



**TEXAS MATH  
SOLUTION**

# Geometry

**Teacher's  
Implementation Guide  
Skills Program Edition  
SY 2022-2023**

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Teacher's Implementation Guide

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## Our Manifesto

**WE BELIEVE** that better math education is important for all students. It makes it possible for them to develop into creative problem solvers, critical thinkers, life-long learners, and more capable adults.

**WE BELIEVE** that better math education moves beyond memorizing equations or performing on tests. It's about delivering the deep conceptual learning that supports ongoing growth and continued development.

**WE BELIEVE** all students can learn math the right way—really learn and understand it more deeply—when teachers believe in them, expect them to participate, and encourage them to own their learning.

**WE BELIEVE** all teachers can teach math the right way when they really know the math, have the desire and right mindset, and get the resources and support they need to build cultures of collaborative learning.

**WE BELIEVE** our learning solutions and services can help accomplish all of this, and that by working together with educators and communities we serve, we guide the way to better math learning.

# LONG + LIVE + MATH



At Carnegie Learning, we choose the path proven most effective by research and classroom experience. We call that path the Carnegie Learning Way. Follow this code to take a look inside.

# ACKNOWLEDGMENTS

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- Amy Jones Lewis, Senior Director of Instructional Design
- Josh Fisher, Instructional Designer
- Victoria Fisher, Instructional Designer
- Janet Sinopoli, Instructional Designer

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“ Mathematics is so much more than rules and algorithms. It is learning to reason, to make connections, and to make sense of the world. We believe in Learning by Doing™—students need to actively engage with the content if they are to benefit from it. Your classroom environment will determine what type of discourse, questioning, and sharing will take place. Students deserve a safe place to talk, to make mistakes, and to build deep understanding of mathematics. My hope is that these instructional materials help you shift the mathematical authority in your class to your students. Be mindful to facilitate conversations that enhance trust and reduce fear. ”

**Sandy Bartle Finocchi, Chief Mathematics Officer**

“ Your students come to you, not as clean slates, but as messy boards full of knowledge that they have gained in previous math classes and also in the world. The lessons in this book are designed to build off what students already know. I encourage you to build confidence in your students by asking them questions to uncover what they already know, connecting their prior experiences with new ideas, providing them time to make connections and to persevere through problems, and giving only the support necessary to keep them on the right path. ”

**Amy Jones Lewis, Senior Director of Instructional Design**

“ At Carnegie Learning, we have created an organization whose mission and culture is defined by student success. Our passion is creating products that make sense of the world of mathematics and ignite a passion in students. Our hope is that students will enjoy our resources as much as we enjoyed creating them. ”

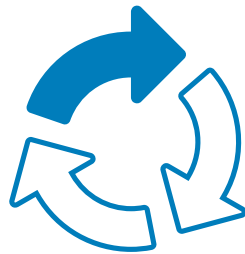
**Barry Malkin, CEO**



At Carnegie Learning, we choose the path proven most effective by research and classroom experience. We call that path the **Carnegie Learning Way**.

## Our Instructional Approach

Carnegie Learning’s instructional approach is based upon the collective knowledge of our researchers, instructional designers, cognitive learning scientists, and master practitioners. It is based on a scientific understanding of how people learn and a real-world understanding of how to apply the science to the classroom. At its core, our instructional approach is based on three simple yet critical components:



### ENGAGE

**Activate student thinking by tapping into prior knowledge and real-world experiences.**

Provide an introduction that generates curiosity and plants the seeds for deeper learning.



### DEVELOP

**Build a deep understanding of mathematics through a variety of activities.**

Students encounter real-world problems, sorting activities, worked examples, and peer analysis—in an environment where collaboration, conversations, and questioning are routine practices.



### DEMONSTRATE

**Reflect on and evaluate what was learned.**

Ongoing formative assessment underlies the entire learning experience, driving real-time adjustments, next steps, insights, and measurements.





## Our Research

Carnegie Learning has been deeply immersed in research ever since it was founded by cognitive and computer scientists from Carnegie Mellon University. Our research extends far beyond our own walls, playing an active role in the constantly evolving field of cognitive and learning science. Our internal researchers collaborate with a variety of independent research organizations, tirelessly working to understand more about how people learn, and how learning is best facilitated. We supplement this information with feedback and data from our own products, teachers,

and students, to continuously evaluate and elevate our instructional approach and its delivery.

## Our Support

We're all in. In addition to our instructional resources, implementing Carnegie Learning in your classroom means you get access to an entire ecosystem of ongoing classroom support, including:

- **Professional Learning:** Our team of Master Math Practitioners is always there for you, from implementation to math academies to a variety of other options to help you hone your teaching practice.
- **Texas Support Center:** We've customized a Support Center just for you and your students. The Texas Support Center provides articles and videos to help you implement the Texas Math Solution, from the basics to get you started to more targeted support to guide you as you scaffold instruction for all learners in your classroom. Visit [www.CarnegieLearning.com/texas-help](http://www.CarnegieLearning.com/texas-help) to explore online and to access content that you can also share with your students and their caregivers.
- **MyCL:** This is the central hub that gives you access to all of the products and resources that you and your students will need. Visit MyCL at [www.CarnegieLearning.com/login](http://www.CarnegieLearning.com/login).
- **LONG + LIVE + MATH:** When you join this community of like-minded math educators, suddenly you're not alone. You're part of a collective, with access to special content, events, meetups, book clubs, and more. Because it's a community, it's constantly evolving! Visit [www.longlivemath.com](http://www.longlivemath.com) to get started.

Scan this code to visit the Texas Support Center and look for references throughout the Front Matter to learn more about the robust resources you will find in the Support Center.



## Our Blend of Learning

The Texas Math Solution delivers instructional resources that make learning math attainable for all students. Learning Together and Learning Individually resources work in parallel to engage students with various learning experiences they need to understand the mathematics at each grade level.

For **Learning Together**, the student textbook is a consumable resource that empowers students to become creators of their mathematical knowledge. This resource is designed to support teachers in facilitating active learning so that students feel confident in sharing ideas, listening to each other, and learning together.

Over the course of a year, based on the recommended pacing, teachers will spend approximately 60% of their instructional time teaching whole-class activities as students learn together.

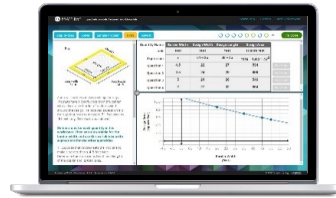
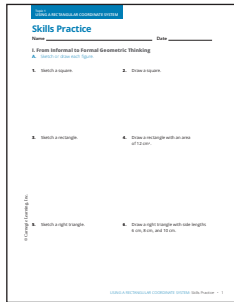
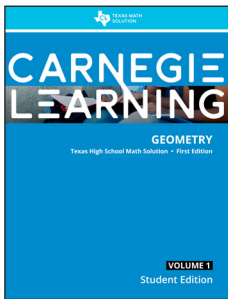
For **Learning Individually**, the Skills Practice provides students the opportunity to engage with problems that target each lesson's skills, concepts, and applications. This resource is designed to target discrete skills for development and mastery, therefore, scaffolding and extension opportunities are provided in the problem sets.

An additional Learning Individually resource is MATHia®, an intelligent software that provides just-in-time support and tracks student progress against fine-grained skills to deliver the right content they need to become proficient with the mathematics.

Over the course of the year, based on the recommended pacing, teachers will spend approximately 40% of their instructional time monitoring students as they work and learn individually.

## Learning Together

## Learning Individually



### TEXTBOOK

I am a record of student thinking, reasoning, and problem solving. My lessons allow students to build new knowledge based upon prior knowledge and experiences, apply math to real-world situations, and learn together in a collaborative classroom.

My purpose is to create mathematical thinkers who are active learners that participate in class.

### SKILLS PRACTICE

I am targeted practice of each lesson's skills, mathematical concepts, and applications for each topic in the student textbook.

My purpose is to provide additional problem sets for teachers to assign as needed for additional practice or remediation.

### MATHia

I am designed to empower students to learn individually at their own pace with sophisticated AI technology that personalizes their learning experiences, while giving teachers real-time insights to monitor student progress.

My purpose is to coach students alongside teachers as students learn, practice, do, and look forward.



Visit the Texas Support Center for additional information on the Learning Individually resources.

# TABLE OF CONTENTS

---

## Module 1: Reasoning with Shapes

### Topic 1: Using a Rectangular Coordinate System

- 1 The Squariest Square  
From Informal to Formal Geometric Thinking
- 2 Hip to Be Square  
Constructing a Coordinate Plane
- 3 Ts and Train Tracks  
Parallel and Perpendicular Lines
- 4 Where Has Polly Gone?  
Classifying Shapes on the Coordinate Plane
- 5 In and Out and All About  
Area and Perimeter on the Coordinate Plane

### Topic 2: Rigid Motions on a Plane

- 1 Put Your Input In, Take Your Output Out  
Geometric Components of Rigid Motions
- 2 Bow Thai  
Translations as Functions
- 3 Staring Back at Me  
Reflections as Functions
- 4 Turn Yourself Around  
Rotations as Functions
- 5 Slide, Flip, Turn: The Latest Dance Craze?  
Translations, Rotations, and Reflections on the Coordinate Plane
- 6 OKEECHOBEE  
Reflectional and Rotational Symmetry

### Topic 3: Congruence Through Transformations

- 1 Elemental  
Formal Reasoning in Euclidean Geometry
- 2 ASA, SAS, and SSS  
Proving Triangle Congruence Theorems
- 3 I Never Forget a Face  
Using Triangle Congruence to Solve Problems

---

## Module 2: Establishing Congruence

### Topic 1: Composing and Decomposing Shapes

- 1 Running Circles Around Geometry  
Using Circles to Make Conjectures
- 2 The Quad Squad  
Conjectures About Quadrilaterals
- 3 Into the Ring  
Constructing an Inscribed Regular Polygon
- 4 Tri- Tri- Tri- and Separate Them  
Conjectures About Triangles
- 5 What's the Point?  
Points of Concurrency

### Topic 2: Justifying Line and Angle Relationships

- 1 Proof Positive  
Forms of Proof
- 2 A Parallel Universe  
Proving Parallel Line Theorems
- 3 Ins and Outs  
Interior and Exterior Angles of Polygons
- 4 Identical Twins  
Perpendicular Bisector and Isosceles Triangle Theorems
- 5 Corners in a Round Room  
Angle Relationships Inside and Outside Circles

### Topic 3: Using Congruence Theorems

- 1 SSS, SAS, AAS, ... S.O.S.!  
Using Triangle Congruence to Determine Relationships Between Segments
- 2 Props to You  
Properties of Quadrilaterals
- 3 Three-Chord Song  
Relationships Between Chords

---

## Module 3: Investigating Proportionality

### Topic 1: Similarity

- 1 Big Little, Big Little  
Dilating Figures to Create Similar Figures
- 2 Similar Triangles or Not?  
Establishing Triangle Similarity Criteria
- 3 Keep It in Proportion  
Theorems About Proportionality
- 4 This Isn't Your Average Mean  
More Similar Triangles
- 5 Run It Up the Flagpole  
Application of Similar Triangles
- 6 Jack's Spare Key  
Partitioning Segments in Given Ratios

### Topic 2: Trigonometry

- 1 Three Angle Measure  
Introduction to Trigonometry
- 2 Going on a Tangent  
Tangent Ratio and Inverse Tangent
- 3 Show Me a Sine  
Sine Ratio and Inverse Sine
- 4 Can I Get a Cosine?  
Cosine Ratio and Inverse Cosine
- 5 Fishing for Complements  
Complement Angle Relationships

---

## Module 4: Connecting Geometric and Algebraic Descriptions

### Topic 1: Circles and Volume

- 1 All Circles Great and Small  
Similarity Relationships in Circles
- 2 A Piece of Pi  
Sectors and Segments of a Circle
- 3 Do Me a Solid  
Building Three-Dimensional Figures
- 4 Get to the Point  
Building Volume and Surface Area Formulas for Pyramids, Cones, and Spheres

### Topic 2: Circles and Cross Sections

- 1 Give Me a Slice  
Cross-Sections
- 2  $X^2$  Plus  $Y^2$  Equals Radius<sup>2</sup>  
Deriving the Equation for a Circle
- 3 A Blip on the Radar  
Determining Points on a Circle

---

## Module 5: Making Informed Decisions

### Topic 1: Independence and Conditional Probability

- 1 What Are the Chances?  
Compound Sample Spaces
- 2 And?  
Compound Probability with *And*
- 3 Or?  
Compound Probability with *Or*
- 4 And, Or, and More!  
Calculating Compound Probability

### Topic 2: Computing Probabilities

- 1 Table Talk  
Compound Probability for Data Displayed in Two-Way Tables
- 2 It All Depends  
Conditional Probability
- 3 Give Me 5!  
Permutations and Combinations
- 4 A Different Kind of Court Trial  
Independent Trials
- 5 What Do You Expect?  
Expected Value





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## End of Course Topic

### Formative Assessment

- 1 Shape Up!  
Performance Task
- 2 Map My Route  
Performance Task
- 3 It's a Bird! It's a Plane! It's ... a Drone?  
Performance Task

### Glossary

In a word, every single piece of Carnegie Learning's Texas Math Solution is intentional. Our instructional designers work alongside our master math practitioners, cognitive scientists, and researchers to intentionally design, draft, debate, test, and revise every piece, incorporating the latest in learning science.

