Where have we been?" and "Where are we going?" Additional components of the Family Guide are, as follows: new notation or strategy taught in the topic, definitions of a few key terms, connection of math to the real world, related standardized test question sample, or talking points for caregivers to support your learning.

We recognize that learning outside of the classroom is crucial to student success at school. While we don't expect families and caregivers to be math teachers, the Family Guides are designed to assist families and caregivers as they talk to you about what you are learning. Our hope is that both you and your family will read and benefit from these guides.


## Students and Caregivers Portal

Research has proven time and again that family engagement greatly improves a student's likelihood of success in school.

The Students \& Caregivers Portal on the Texas Support Center provides:

- Getting to Know Carnegie Learning video content to provide an introduction to the instructional materials and research.
- Articles and quick tip videos offering strategies for how families and caregivers can support student learning.
- Access to instructional resources to support students and caregivers.
To access new content and resources, visit the Students and Caregivers Portal on the Texas Support Center at https://www.CarnegieLearning.com/texas-help/studentscaregivers/