## Intentional Mathematics Design

Carnegie Learning's Texas Math Solution is thoroughly and thoughtfully designed to ensure students build the foundation they'll need to experience ongoing growth in mathematics.

### Mathematical Coherence

The arc of mathematics develops coherently, building understanding by linking together within and across grades, so students can learn concepts more deeply and apply what they've learned to more complex problems going forward.

### Mathematical Process Standards

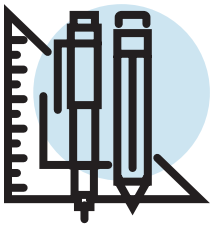
Carnegie Learning is organized around the Mathematical Process Standards to encourage experimentation, creativity, and false starts, which is critical if we expect students to tackle difficult problems in the real world, and persevere when they struggle.

### Multiple Representations

Carnegie Learning recognizes the importance of connecting multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

### Transfer

Carnegie Learning focuses on developing transfer. Doing A and moving on isn't the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.



# Texas Math Solution Overview

The instructional materials in the Carnegie Learning Texas Math Solution cover functions, figures, and data sets, from their fundamental concepts to the connections between them. We think about these interrelated ideas in a holistic way to integrate students' understanding with their developing habits of mind.

## WHAT ARE THE CARNEGIE LEARNING TEXAS MATH SOLUTION GUIDING PRINCIPLES?

The Texas Math Solution has been strongly influenced by scientific research into the learning process and student motivations for academic success. Its guiding principles are active learning, discourse through collaboration, and personalized learning.

Our classroom activities emphasize active learning and making sense of the mathematics, and we ask deep questions that require students to thoroughly understand the material.

### Active Learning

The research makes it clear that students need to actively engage with content in order to benefit from it. Studies show that as instruction moves up the scale from entirely passive to fully interactive, learning becomes more robust. All of the activities we provide for the classroom encourage students to be thoughtful about their work, to consider hypotheses and conclusions from different perspectives, and to build a deep understanding of mathematics. The format of the student text, as a consumable workbook, supports active instruction.

### Discourse through Collaborative Learning

Effective collaboration encourages students to articulate their thinking, resulting in self-explanation. Reviewing other students' approaches and receiving feedback on their own provides further metacognitive feedback. Collaborative problem-solving encourages an interactive instructional model, and we have looked to research to provide practical guidance for making collaboration work. The collaborative tasks within our lessons are designed to promote active dialogue centered on structured activities.

### Personalized Learning

One of the ways to build intrinsic motivation is to relate tasks to students' existing interests. Research has proven that problems that capture student interests are more likely to be taken seriously. In the textbook, problems often begin with the students' intuitive understanding of the world and build to an abstract concept, rather than the other way around.

## HOW IS THE CONTENT DEVELOPED IN A MATHEMATICALLY COHERENT WAY?

Throughout the high school math courses of the Texas Math Solution, students examine and investigate functions, figures, and data sets. Within each category, we strive to extend and connect students' experience in middle school around the critical mathematical ideas of transformation, equivalence and congruence, and proportionality and similarity.

### Functions • Figures • Data Sets

Transformation  
Equivalence  
& Congruence  
Proportionality  
& Similarity

## Transformation

Transforming functions and figures builds from an understanding of the fundamental behaviors of translations, rotations, reflections, and dilations. These behaviors apply in the same ways to different function types in algebra and to geometric figures on the plane. Understanding the structure of transformations leads to connections across multiple domains in multiple courses.

## Equivalence & Congruence

Equivalence is approached in two ways. First, understanding equivalence using multiple relationships of the same function or data set reveals different properties or key characteristics. Second, understanding equivalence in terms of expressions allows students to compose and decompose equations, make sense of solutions, and solve problems. Congruence is treated similarly: understanding congruence using rigid motions highlights key characteristics that are true for both figures, which leads to establishing triangle congruence criteria, an important underpinning for formal proof. The concept of equivalence is extended to the analysis of data, where students learn the critical skill of representing data in equivalent but differently useful ways, enabling them to make analyses and decisions.

## Proportionality & Similarity

Developing proportional reasoning is a life-long journey that begins in middle school: from ratios and proportions to understanding how linear functions relate to sequences with common differences and how exponential functions relate to sequences with common ratios. Exploring dilations and the relationships that hold true in similar figures develops spatial reasoning. Analyzing similarity in right triangles extends to right triangle trigonometry, connecting the algebra and geometry domains.

## HOW IS THE MATHEMATICS CONTENT DELIVERED TO PROMOTE PRODUCTIVE MATHEMATICAL PROCESS STANDARDS?

Students deserve math learning that develops them into creative problem solvers, critical thinkers, life-long learners, and more capable adults, while teachers deserve materials that will support them in bringing learning to life. There are three organizing principles that guide these instructional materials.

### Seeing Connections

Activities make use of models—e.g., real-world situations, graphs, diagrams, and worked examples—to help students see and make connections between different topics. In each lesson, learning is linked to prior knowledge and experiences so that students build their new understanding on the firm foundation of what they already know. We help students move from concrete representations and an intuitive understanding of the world to more abstract representations and procedures. Activities thus focus on real-world situations to demonstrate the usefulness of mathematics.

### Exploring Structure

Questions are phrased in a way that promotes analysis, develops higher-order-thinking skills, and encourages the seeking of mathematical relationships. Students inspect a given function, figure, or data set, and in each case, they are asked to discern a pattern or structure. We want students to become fluent in seeing how the structure of each representation—verbal, graphic, numerical, and algebraic—reveals properties of the function it defines. We want students to become fluent at composing and decomposing expressions, equations, and data sets. We want them to see how the structure of transformations applies to all function types and rigid motions. As students gain proficiency in manipulating structure, they become capable of comparing, contrasting, composing, decomposing, transforming, solving, representing, clarifying, and defining the characteristics of functions, figures, and data sets.

### Reflecting and Communicating

A student-centered approach focuses on students thinking about and discussing mathematics as active participants in their own learning. Through articulating their thinking in conversations with a partner, in a group, or as a class, students integrate each piece of new knowledge into their existing cognitive structure. They use new insights to build new connections. Through collaborative tasks and the examination of peer work—both within their groups and from examples provided in the lessons—students give and receive feedback, which leads to verifying, clarifying, and/or improving the strategy.

# CONTENT AND ALIGNMENT

## Geometry Content at a Glance

This Year at a Glance highlights the sequence of topics and the number of blended instructional days (1 day is a 45-minute instructional session) allocated for Geometry in the Texas Math Solution. The suggested pacing information includes time for assessments, providing you with an instructional map that covers 180 days of the school year. As you set out at the beginning of the year, we encourage you to still modify this plan as necessary to meet the range of needs for your students.

### Texas Geometry: Year at a Glance

\*1 Day Pacing = 45-minute Session

Module	Topic	Pacing	TEKS
Process Standards are embedded in every module: G.1A, G.1B, G.1C, G.1D, G.1E, G.1F, G.1G			
1 Reasoning with Shapes	1: Using a Rectangular Coordinate System	19	G.2B, G.2C, G.3C, G.4A, G.5A, G.5B, G.5C, G.9B, G.10B, G.11A, G.11B
	2: Rigid Motions on a Plane	19	G.3A, G.3B, G.3C, G.3D, G.5B, G.6A, G.6C
	3: Congruence Through Transformations	12	G.2B, G.4A, G.4B, G.4C, G.4D, G.6B, G.6C
		50	
2 Establishing Congruence	1: Composing and Decomposing Shapes	18	G.4A, G.4B, G.5A, G.5B, G.5C, G.5D, G.6E, G.9B, G.12A
	2: Justifying Line and Angle Relationships	21	G.3B, G.4A, G.4B, G.5A, G.5B, G.5C, G.6A, G.6B, G.6D, G.9B, G.12A
	3: Using Congruence Theorems	9	G.5A, G.5B, G.5C, G.6B, G.6E, G.12A
		48	
3 Investigating Proportionality	1: Similarity	20	G.2A, G.2B, G.3A, G.4C, G.5B, G.5C, G.6D, G.7A, G.7B, G.8A, G.8B
	2: Trigonometry	13	G.9A, G.9B
		33	
4 Connecting Geometric and Algebraic Descriptions	1: Circles and Volume	13	G.10A, G.10B, G.11B, G.11C, G.11D, G.12B, G.12C, G.12D
	2: Circles and Cross Sections	8	G.2B, G.10A, G.12E
		21	
5 Making Informed Decisions	1: Independence and Conditional Probability	8	G.13C, G.13D
	2: Computing Probabilities	11	G.13A, G.13B, G.13C, G.13D, G.13E
		19	
End of Course Formative Assessment	Performance Tasks	9	G.2B, G.6B, G.9A, G.9B, G.11B
		9	
Total Days:		180	

# CONNECTING CONTENT AND PRACTICE

Each lesson of the Texas Math Solution has the same structure. This consistency allows both you and your students to track your progress through each lesson. Key features of each lesson are noted.

## Lesson Structure

### ENGAGE

#### Establishing Mathematical Goals to Focus Learning

Create a classroom climate of collaboration and establish the learning process as a partnership between you and students.

Communicate continuously with students about the learning goals of the lesson to encourage self-monitoring of their learning.

Visit the Texas Support Center for additional guidance on how to foster a classroom environment that promotes collaboration and communication.



### LESSON STRUCTURE

Each lesson has the same structure. Key features are noted.

The diagram shows a lesson page for 'The Quad Squad' (Lesson 2). The title is 'The Quad Squad' with the subtitle 'Conjectures About Quadrilaterals'. The page is divided into sections: 'Warm Up' with four quadrilateral diagrams, 'Learning Goals' with three bullet points, and 'Key Terms' with four terms. A circled '2' in the bottom left corner indicates a question: 'You have classified quadrilaterals by their side measurements and side relationships. What conjectures can you make about different properties of quadrilaterals?'. The page number '1' is in the bottom right corner.

#### 1. Learning Goals

Learning goals are stated for each lesson to help you take ownership of the learning objectives.

#### 2. Connection

Each lesson begins with a statement connecting what you have learned with a question to ponder.

Return to this question at the end of this lesson to gauge your understanding.



Mathematics is the science of patterns. So, we encourage students throughout this course to notice, test, and interpret patterns in a variety of ways—to put their “mental tentacles” to work in every lesson, every activity. Our hope is that this book encourages you to do the same for your students, and create an environment in your math classroom where productive and persistent learners develop and thrive.

Josh Fisher, Instructional Designer



### Activating Student Thinking

Your students enter each class with varying degrees of experience and mathematical success. The focus of the Getting Started is to tap into prior knowledge and real-world experiences, to generate curiosity, and to plant seeds for deeper learning. Pay particular attention to the strategies students use, for these strategies reveal underlying thought processes and present opportunities for connections as students proceed through the lesson.

### Supporting Emergent Bilingual Students

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they complete the Getting Started activities in each lesson.

#### 3. Getting Started

Each lesson begins with Getting Started. When working on Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.

3

**GETTING STARTED**

#### Cattywampus

A quadrilateral may be convex or concave. The quadrilaterals you are most familiar with—trapezoids, parallelograms, rectangles, rhombi, and squares—are convex. A convex polygon contains all of the line segments connecting any pair of points. It is concave if and only if at least one of its interior angles is greater than  $180^\circ$ .


Think

about:


Why can a concave quadrilateral have only one angle greater than  $180^\circ$ ?

Consider the two quadrilaterals shown. A quadrilateral has exactly two diagonals.

Convex



Concave



1. Draw the diagonals in the two quadrilaterals shown. What do you notice?
2. Make a conjecture about the diagonals of a convex quadrilateral and about the diagonals of a concave quadrilateral.

The diagonals of any convex quadrilateral create two pairs of vertical angles and four linear pairs of angles.

3. Label the vertices of the convex quadrilateral as well as the point of intersection of the diagonals. Identify each pair of vertical angles and each linear pair of angles.

2 • TOPIC 1: Composing and Decomposing Shapes

FM-14 • Lesson Structure

Connecting Content and Practice • FM-23

## DEVELOP

### Aligning Teaching to Learning

Students learn when they are actively engaged in a task: reasoning about the math, writing their solutions, justifying their strategies, and sharing their knowledge with peers.

Support productive struggle by allowing students time to engage with and persevere through the mathematics.

Support student-to-student discourse as well as whole-class conversations that elicit and use evidence of student thinking.

**4** **ACTIVITY 2.1** Quadrilaterals Formed Using Concentric Circles

Let's explore the diagonals of different convex quadrilaterals. Consider a pair of concentric circles with center  $A$ . Diameter  $BC$  is shown.

**ACTIVITY 2.2** Quadrilaterals Formed Using a Circle

In the previous activity, you drew quadrilaterals using a pair of concentric circles. Now let's draw quadrilaterals using only one circle. Circle  $P$  with diameter  $QR$  is shown.

**ACTIVITY 2.3** Making Conjectures About Quadrilaterals

In the previous two activities, you used the properties of the diagonals to discover each member of the quadrilateral family. You investigated the relationships between the diagonals of quadrilaterals.

**1. Make a conjecture about the diagonals of the described quadrilaterals. Explain your reasoning using examples.**

- parallelograms
- rectangles
- quadrilaterals with pairs of adjacent congruent sides

6 •

LESSON 2: The Quad Squad • 9

**4. Activities**  
You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

**Remember:**

- It's not just about answer-getting. The process is important.
- Making mistakes are a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, worked examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

Lesson Structure • FM-15



### Supporting Emergent Bilingual Students

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they engage in mathematical discourse throughout each lesson.



## DEMONSTRATE

### Ongoing Formative Assessment Drives Instruction

For students to take responsibility for their own learning, they need to be encouraged to self-assess. Students can use the Talk the Talk to monitor their own progress towards mastering the learning goals. Listen and review their answers and explanations and provide feedback to help them improve their understanding.

As you plan the next lesson, consider the connections you can make to build off the strengths or fill any gaps identified from this formative assessment.

#### 5. Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

NOTES


**5 TALK the TALK**

**Zukei, Don't Bother Me**


Remember, a Zukei puzzle is a Japanese logic puzzle in which a grid is presented with a number of points shown at different intersections. Each grid is presented along with the name of a geometric figure. The goal of the puzzle is to determine which points on the grid are the vertices of the named geometric figure.

1. For each Zukei puzzle, identify and connect the vertices that form each shape. There is only one correct answer.


a. Square




b. Rectangle




c. Rhombus



d. Parallelogram



e. Trapezoid



16 • TOPIC 1: Composing and Decomposing Shapes

FM-16 • Lesson Structure

## Student Lesson Overview Videos

Each lesson has a corresponding lesson overview video(s) for students to utilize and reference to support their learning. The videos provide an overview of key concepts, strategies, and/or worked examples from the lessons.



# Assignment

An intentionally designed Assignment follows each lesson.

There is one Assignment per lesson. Lessons often span multiple days. Be thoughtful about which portion of the Assignment students can complete based on that day's progress.

The **Stretch** section is not necessarily appropriate for all learners. Assign this to students who are ready for more advanced concepts.

The **Review** section provides spaced practice of concepts from the previous lesson and topic and of the fluency skills important for the course.

## ASSIGNMENT

**Assignment**      LESSON 2: The Quad Squad

**6 Write**  
Define each term in your own words. Use the words *diagonal*, *interior angle*, and *midsegment* in your definitions.

- kite
- isosceles trapezoid
- cyclic quadrilateral

**8 Practice**

- Determine which quadrilateral each letter in the diagram represents using the list shown.

Kites	Square
Rectangles	Parallelogram
Rhombi	Isosceles trapezoid


2. State as many properties as you can for each quadrilateral.

a. Rectangle	b. Square
c. Kite	d. Parallelogram
e. Rhombus	f. Isosceles trapezoid

3. Describe how to construct each quadrilateral.

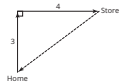
- Square WXYZ given diagonal  $\overline{WY}$ .
- Parallelogram RSTU given diagonal  $\overline{RT}$ .

**9 Stretch**  
Create a Zukei puzzle for an isosceles trapezoid in which the bases do not lie on the grid lines. Use a minimum of 10 dots. Make sure that your puzzle has only one correct answer.




**10 Review**

- Write a conjecture about alternate interior angles. Draw an example to test your conjecture.
- Draw examples of inscribed angles that intercept the same arc of a circle. What conjecture can you make about the measures of the inscribed angles?
- Jay walks 3 blocks north and then 4 blocks east to get to the store. If he walks straight back home, how far does Jay walk in all?

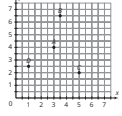


4. TV screen sizes are given by their diagonal measure from a top corner to the opposite bottom corner. What is the approximate size of this TV to the nearest inch?



5. Use the coordinate plane to approximate each distance. Write each answer as a decimal to the nearest hundredth.

- The distance between point A and point C
- The distance between point D and point E



**6. Write**  
Reflect on your work and clarify your thinking.

**7. Remember**  
Take note of the key concepts from the lesson.

**8. Practice**  
Use the concepts learned in the lesson to solve problems.

**9. Stretch**  
Ready for a challenge?

**10. Review**  
Remember what you've learned by practicing concepts from previous lessons and topics.

Assignment • FM-17

# Topic Summary

A Topic Summary is provided for students at the end of each topic. The Topic Summary lists all key terms of the topic and provides a summary of each lesson. Each lesson summary defines key terms and reviews key concepts, strategies, and/or worked examples.

## Using a Rectangular Coordinate System Summary

### KEY TERMS

- sketch
- draw
- conjecture
- auxiliary line
- construct
- compass
- straightedge
- point
- line
- line segment
- midpoint
- segment bisector
- perpendicular bisector
- diagonal
- transformation
- rigid motion
- translation
- reflection
- rotation
- Distance Formula
- Midpoint Formula
- composite figure
- regular polygon

### LESSON 1

#### The Squariest Square

When you **sketch** a geometric figure, you draw a rough picture of it. When you **draw** geometric figures, you use a coordinate plane to draw exact figures.

A **conjecture** is a mathematical statement that you think is true. You can move from mathematical statements to conjectures by looking for patterns.

An **auxiliary line** is a line drawn in a diagram to help in solving a problem. For example, the dashed line through point C is an auxiliary line drawn to bisect the interior angles of a triangle.

### LESSON 2

#### Hip to Be Square

When you **construct** geometric figures, you create exact figures without measurements, using only a **compass** and a **straightedge**. A compass is a tool used to create arcs and circles. A straightedge is a ruler with no numbers.

A **point** is described simply as a location. A point in geometry has no size or shape, but it is often represented using a dot. In a diagram, a point can be labeled using a capital letter. A **line** is described as a straight, continuous arrangement of an infinite number of points. A line has an infinite length, but no width. Arrowheads are used to indicate that a line extends infinitely in opposite directions. In a diagram, a line can be labeled with a lowercase letter positioned next to the arrowhead. A **line segment** is a part of a line between two points on the line, called the endpoints. A distance along a line is the length of a line segment connecting two points on the line. A line segment  $\overline{AB}$  has the distance  $AB$ .

The **midpoint** of a segment is the point that divides the segment into 2 congruent segments. A **segment bisector** is a line, line segment, or ray that divides a line segment into two line segments of equal length. The basic geometric construction used to locate a midpoint of a line segment is called bisecting a line segment.

You can use patty paper to bisect a line segment.

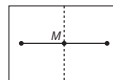
Draw a line on the paper.



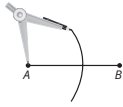
Fold the paper so the endpoints of the line segment lie on top of each other.



Open the paper. The crease represents the segment bisector, and the midpoint is located where the crease intersects the line segment.

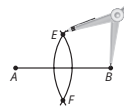


To construct a segment bisector using only a compass and straightedge, you make use of the fact that all the radii of a circle have an equal length.



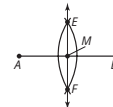
**Construct an Arc**

Open the radius of the compass to more than half the length of  $\overline{AB}$ . Use endpoint A as the center and construct an arc.



**Construct Another Arc**

Keep the compass radius and use point B as the center as you construct an arc. Label the points formed by the intersection of the arcs point E and point F.



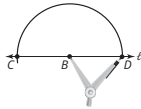
**Construct a Line**

Connect points E and F. Line segment EF is the segment bisector of  $\overline{AB}$ . The point M represents the midpoint of  $\overline{AB}$ .

Line EF bisects  $\overline{AB}$ . Point M is the midpoint of  $\overline{AB}$ .

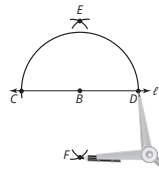
A **perpendicular bisector** is a line, line segment, or ray that bisects a line segment and is also perpendicular to the line segment.

You can use a compass and straightedge to create a perpendicular bisector.



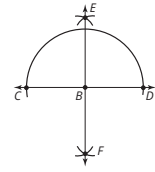
**Construct an Arc**

Use B as the center and construct an arc. Label the intersections points C and D.



**Construct other Arcs**

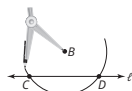
Open the compass larger than the radius. Use C and D as centers and construct



**Construct a Line**

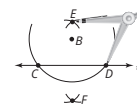
Use a straightedge to connect points E and F. Line EF is perpendicular

You can also construct a perpendicular line through a point not on a line.



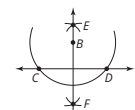
**Construct an Arc**

Use B as the center and construct an arc. Label the intersections points C and D.



**Construct other Arcs**

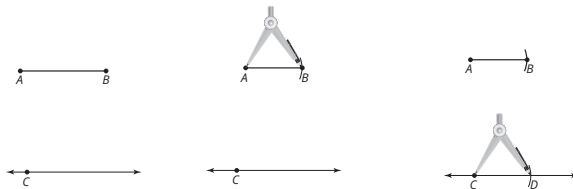
Open the compass larger than the radius. Use C and D as centers and construct arcs above and below the line. Label the intersection points E and F.



**Construct a Line**

Use a straightedge to connect points E and F. Line EF is perpendicular to  $\overline{CD}$ .

You can duplicate a line segment by constructing an exact copy of the original line segment.



**Construct a Starter Line**

Use a straightedge to construct a starter line longer than  $\overline{AB}$ . Label point C on the line

**Measure Length**

Set your compass to the length AB.

**Copy Length**

Place the compass to C max point D on the new segment

Line segment CD is a duplicate of  $\overline{AB}$ .

# Problem Types You Will See

Lessons include a variety of problem types to engage students in reasoning about the math.

## PROBLEM TYPES YOU WILL SEE

### Worked Example

#### When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

#### Ask Yourself:

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

### Worked Example

Consider  $\triangle ABC$  and  $\triangle ADE$  shown. They are both  $45^\circ$ - $45^\circ$ - $90^\circ$  triangles.

$$\frac{\text{leg length of } \triangle ADE}{\text{hypotenuse length of } \triangle ABC}$$

Triangle  $ABC$  is similar to  $\triangle ADE$  by the AA Similarity Theorem.

Therefore, the lengths of the corresponding sides are proportional.

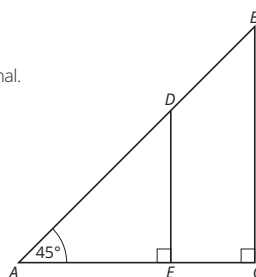
$$\frac{AE}{AC} = \frac{AD}{AB}$$

You can rewrite the proportion.

side length adjacent to  $\angle A$

$$\frac{AE}{AD} = \frac{AC}{AB}$$

length of hypotenuse



So, given the same reference angle measure, the ratio  $\frac{\text{side length adjacent to reference angle}}{\text{length of hypotenuse}}$  is constant in similar right triangles.

### Who's Correct?

#### When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or not correct.

#### Ask Yourself:

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?



5. Jun says that the sine and cosecant value of every acute angle is less than 1. Todd says that the sine value of every acute angle is less than 1, but the cosecant value is greater than 1. Who is correct? Explain your reasoning.

FM-18 • Problem Types You Will See

### Worked Examples

Research shows students learn best when they are actively engaged with a task. Many students need a model to know how to engage effectively with Worked Examples. Students need to be able to question their understanding, make connections with the steps, and ultimately self-explain the progression of the steps and the final outcome. Worked Examples provide a means for students to view each step taken to solve the example problem. The questions that follow are designed to serve as a model for self-questioning and self-explanations. They represent and mimic an internal dialogue about the mathematics and the strategies. This approach doesn't allow students to skip over the example without interacting with it, thinking about it, and responding to the questions. This approach will help students develop the desired habits of mind for being conscientious about the importance of steps and their order.

### Who's Correct?

"Who's Correct?" problems are an advanced form of correct vs. incorrect responses. In this problem type, students are not given who is correct. Students have to think more deeply about what the strategies really mean and whether each of the solutions makes sense. Students will determine what is correct and what is incorrect, and then explain their reasoning. These types of problems will help students analyze their own work for errors and correctness.

## Thumbs Up/ Thumbs Down

Thumbs Up problems provide a framework that allows students the opportunity to analyze viable methods and problem-solving strategies. Questions are presented to help students think deeper about the various strategies, and to focus on an analysis of correct responses. Research shows that only providing positive examples does not eliminate some of the things students may think; it is also efficient to show negative examples. From the incorrect responses, students learn to determine where the error in calculation is, why the method is an error, and also how to correct the method to correctly calculate the solution.

### Thumbs Up

#### When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.

#### Ask Yourself:

- Why is this method correct?
- Have I used this method before?

*Gabriel*



The side length ratios of the opposite side to the hypotenuse or the adjacent side to the hypotenuse is a percent. If the ratio is approximately 0.70, that means the length of the side is about 70% the length of the hypotenuse.

### Thumbs Down

#### When you see a Thumbs Down icon:

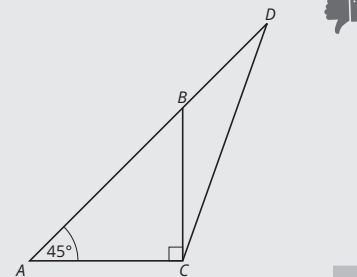
- Take your time to read through the incorrect solution.
- Think about what error was made.

#### Ask Yourself:

- Where is the error?
- Why is it an error?
- How can I correct it?

*Alicia*

The ratio  $\frac{BC}{AB}$  is equal to the ratio  $\frac{DC}{AD}$ , because the ratio  $\frac{\text{side opposite } \angle A}{\text{hypotenuse}}$  is the same for both  $\triangle ABC$  and  $\triangle ADC$ , given the reference angle A, which is  $45^\circ$ .



# Promoting Self-Reflection

## Thought Bubbles

Look for these icons as you journey through the textbook. Sometimes they will remind you about things you already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are here to help and guide your learning.



Side notes are included to provide helpful insights as you work.

## Thought Bubbles

Thought bubbles are embedded throughout the Texas Math Solution promote productive reflection by reminding students to stop and think. This feature is used in a variety of ways: it may remind students to recall a previous mathematical concept, help students develop expertise to think through problems, and occasionally, present a fun fact.



A mathematician is an artist who works with patterns. I think the beauty of mathematics lies in the new connections you can make to express the patterns around you, no matter your age. The art is in the process, not the outcome. When we can get students to see the beauty of the mathematics, and equip them with the tools to express themselves mathematically, then we can truly create critical thinkers.



Victoria Fisher, Instructional Designer

# Mathematical Process Standards

## Note

Each lesson provides opportunities for students to think, reason, and communicate their mathematical understanding. However, it is your responsibility as a teacher to recognize these opportunities and incorporate these practices into your daily rituals. Expertise is a long-term goal, and students must be encouraged to apply these practices to new content throughout their school career.

## MATHEMATICAL PROCESS STANDARDS

### Texas Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The “I can” expectations listed below align with the TEKS Mathematical Process Standards and encourage students to develop their mathematical learning and understanding.

#### ► Apply mathematics to problems arising in everyday life, society, and the workplace.

I can:

- use the mathematics that I learn to solve real world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

#### ► Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

I can:

- explain what a problem “means” in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

FM-20 • Mathematical Process Standards



## Supporting Students to Use Mathematical Tools

Visit the Texas Support Center for strategies to support students as they use mathematical tools, including formula charts and reference sheets.



► **Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.**

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

► **Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.**

I can:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

► **Create and use representations to organize, record, and communicate mathematical ideas.**

I can:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

Mathematical Process Standards • FM-21

### Note

When you are facilitating each lesson, listen carefully and value diversity of thought, redirect students' questions with guiding questions, provide additional support with those struggling with a task, and hold students accountable for an end product. When students share their work, make your expectations clear, require that students defend and talk about their solutions, and monitor student progress by checking for understanding.

There is one more page of mathematical process standards that is not provided here, but is available in the Student Textbook Front Matter.

## Supporting ALL Learners

Visit the Texas Support Center for facilitation strategies to support ALL students as they engage in the Mathematical Process Standards.



# Academic Glossary

## Language Expectations

It is critical for students to possess an understanding of the language of their text. Students must learn to read for different purposes and write about what they are learning. Encourage students to become familiar with the key words and the questions they can ask themselves when they encounter these words.

It is our recommendation to be explicit about your expectations of language used and the way students write responses throughout the text. Encourage students to answer questions with complete sentences. Complete sentences help students reflect on how they arrived at a solution, make connections between topics, and consider what a solution means both mathematically as well as in context.

## ACADEMIC GLOSSARY

There are important terms you will encounter throughout this book. It is important that you have an understanding of these words as you get started on your journey through the mathematical concepts. Knowing what is meant by these terms and using these terms will help you think, reason, and communicate your ideas.

Visit the Students & Caregivers Portal on the Texas Support Center at [www.CarnegieLearning.com/texas-help](http://www.CarnegieLearning.com/texas-help) to access the Mathematics Glossary for this course anytime, anywhere.



### ANALYZE

#### Definition

To study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

#### Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

### Related Phrases

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

### EXPLAIN YOUR REASONING

#### Definition

To give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

#### Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

### Related Phrases

- Show your work
- Explain your calculation
- Justify
- Why or why not?

Academic Glossary • FM-23



## Supporting Students at Varying Levels of Language Proficiency

Visit the Texas Support Center for guidance on how to leverage the Academic Glossary to support students at varying levels of language proficiency.

#### Related Phrases

- Show
- Sketch
- Draw
- Construct
- Create
- Plot
- Graph
- Write an equation
- Complete the table

## REPRESENT

### Definition

To display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

### Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

#### Related Phrases

- Predict
- Approximate
- Expect
- About how much?

## ESTIMATE

### Definition

To make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

### Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

#### Related Phrases

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

## DESCRIBE

### Definition

To represent or give an account of in words. Describing communicates mathematical ideas to others.

### Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

FM-24 • Academic Glossary

### Ask Yourself

The Ask Yourself questions help students develop the proficiency to explain to themselves the meaning of problems.

### Real-World Context

Real-world contexts confirm concrete examples of mathematics. The scenarios in the lessons help students recognize and understand that quantitative relationships seen in the real world are no different than quantitative relationships in mathematics. Some problems begin with a real-world context to remind students that the quantitative relationships they already use can be formalized mathematically. Other problems will use real-world situations as an application of mathematical concepts.

## Mathematics Glossary

A course-specific mathematics glossary is available for students to utilize and reference during their learning. Definitions and examples of key terms are provided in the glossary.

# The Modeling Process

Modeling is the process of choosing appropriate mathematical tools to analyze and understand real-world phenomena and to make decisions accordingly. The Modeling Process provides a structure to help students become better problem solvers. In the textbook, students will encounter activities that explicitly guide them through the four steps of the Modeling Process. As they progress through high school mathematics, they should start to use this process intuitively.

## Notice and Wonder

Gather information, notice patterns, and formulate mathematical questions about what you notice.

## Organize and Mathematize

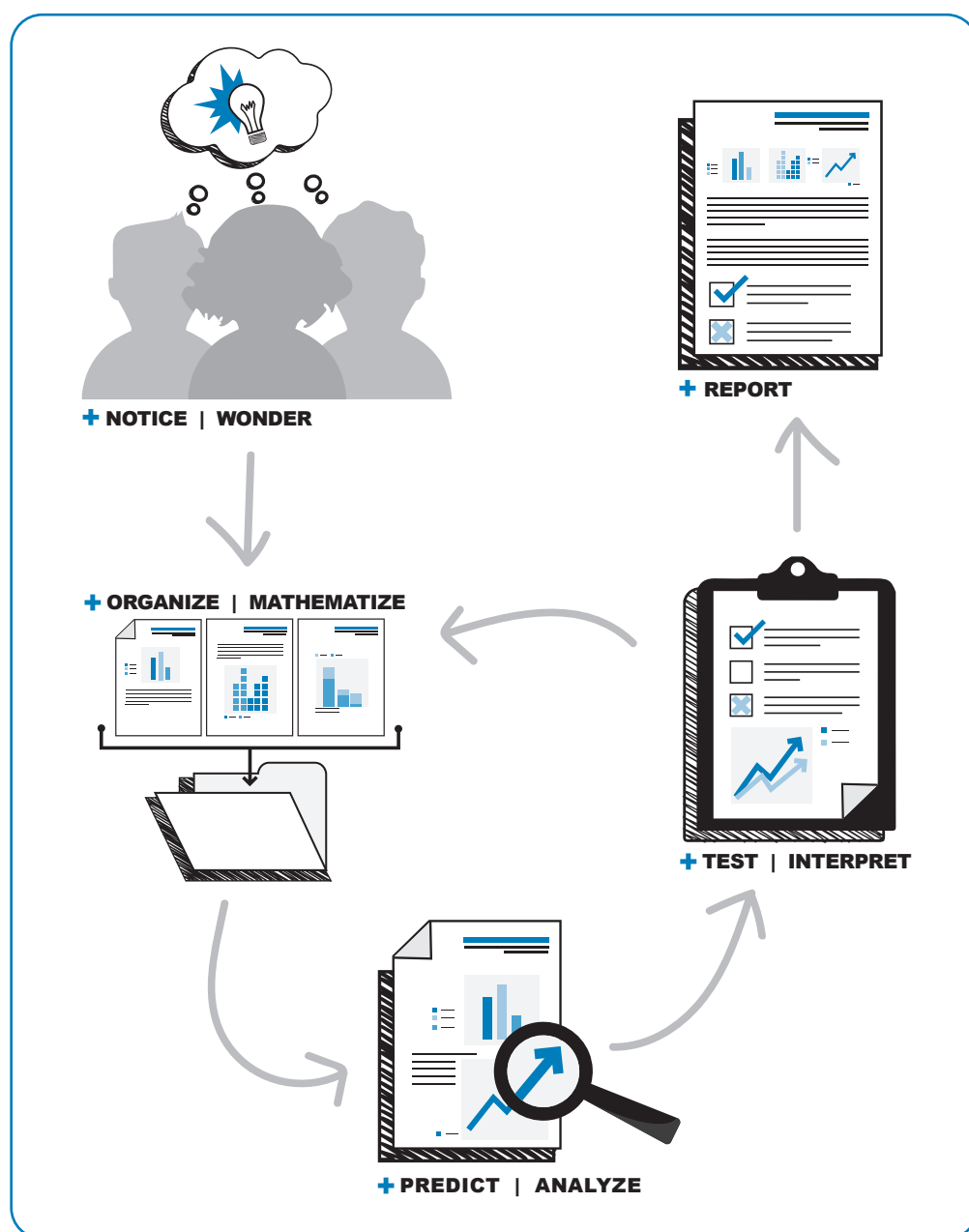
Organize your information and represent it using mathematical notation.

## Predict and Analyze

Extend the patterns created, complete operations, make predictions, and analyze the mathematical results.

## Test and Interpret

Interpret your results and test your mathematical predictions in the real world. Make adjustments as necessary.



## Teacher's Implementation Guide

The Teacher's Implementation Guide (TIG) is designed to fully support a wide range of teachers implementing our materials: from first-year teachers to 30-year veterans and from first-time Carnegie Learning users to master practitioners.

One goal in developing the TIG was to make our instructional design apparent to the users.

The lessons of each topic were written to be accessible to the full range of learners. With every instructional decision you make, keep in mind your mathematical objectives for the topic and module and the course. Plan each lesson by thinking about how you will create access for your particular group of students, maintain access and pace throughout the lesson, and assess their understanding along the way. We recommend that you do the math in each topic before implementing the activities with your specific group of students.

### WHAT MAKES THIS TIG USEFUL?

#### Effective Lesson Design

Each lesson has a consistent structure for teachers and students to follow. The learning experiences are engaging and effective for students.

#### Pacing

Each course is designed to be taught in a 180-day school year. Pacing suggestions are provided for each lesson. Each day in the pacing guide is equivalent to about a 45-minute instructional session.

#### Instructional Supports

Guiding questions are provided for teachers to use as they're circulating the room, as well as differentiation strategies, common student misconceptions, and student look-fors.

#### Clearly Defined Mathematics

The content and instructional goals are clearly described at the module, topic, lesson, and activity levels.

The TIG is critical to understanding how the mathematics that students encounter should be realized in the classroom. The TIG describes the depth of understanding that students need to develop for each standard and a pathway for all learners to be successful. It provides differentiation strategies to support students who struggle, to extend certain activities for students who are advanced in their understanding of the content, and to support emergent bilingual students.

Visit the Texas Support Center at [www.carnegielearning.com/texas-help](http://www.carnegielearning.com/texas-help).

[CarnegieLearning.com/texas-help](http://www.carnegielearning.com/texas-help) for additional resources to support you anytime, anywhere.



# Module and Topic Overviews

“Teachers must first develop their ideas about where the curriculum program is going mathematically (curriculum vision) before deciding whether the curriculum materials will help them reach that mathematical goal (curriculum trust)” (Drake & Sherin, 2009, p. 325).

You are responsible for teaching the essential concepts associated with a particular course. You need to understand how activities within lessons build to achieve understanding within topics, and how topics build to achieve understanding throughout the course. In the Texas Math Solution, Carnegie Learning seeks to establish a shared curriculum vision with you.

## Module Overview

Each module begins with an overview that describes the reasoning behind the name, the mathematics being developed, the connections to prior learning, the connections to future learning.

## Topic Overview

A Topic Overview describes how the topic is organized, the entry point for students, how a student will demonstrate understanding, why the mathematics is important, how the activities promote expertise in the mathematical process standards, what materials are needed, examples of new tools and notations, and more detailed information to help with pacing.

The screenshot shows a curriculum overview page for 'Using a Rectangular Coordinate System' (Topic 1 Overview) under 'Module 1 Overview: Reasoning with Shapes'. The page features a blue header with the Carnegie Learning (CL) logo and a Texas state outline. The main content area is white with blue accents. It includes a quote at the top, a 'Why is this Reasoning?' section with a grid icon, and several paragraphs of text describing the topic's content and student activities. A footer at the bottom right reads 'TOPIC 1: Using a Rectangular Coordinate System · 1'.

**Module 1 Overview**  
Reasoning with Shapes

"In fact, as one of the compass is involved a transformations, and The whole developm ones, the geometry t remarks with surpris square! experiences square." (Developing B

**Using a Rectangular Coordinate System**  
Topic 1 Overview

**How is Using a Rectangular Coordinate System organized?**

In this topic, students investigate the properties of squares and use transformations of squares to construct a coordinate plane. They develop strategies for determining the perimeters and areas of rectangles, triangles, parallelograms, and composite plane figures on the coordinate plane. Students also explore the effects of proportional and non-proportional changes to the dimensions of a plane figure on its perimeter and area.

Students begin the course exploring the nature of geometric reasoning. They first analyze three angles formed in a diagram comprising three congruent, adjacent squares. Students use measuring tools to determine the sum of the angle measures and then compare their results with their classmates' results. They learn that investigating with measuring tools is an informal strategy to look at the structure of a figure and develop ideas about its characteristics. Considering a conjecture about the sum of the angles, students then investigate the angle measures with a differently-sized version of the diagram. Students learn that considering more cases is an important step in geometric reasoning. They then use patty paper to explore the diagram without measuring tools, a step towards more formal reasoning. While stopping short of a formal geometric proof, students consider how adding auxiliary lines is helpful in seeing relationships among lines and angles in the diagram. More advanced students should be encouraged to write a formal proof.

Next, students consider the structure of the coordinate plane. Recognizing that it is composed of parallel and perpendicular lines, students construct a coordinate plane using transformations of a square. To do this, students learn basic constructions—constructing perpendicular lines, constructing parallel lines, and duplicating segments. Students use transformations to explore the relationship between perpendicular lines. They use translations to prove that the slopes of parallel lines are equal; they use rotations to prove that if two lines are perpendicular, then the slopes of the lines are negative reciprocals. Students write the equations of lines parallel or perpendicular to given lines at given points.

Students then use a Venn diagram to review properties of triangles and quadrilaterals. To determine the side lengths of figures on the coordinate plane, students derive the Distance Formula from the Pythagorean Theorem. They use the slope criteria for parallel and perpendicular lines to verify parallel sides and perpendicular angles where appropriate. They combine these skills to classify triangles and quadrilaterals on the coordinate plane and to compose a quadrilateral given three of the vertices. The term *midpoint* is defined, and students use the Midpoint Formula to investigate the pattern of shapes that is formed by connecting the midpoints of the sides.

Students continue to practice using the Distance Formula when calculating the area and perimeter of triangles, rectangles and composite figures on the coordinate plane in real-world scenarios.

TOPIC 1: Using a Rectangular Coordinate System · 1

# Facilitation Notes

For each lesson, you are provided with detailed facilitation notes to fully support your planning process. This valuable resource provides point-of-use support that serves as your primary resource for planning, guiding, and facilitating student learning.

## 1. Materials

Materials required for the lesson are identified.

## 2. Lesson Overview

The Lesson Overview sets the purpose and describes the overarching mathematics of the lesson, explaining how the activities build and how the concepts are developed.

## 3. TEKS Addressed

The focus TEKS for each lesson are listed. Carnegie Learning recognizes that some lessons could list several TEKS based on the skills needed to complete the activities, however, the TEKS listed are what the lesson is focused on developing and mastering.

## 4. ELPS Addressed

The English Language Proficiency Standards for each lesson are listed. As you plan, consider these ELPS and determine the instructional strategies that you will use to meet these ELPS.

## 5. Essential Ideas

These statements are derived from the standards and state the concepts students will develop.

1

## 2

# Hip to Be Square

Constructing a Coordinate Plane

**MATERIALS**

- Compasses
- Patty paper
- Straightedges

**2 Lesson Overview**  
Students consider how a coordinate plane can be constructed using squares. They start by completing geometric constructions using patty paper or a compass and a straightedge. They analyze worked examples to construct perpendicular lines, perpendicular bisectors, and duplicated line segments. Students construct a square and then describe how rigid motions can be applied to create a coordinate plane. They then describe rigid motions that can be used to create two-dimensional shapes on a coordinate plane. Students also relate a sequence of translations to the slope of a line.

**3 Geometry**  
**Coordinate and Transformational Geometry**  
**(3) The student uses the process skills to generate and describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity).**  
**The student is expected to:**  
(C) identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane.

**Logical Argument and Constructions**  
**(5) The student uses constructions to validate conjectures about geometric figures.**  
**The student is expected to:**  
(B) construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, the perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line using a compass and a straightedge.  
(C) use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships.

**4 ELPS**  
1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

**5 Essential Ideas**

- When you construct geometric figures, you create exact figures using only a compass and straightedge or patty paper.
- The midpoint of a segment is a point that divides the segment into two congruent segments.

LESSON 2: Hip to Be Square • 1

## 6. Lesson Structure

This section highlights how the parts of the lesson fit within the instructional design: Engage, Develop, and Demonstrate. A summary of each activity included.

## 7. Pacing

Lessons often span more than one 45-minute class period. Suggested pacing is provided for each lesson so that the entire course can be completed in a school year.

- A segment bisector is a line, line segment, or ray that divides a line segment into two line segments of equal length.
- A perpendicular bisector is a line, line segment, or ray that bisects a line segment and is also perpendicular to the line segment.
- Any point on a perpendicular bisector is equidistant to the endpoints of the original segment it bisects.
- The diagonals of a square are congruent, bisect each other, are perpendicular to one another, and bisect the angles of the square.
- A coordinate plane can be created by constructing a square and applying rigid motion transformations to the square.

## 6 Lesson Structure and Pacing: 2 Days 7

### Day 1

#### Engage

##### Getting Started: Getting Back in Shape

Students distinguish between sketching, drawing, and constructing geometric figures. They draw a right angle and explain their method.

#### Develop

##### Activity 2.1: Constructing a Perpendicular Line

Students view the coordinate plane as a geometric figure formed by two perpendicular lines, the  $x$ -axis and  $y$ -axis, and a composition of an infinite number of uniform squares, each with the dimension of one unit. They use patty paper to create a line perpendicular to a given segment. Students also analyze worked examples that provide steps to construct a line perpendicular to a given line through a point on the given line and through a point not on the given line. They practice these constructions and then conjecture about the distance from any point on a perpendicular bisector of a line segment to the endpoints of the segment.

### Day 2

##### Activity 2.2: Constructing a Square

Students analyze a worked example that demonstrates the use of construction tools to duplicate a line segment. They practice this construction and then construct a square with a given side length. Finally, they make conjectures about the segments and angles formed by the diagonals of a square and use patty paper to justify their conjectures.

##### Activity 2.3: Rigid Motions

Students recall the definitions of *rigid motion*, *translation*, *reflection*, and *rotation*. They analyze one strategy to construct a coordinate plane using translations and then describe other methods. Two-dimensional figures are drawn on the coordinate plane, and students describe how the shapes could be formed using transformations of line segments.

#### Demonstrate

##### Talk the Talk: Walking on a Thin Line

Students describe a sequence of transformations that can be performed on a given line segment that results in a line. They also determine the slope of the line and write its equation.

2 • TOPIC 1: Using a Rectangular Coordinate System



8

## Getting Started: Getting Back in Shape

ENGAGE

9

### Facilitation Notes

In this activity, students distinguish between sketching, drawing, and constructing geometric figures. They draw a right angle and explain their method.

Ask a student to read the introduction and definitions aloud.  
Discuss as a class.

Have students work with a partner or in a group to complete Question 1.  
Share responses as a class.

10

### Misconceptions

- Students may trace the corner of a piece of patty paper and assume it is a right angle. Discuss the issues with that assumption.
- Students may sketch the angle. Remind them that drawing the angle means that they can use tools for precision.

### Questions to ask

- Do the lengths of the rays make a difference?
- Do the rays need to be the same length?
- What are some examples of right angles that you see in the classroom?

11

12

### Summary

Sketches of geometric figures are created without tools. Drawings are created with measuring tools.

## Activity 2.1 Constructing a Perpendicular Line



DEVELOP

### Facilitation Notes

In this activity, students view the coordinate plane as a geometric figure formed by two perpendicular lines, the  $x$ -axis and  $y$ -axis, and as the composition of an infinite number of uniform squares, each with the dimension of one unit. They use patty paper to create a line perpendicular to a given segment. Students also analyze worked examples that provide steps to construct a line perpendicular to a given line through a point on the given line and through a point not on the given line. They practice these constructions and then conjecture about the distance from any point on a perpendicular bisector of a line segment to the endpoints of the segment.

LESSON 2: Hip to Be Square • 3

## 8. Facilitation Notes by Activity

A detailed set of guidelines walks the teacher through implementing the Getting Started, Activities, and Talk the Talk portions of the lesson. These guidelines include an activity overview, grouping strategies, guiding questions, possible student misconceptions, differentiation strategies, student look-fors, and an activity summary.

## 9. Activity Overview

Each set of Facilitation Notes begins with an overview that highlights how students will actively engage with the task to achieve the learning goals.

## 10. Misconceptions

Common student misconceptions are provided in places where students may overgeneralize mathematical relationships or have confusion over the vocabulary used. Suggestions are provided to address the given misconception.

## 11. White Space

The white space in each margin is intentional. Use this space to make additional planning notes or to reflect on the implementation of the lesson.

## 12. Summary

The summary brings the activity to closure. This statement encapsulates the big mathematical ideas of the particular activity.

### 13. Differentiation Strategies

To assist all students, instructional strategies are provided that benefit the full range of learners.

### 14. Differentiation Strategies

To extend an activity for students who are ready to advance beyond the scope of the activity, additional challenges are provided.

### 15. As Students Work, Look For

These notes provide specific language, strategies, and/or errors to look and listen for as you circulate and monitor students working in pairs or groups. You can incorporate these ideas when students share their responses with the class.

13

#### Differentiation strategies

- To scaffold support, have them use construction tools to do the worked examples for themselves throughout the lesson.
- To assist all students,
  - Allow them to use either patty paper or a compass and straightedge to complete constructions, unless the specific tools to be used are noted in the question.
  - Provide options for placing patty paper constructions in the textbook. Suggest students either staple their patty paper in the book or transfer the solution from their patty paper to the diagram in the textbook. To transfer a solution, trace the solution on the back of the patty paper using a pencil, align the diagram on the patty paper with the diagram in the textbook, and then retrace the answer on the patty paper so that the lead transfers to the diagram in the textbook.

#### Differentiation strategies

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Ask a student to read the introduction. Discuss as a class. Analyze the worked example as a class.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

14

#### Differentiation strategies

- To extend the activity, prior to starting the activity, have students draw a line segment on patty paper using a straightedge and then create a line perpendicular to the segment without using any other tools. They can then refer to the worked example to verify their procedure.
- To scaffold support,
  - Discuss the meaning of the term *bisect*. The root word *sect* means to cut, as in dissect. The prefix *bi-* means two, as in a bicycle having two wheels. Therefore, the term *bisect* means to cut into two equal parts.
  - Review the terms *equal* and *congruent*. The term *equal* is used when referring to numbers; for example,  $JK = KL$ , the measure of  $\overline{JK}$  equals the measure of  $\overline{KL}$ , or  $JK = 5$  cm. The term *congruent* is used when referring to figures; for example,  $\overline{JK} \cong \overline{KL}$ .

#### As students work, look for

A crease in the student's patty paper that appears perpendicular to the drawn line segment. If the crease does not appear to intersect the line segment at right angles, the line segment was not aligned on top of itself when making the fold. If they are constructing a perpendicular bisector, the endpoints should align when making the fold.

#### Questions to ask

- How are a midpoint and segment bisector related? How are they different?

15

4 • TOPIC 1: Using a Rectangular Coordinate System

### Note: Alternative Grouping Strategies

Differentiation strategies that provide other grouping strategies, such as whole class participation and the jigsaw method, are sometimes recommended for specific activities. These are listed as Differentiation Strategies.

More information about grouping strategies is available online in the Texas Support Center at [www.CarnegieLearning.com/texas-help](http://www.CarnegieLearning.com/texas-help).

- Does a midpoint always lie on the segment bisector? Why?
- Can a line segment have more than one midpoint?
- Does the line segment have to have a horizontal orientation on the patty paper for this to work? Could you have drawn a line segment with a vertical or diagonal orientation?
- When bisecting the line segment, is there more than one way to fold the paper so that the endpoints of the line segment lie on top of each other?
- Did Thomas place the endpoints of the line segment on top of each other before folding the crease in the patty paper? What did he do instead?
- How do you know the two line segments created by the midpoint and segment bisector are congruent?

Ask a student to read the definition following Question 3. Analyze the worked example as a class.

16

Have students work with a partner or in a group to complete Questions 4 through 6. Share responses as a class.

**As students work, look for**

- Placement of the compass on points of the line other than the endpoints.
- Arcs that are less than half the distance between the two endpoints.

**Questions to ask**

- Explain the steps to create a perpendicular bisector.
- Are  $\overline{BC}$  and  $\overline{BD}$  both radii of circle  $B$ ? How does this prove that  $B$  is the midpoint of  $\overline{CD}$ ?
- How did you determine the setting of the compass needed to draw the first arc?
- Did you change the setting of the compass before you drew the second arc? Does it matter?
- What is the difference in the first step when constructing a segment bisector and the first step when constructing a perpendicular line through a point on the line?

Have students work with a partner or in a group to analyze the worked example and complete Questions 7 and 8. Share responses as a class.

**As students work, look for**

Confusion regarding the initial setting of the compass. The setting is not specified because any length for the radius is acceptable.

LESSON 2: Hip to Be Square • 5

## 16. Grouping Strategies

Suggestions appear to help chunk each activity into manageable pieces and establish the cadence of the lesson.

Learning is social. Whether students work in pairs or in groups, the critical element is that they are engaged in discussion. Carnegie Learning believes, and research supports, that student-to-student discourse is a motivating factor; it increases student learning and supports ongoing formative assessment. Additionally, it provides students with opportunities to have mathematical authority.

Working collaboratively can, when done well, encourage students to articulate their thinking (resulting in self-explanation) and also provides metacognitive feedback (by reviewing other students' approaches and receiving feedback on their own).

The student discussion is then transported to a classroom discussion facilitated by the teacher to guarantee all necessary mathematics is addressed, once again, with the same benefits of discussion.

## 17. Questions to Ask

The overarching questioning strategies throughout each lesson promote analysis and higher-order thinking skills beyond simple yes or no responses.

These questions can be used to gather information, probe thinking, make the mathematics explicit, and encourage reflection and justification as students are working together or when they are sharing responses as a class. These questions are an embedded formative assessment strategy to provide feedback as students are actively engaged in learning.

17

### Questions to ask

- What suggestion would you give to Aaron?
- How do you know when the setting on the compass is greater than half the length of  $\overline{RS}$ ?
- If Aaron's compass setting is equal to exactly half the length of  $\overline{RS}$ , what would happen? Would the arcs intersect?
- What is the purpose of drawing the first arc?
- Why can't you bisect a line?

## Activity 2.2 Constructing a Square



### Facilitation Notes

In this activity, students analyze a worked example which demonstrates the use of construction tools to duplicate a line segment. They practice this construction and then construct a square with a given side length. They make conjectures about the segments and angles formed by the diagonals of a square.

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6 • TOPIC 1:

Have students work with a partner or in a group to complete Questions 4 through 6. Share responses as a class.

### Differentiation strategy

To scaffold support for constructing the square, suggest they extend  $\overline{JK}$  on both sides to construct a perpendicular bisector at point  $J$  and at point  $K$ .

### Questions to ask

- How did you construct the right angles for your square?
- How did you get the right angles to appear at the vertices instead of at the midpoint of each side?
- How did you get the side lengths to be the same for all four sides?
- How was patty paper used to verify the constructed figure is a square?
- Why isn't it enough to know that the four sides are congruent?
- How did you use patty paper to verify that the angles are right angles?
- Are the diagonals congruent? How do you know?
- Do the diagonals of a square bisect each other? How can you check?
- Are the diagonals of a square perpendicular to each other?
- Do the diagonals of a square bisect the angles at each vertex of the square?
- Do the diagonals of a square form 4 congruent isosceles right triangles?
- How do you know the triangles formed are right? Isosceles? Congruent?

### Summary

A line segment can be duplicated using patty paper or construction tools. The diagonals of a square are congruent, bisect each other, are perpendicular to one another, and bisect the angles of the square.

## Activity 2.3 Rigid Motions



### Facilitation Notes

In this activity, students recall the definitions of *rigid motion*, *translation*, *reflection*, and *rotation*. They analyze one way to construct a coordinate plane, using translations and students are instructed to describe other ways. Two-dimensional figures are drawn on coordinate planes and students describe how the shapes could be formed using transformations of line segments.

8 • TOPIC 1: Using a Rectangular Coordinate System

## Note

Differentiation strategies are provided that will ensure all students acquire the knowledge of the activity. These strategies provide flexibility within the lesson to allow for varying student acquisition and demonstration of learning. These strategies provide suggestions for all students, including those with learning strengths or learning gaps.

Ask a student to read the introduction and definitions aloud. Discuss as a class.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

**As students work, look for**

Connections made between the properties of a square and the transformations of the square to create a coordinate plane.

**Questions to ask**

- If Felipe had translated the shape a certain number of times, how would you describe the shape?
- How would you describe the shape if Felipe had translated it an infinite number of times?
- If Felipe had translated the shape right, then taken the shape a certain number of times, would you be able to describe the shape?
- Can you describe a four-step transformation that would produce the shape?
- Did you reflect any segments over an axis? Which axis?
- Did you reflect any segments over a horizontal or vertical line? What is the equation of the line?
- Did you use rotation to produce any shape? Which one? How many degrees rotation?

**Questions to ask**

- Did you start with a line segment constructed in one square or more than one square?
- Does it matter which line segment you use to perform the first translation?
- What are the endpoints of the line segment you used first?
- Did you translate the initial line segment up or down? To the left or right?
- If you translate the initial line segment up first, what effect does it have on the horizontal translation?
- If you translate the initial line segment down first, is the second translation to the left or to the right?
- What is the definition of *slope*?
- How do you determine the slope of the line on the coordinate plane?
- What is the slope of the line on the coordinate plane?
- Is the slope of the line 1 or  $-1$ ? How do you know?
- What information do you need to determine the equation of the line?
- Does the graph of the line pass through the origin?
- What are the coordinates of the  $y$ -intercept?
- How can the equation of a line be written using the slope and  $y$ -intercept?

**Summary**

A sequence of translations can be related to the slope of a line.

**Note**

Talk the Talk helps you to assess student learning and to make decisions about helpful connections you need to make in future lessons.

“Position yourself to take full advantage of the richness of the mathematics addressed in the textbook. The Facilitation Notes provide guidance to reach each student from their current level of understanding to advance to the next stage. Place yourself in the position of the student by experiencing the textbook activities prior to class. Realize your role in the classroom—empower your students! Step back and let them do the math with confidence in their role as learner and your role as facilitator of learning.”

Janet Sinopoli, Instructional Designer

# Supporting Emergent Bilingual Students

Throughout instruction, ELL tips are placed for teachers at point-of-use on the mini-lesson page in the TIG. They provide additional modifications to support this special population.

These tips:

- Inform teachers of potential learning obstacles specific to the lesson.
- Provide engaging activities for learning and assessment.
- Reinforce newly acquired mathematical language to gain an increasing level of comprehension of English.
- Introduce students to language needed to understand a specific context.

Students internalize new content language by using and reusing it in meaningful ways in a variety of different speaking activities that build concept and language attainment.

## For More Support

Visit the Texas Support Center for many more resources to support you and your students who are emergent bilingual students.



Emergent bilingual students often face multiple challenges in the mathematics classroom beyond language development skills, including a lack of confidence, peer-to-peer understanding, and building solid conceptual mastery. The Carnegie Learning Texas Math Solution seeks to support emergent bilingual students as they develop skills in both mathematics and language.

**ACTIVITY**  
5.5

**Proportional and Non-Proportional Changes in Dimensions**

In the activity *Perimeter and Area of Figures on the Coordinate Plane*, you investigated how doubling one or both of the dimensions of a figure affected its area. Now let's investigate how both proportional and non-proportional changes in a figure's dimensions affect its perimeter and area.

**1. Consider the following rectangles with the dimensions shown.**

Rectangle 1

Rectangle 2

Rectangle 3

**Complete the table to determine how doubling or tripling each rectangle's base and height affects its perimeter and area. The information for Rectangle 1 has been done for you.**

		Original Rectangle	Rectangle Formed by Doubling Dimensions	Rectangle Formed by Tripling Dimensions
Rectangle 1	Linear Dimensions	$b = 5$ in., $h = 4$ in.	$b = 10$ in., $h = 8$ in.	$b = 15$ in., $h = 12$ in.
	Perimeter (in.)	$2(5 + 4) = 18$	$2(10 + 8) = 36$	$2(15 + 12) = 54$
	Area (in. <sup>2</sup> )	$5(4) = 20$	$10(8) = 80$	$15(12) = 180$
Rectangle 2	Linear Dimensions			
	Perimeter (in.)			
	Area (in. <sup>2</sup> )			
Rectangle 3	Linear Dimensions			
	Perimeter (in.)			
	Area (in. <sup>2</sup> )			

LESSON 5: In and Out and All About • 25

**ELL Tip**

Ask students what they think is meant by a *proportional change*. Some students may use the term *scale factor*. Encourage students to ask other students when unfamiliar terms are used. Guide students to conclude that a *proportional change* is a result of a multiplication. Then ask if this multiplication can only apply to one dimension or if it should apply to two dimensions.

LESSON 5: In and Out and All About • 47

**Answers**

- Rectangle 2 (Original Rectangle)  
Linear Dimensions:  
 $b = 6$  in.;  $h = 2$  in.  
Perimeter (in.):  
 $2(6 + 2) = 16$   
Area (in.<sup>2</sup>):  $6(2) = 12$   
Rectangle 2 (Rectangle Formed by Doubling Dimensions)  
Linear Dimensions:  
 $b = 12$  in.;  $h = 4$  in.  
Perimeter (in.):  
 $2(12 + 4) = 32$   
Area (in.<sup>2</sup>):  $12(4) = 48$   
Rectangle 2 (Rectangle Formed by Tripling Dimensions)  
Linear Dimensions:  
 $b = 18$  in.;  $h = 6$  in.  
Perimeter (in.):  
 $2(18 + 6) = 48$   
Area (in.<sup>2</sup>):  $18(6) = 108$   
Rectangle 3 (Original Rectangle)  
Linear Dimensions:  
 $b = 3$  in.;  $h = 3$  in.  
Perimeter (in.):  
 $2(3 + 3) = 12$   
Area (in.<sup>2</sup>):  $3(3) = 9$   
Rectangle 3 (Rectangle Formed by Doubling Dimensions)  
Linear Dimensions:  
 $b = 6$  in.;  $h = 6$  in.  
Perimeter (in.):  
 $2(6 + 6) = 24$   
Area (in.<sup>2</sup>):  $6(6) = 36$   
Rectangle 3 (Rectangle Formed by Tripling Dimensions)  
Linear Dimensions:  
 $b = 9$  in.;  $h = 9$  in.  
Perimeter (in.):  
 $2(9 + 9) = 36$   
Area (in.<sup>2</sup>):  $9(9) = 81$

# Assessments

Both formative and summative assessments are an integral part of information gathering. Formative assessment tools are provided throughout each lesson, providing you with ongoing feedback of student performance and encouraging students to monitor their own progress. Ongoing formative assessment underlies the entire learning experience, driving real-time adjustments, next steps, insights, and measurements.

End of topic summative assessments are provided to measure student performance on a clearly denoted set of standards. For certain topics that extend longer than four instructional weeks, a mid-topic summative assessment is also provided.

## Enhanced End of Topic Assessment

There are three problem type sections per assessment. Multiple-choice questions, open-response questions, and griddable response questions prepare students for enhanced standardized tests.

The answer key provides teachers with the TEKS aligned to each question, as well as sample answers for open-response and griddable response questions.

**Topic 1**  
USING A RECTANGULAR COORDINATE SYSTEM


### Enhanced End of Topic Assessment

Name \_\_\_\_\_ Date \_\_\_\_\_

**Part A: Multiple-Choice Questions**

**TEKS 6.5B**

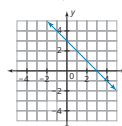
1. What is the first step in constructing the perpendicular bisector of line segment AB?



- Place the point of the compass at point B, and draw an arc between points A and B.
- Place the point of the compass at point A, and draw an arc between points A and B.
- Place the point of the compass at the midpoint of the distance from A to B, and draw an arc perpendicular to line segment AB.
- Use your straightedge to draw a line perpendicular to line segment AB.

**TEKS 6.2C**

2. Consider the graphed equation  $y = -x + 2$  and the line  $l$  shown below. Line  $l$  is parallel to the graphed equation.



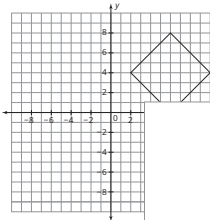
- $y = x - 1$
- $y = -x - 1$
- $y = -x + 4$
- $y = -x + 2$

**Topic 1**  
USING A RECTANGULAR COORDINATE SYSTEM

**Part B: Open-Response Questions**

**TEKS 6.3C**

6. The figure shown was constructed using rigid motions, starting with line segments constructed in one or more squares. Describe a sequence of transformations of a figure that could produce the resulting shape.



**Sample answer:**  
Start with the line segment with segment  $90^\circ$  clockwise. Then r

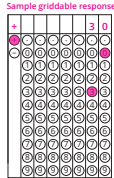
**Topic 1**  
USING A RECTANGULAR COORDINATE SYSTEM

**Part C: Griddable Response Questions**  
Record your answers and fill in the bubbles.

**TEKS 6.9B**

13. Two sides of a triangle measure 18 in. and 24 in. What is the length of the third side if the side lengths are a Pythagorean triple?


**30 in.**



**Sample griddable response:**

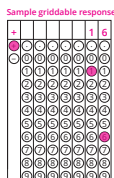
**TEKS 6.10B**

14. A rectangle has the dimensions shown below.



The length of each side is quadrupled. How many times larger is the new area when compared to the original?

**16 times**



**Sample griddable response:**



# End of Course Topic

The End of Course Topic is the final topic of the course which includes a collection of problem-based performance tasks that are aligned with selected priority math standards of the course. This final topic provides students an additional opportunity to demonstrate their ability to make sense of multi-step, real-world problems, communicate their thinking, represent solutions, and justify their reasoning on content aligned with these selected math standards.

## Performance Tasks

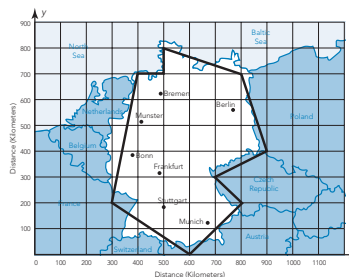
Each performance task is a formative assessment tool that allows students to demonstrate their learning of the selected course content. At the end of each task, a section titled “Your Work Should Include” lists the categories and the corresponding max scoring points from the grading rubric.

1

### PERFORMANCE TASK

#### Shape Up!

Sahil is using a map to estimate the area of Germany. He thinks the country could best be outlined by a nonagon. He draws the polygon shown to approximate its shape.



1. Use Sahil's polygon to estimate the percent of Germany's border that is coastline. Show your work.
2. Consider the population of Germany.
  - a. Estimate the population of Germany if there are approximately 237 people per square kilometer. Explain your process.
  - b. How does your approximation compare to Germany's current estimated population of 82.69 million people?

#### Your work should include:

- Approximate distance of the Germany's border that is coastline with supporting work. (3 points)
- Approximate distance of Germany's border with supporting work. (3 points)
- Approximate percent of the border of Germany that is coastline. (1 point)
- Approximate area of Germany with a supporting explanation. (3 points)
- Approximate population of Germany. (2 points)
- Comparison of the approximate population to the current estimated population. (1 point)



# Grading Rubric

The grading rubric is for students and teachers to set clear expectations for how each completed performance task will be evaluated. Students should use the rubric to guide their work and self-monitor their progress. Teachers should use the rubric to evaluate and provide feedback for the completed performance task.

**RUBRIC: 13 TOTAL POINTS**

	0 points	1 point	2 points	3 points
<b>Approximate Distance of Coastline</b>	No distance provided.	A correct distance with no supporting work or supporting work with more than a minor error.	A correct distance with some supporting work or an incorrect distance due to a minor error in supporting work.	A correct distance with complete supporting work.
<b>Approximate Distance of Border</b>	No distance provided.	A correct distance with no supporting work or supporting work with more than a minor error.	A correct distance with some supporting work or an incorrect distance due to a minor error in supporting work.	A correct distance with complete supporting work.
<b>Approximate Percent</b>	An incorrect percent or no percent provided.	A correct percent based on calculations.	N/A	N/A
<b>Approximate Area</b>	No area provided.	A correct area with no supporting work or supporting work with more than a minor error.	A correct area with a partial explanation or an incorrect area due to a minor error with a partial explanation.	A correct area with a complete explanation.
<b>Approximate Population</b>	No population provided.	A correct population with no explanation or supporting work with an error.	A correct population with an accurate explanation.	N/A
<b>Comparison of Current Estimated and Approximate Populations</b>	No comparison provided.	A comparison is provided.	N/A	N/A

# Teacher's Implementation Guide

The Teacher's Implementation Guide for the End of Course Topic contains a performance task overview, list of aligned TEKS and ELPS, essential ideas, facilitation notes which describe how to pace the two-day performance task, sample answer, and grading rubric.

**1**

**Performance Task**

Shape Up!

**MATERIALS**

- Graphing technology
- Allow students to have access to any additional materials that may assist in the completion of this task.

**Performance Task Overview**

Students are shown a map of Germany on a scaled coordinate plane along with an approximate border of Germany that has been outlined by a nonagon. They use the Distance Formula to approximate the percent of the border of Germany that is coastline. Students then approximate the area of Germany using the dimensions on the coordinate plane. They use their area calculation to estimate the population of Germany given the number of people in Germany per square kilometer and then compare their approximation to the current estimated population of Germany.

**Geometry Coordinate and Transformational Geometry**

(2) The student uses the process skills to understand the connections between algebra and geometry and uses the one- and two-dimensional coordinate systems to verify geometric conjectures. The student is expected to:

(B) derive and use the distance, slope, and midpoint formulas to verify geometric relationships, including congruence of segments and parallelism or perpendicularity of pairs of lines.

**Two-Dimensional and Three-Dimensional Figures**

(1) The student uses the process skills in the application of formulas to determine measures of two- and three-dimensional figures. The student is expected to:

(B) determine the area of composite two-dimensional figures comprised of a combination of triangles, parallelograms, trapezoids, kites, regular polygons, or sectors of circles to solve problems using appropriate units of measure.

**ELPS**

1A, 1B, 1C, 1H, 2A, 2B, 2D, 2E, 3A, 3B, 3C, 3D, 3E, 3G, 3H, 4E, 4F, 4I, 4J, 5F, 5G

**Essential Ideas**

The Distance Formula states that the distance between points  $(x_1, y_1)$  and  $(x_2, y_2)$  on a coordinate plane is given by the equation  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ .

END OF COURSE TOPIC: Performance Task 1 - 1

2a. The population of Germany is approximately 71,100,000.

Area A: 15,000    Area B: 25,000    Area C: 150,000    Area D: 30,000  
Area E: 15,000    Area F: 10,000    Area G: 5,000    Area H: 10,000

The area of Germany is approximately 300,000 square kilometers. I divided the nonagon into 6 triangles and 2 rectangles and calculated the area of each shape. Then I added all of the areas together.

To determine the number of people in Germany, I multiplied the population density of 237 people per square kilometer multiplied by the area in square kilometers.

$$237 \text{ people} \times 300,000 \text{ km}^2 = 71,100,000 \text{ people}$$

2b. My approximation of the population of Germany was 14% lower than the current estimated population. This is not surprising due to the fact that the polygon used to estimate the area of the country was not very exact.

$$\frac{71,100,000 - 82,600,000}{82,600,000} = \frac{-11,500,000}{82,600,000} \approx -0.14$$

Similar to the other topics in this course, the End of Course Topic also has a Topic Family Guide for students and caregivers, and a Topic Overview for teachers. The End of Course Topic does not include an end of topic assessment since each performance task is a formative assessment.

## GETTING READY

Carnegie Learning recognizes that it is the classroom teachers who make the material come alive for students, transforming the way math is taught. Implementation requires integrating learning together and learning individually.

### PREPARE YOURSELF

## Prepare for Learning Together

The most important first step you can take in preparing to teach with these instructional materials is to become comfortable with the mathematics.

- Read through the Module 1 Overview and the Topic 1 Overview.
- Do the math of the first Topic, and consider the facilitation notes.
- Prepare team-building activities to intentionally create a student-centered environment.

## Prepare for Learning Individually

Plan how you will utilize Skills Practice as a Learning Individually resource. Then, determine how you will introduce Skills Practice to students. Explain to them the benefits of working individually and why practice is important.

- Read through Module 1 Topic 1 Skills Practice.
- Determine which problem sets align with the activities in the corresponding student lessons.
- Based on student performance in the lesson, be prepared to assign the class, small groups of students, or individual students different problem sets to practice skills to develop mastery.

Plan how you will introduce students to MATHia. Explain to them the benefits of working individually and why practice is important.

- Test out the computers or tablets that your students will be using.
- Verify your classes have been set up in Teacher's Toolkit with correct MATHia content assigned. Or manually set up your classes in Teacher's Toolkit if applicable.
- Use the Content Browser in Teacher's Toolkit to explore the content students are assigned.
- Be prepared to demonstrate how students will access and log into MATHia.

## Prepare the Environment

The classroom is often considered the third teacher. Consider how to create a learning environment that engages students and fosters a sense of ownership. The use of space in your classroom should be flexible and encourage open sharing of ideas.

- Consider how your students are going to use the consumable book. It is the student's record of their learning. Many teachers have students move an entire topic to a three-ring binder as opposed to carrying the entire book.
- Arrange your desks so students can talk and collaborate with each other.
- Prepare a toolkit for groups to use as they work together and share their reasoning (read the materials list in each Topic Overview).
- Consider where you will display student work, both complete and in-progress.
- Create a word wall of key terms used in the textbook.

PREPARE  
YOUR CLASSROOM

## Prepare the Learners

If you expect students to work well together, they need to understand what it means to collaborate and how it will benefit them. It is important to establish classroom guidelines and structure groups to create a community of learners.

- Facilitate team-building activities and encourage students to learn each others' names.
- Set clear expectations for how the class will interact:
  - Their text is a record of their learning and is to be used as a reference for any assignments or tests you give.
  - They will be doing the thinking, talking, and writing in your classroom.
  - They will be working and sharing their strategies and reasoning with their peers.
  - Mistakes and struggles are normal and necessary.

PREPARE  
YOUR STUDENTS

## Prepare the Support

- Prepare a letter to send home on the first day. Visit the Texas Support Center for a sample letter.
- Encourage families and caregivers to read the introduction of the textbook.
- Ensure that families and caregivers receive the module Family and Caregiver Guide at the start of each module. They should also receive the topic Family Guide at the start of the first topic and each subsequent topic.
- Consider a Family Math Night some time within the first few weeks of the school year.
- Encourage families and caregivers to explore the Students & Caregivers Portal on the Texas Support Center at [www.CarnegieLearning.com/texas-help/students-caregivers](http://www.CarnegieLearning.com/texas-help/students-caregivers).

PREPARE FAMILIES  
AND CAREGIVERS



# Students and Caregivers Portal

Research has proven time and again that family engagement greatly improves a student's likelihood of success in school.

The Students & Caregivers Portal on the Texas Support Center provides:

- Getting to Know Carnegie Learning video content to provide an introduction to the instructional materials and research.
- Articles and quick tip videos offering strategies for how families and caregivers can support student learning. **Visit the Texas Support Center regularly to access new content and resources for students and caregivers as they learn mathematics in a variety of environments outside of the classroom.**



## MODULE FAMILY AND CAREGIVER GUIDES

Each module has a Family and Caregiver Guide available through the Students & Caregivers Portal on the Texas Support Center. Each module guide of the course will provide a different highlight of the academic glossary, description and examples of TEKS Mathematical Process Standards, and an overview of a different component of our instructional approach known as The Carnegie Learning Way. Also included is a module overview of content, specific key terms, visual representations, and strategies students are learning in each topic of the module.

The purpose of the Family and Caregiver Guide is to bridge student learning in the classroom to student learning at home. The goal is to empower families and caregivers to understand the concepts and skills learned in the classroom so that families can review, discuss, and solidify the understanding of these key concepts together. Videos will also be available on the Students & Caregivers Portal to provide added support.

The image displays three pages from the 'MODULE 1 FAMILY AND CAREGIVER GUIDE' for 'Reasoning with Shapes'. The pages are numbered 1, 2, and 3.

- Page 1:** Contains the title 'MODULE 1 FAMILY AND CAREGIVER GUIDE', the topic 'Reasoning with Shapes', and an introduction. It includes a 'Module Introduction' section explaining that mathematics is a connected set of ideas and that students should connect what they know about lines on the coordinate plane to verify simple geometric theorems. It also features an 'Academic Glossary' table with terms like 'Reflection' and 'Rotation'.
- Page 2:** Features a 'TABLE OF CONTENTS' for the module, listing pages for 'Module Introduction', 'Topic 1', 'Topic 2', 'Topic 3', and 'Topic 4'. It also includes a 'DEVELOP' section with a purpose to 'Build a deep understanding of mathematics through different contexts' and 'DEMONSTRATE' section with a purpose to 'Reflect on and describe what was learned'.
- Page 3:** Focuses on 'TOPIC 3' and 'Congruence Through Transformations'. It includes a table with 'TOPIC 3' and 'TOPIC 4' columns, a 'Key Terms' list (Point, Line Segment, Reflection, Distance Formula, Midpoint Formula, Angle, Transformation, Regular Polygon, Right Triangle), and a diagram of a triangle with its medians intersecting at a point (centroid).

## TOPIC FAMILY GUIDES

Each topic contains a Family Guide that provides an overview of the mathematics of the topic, how that math is connected to what students already know, and how that knowledge will be used in future learning. It also incorporates an illustration of math from the real world, a sample standardized test question, talking points, and a few of the key terms that students will learn.

We recognize that learning outside of the classroom is crucial to students' success at school. While we don't expect families and caregivers to be math teachers, the Family Guides are designed to assist families and caregivers as they talk to their students about what they are learning. Our hope is that both the students and their caregivers will read and benefit from the guides.

Carnegie Learning Family Guide Geometry

### Module 1: Reasoning with Shapes

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**TOPIC 1: USING A RECTANGULAR COORDINATE SYSTEM**

Students begin this topic by investigating a geometry puzzle which stimulates the need to measure and then prove that three angles in a diagram sum to  $90^\circ$ . Students then review the properties of squares and rigid motions and use constructions to build a rectangular coordinate system by creating and transforming squares. Students then study parallel and perpendicular line relationships on the coordinate plane, classify polygons on the coordinate plane, and determine the area and perimeter of shapes on the coordinate plane.


**Where have we been?**  
Students have performed rigid motion transformations of geometric objects in middle school and have explored the properties of triangles, quadrilaterals, and regular polygons. They have studied informal demonstrations of geometric congruence using parallel lines and have a wealth of experience with the coordinate plane from elementary school through middle school.

**Where are we going?**  
In this topic, students are introduced to making conjectures—a theme that will continue into the early parts of the next topic. Students use what they have learned in previous courses to ask formal questions.

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#### Using Squares to Show the

The diagram shows a diagonal drawn in a 1 unit up and right 1. The figure composed of these squares is rotated counter-clockwise to produce the shaded square.



The squares constructed can be those of a coordinate system where the slopes of perpendicular lines are negative reciprocals.

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#### The Bermuda Triangle

One of the most famous stretches of ocean in the Atlantic is an area between the United States, Puerto Rico, and Bermuda known as the Bermuda Triangle.



A heavily traveled area by planes and ships, it has become famous because of the many stories about ships and planes lost or destroyed as they moved through the Triangle.

For years, the Bermuda Triangle was suspected of having mysterious, supernatural powers that fatally affected all who traveled through it. Others believed natural phenomena, such as human error and dangerous weather, are to blame for the incidents.

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<p><b>Talking Points</b></p> <p>Coordinate geometry can be an important topic to know about for college admissions tests.</p> <p>Here is an example of a sample question:</p> <p><b>In the <math>xy</math>-plane, a triangle has vertices at <math>(5, 0)</math>, <math>(\sqrt{2}, 0)</math>, and <math>(2, \sqrt{10})</math>. What is the approximate area of the triangle?</b></p> <p>You can think of the base as the horizontal line segment. Its length is <math>5 - \sqrt{2}</math>, and the height is <math>\sqrt{10}</math>. So, the area is</p> $\frac{1}{2}(\sqrt{10})(5 - \sqrt{2}) \approx 5.67$ <p>So, the area of the triangle is approximately 5.67 square units.</p>	<p><b>Key Terms</b></p> <p><b>conjecture</b> A conjecture is a mathematical statement that appears to be true, but has not been formally proved.</p> <p><b>transformation</b> A transformation is the mapping, or movement, of the points of a figure on a plane according to a common action or operation.</p> <p><b>Distance Formula</b> The Distance Formula states that if <math>(x_1, y_1)</math> and <math>(x_2, y_2)</math> are two points on the coordinate plane, then the distance <math>d</math> between them is given by <math>d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}</math>.</p>
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2 • TOPIC 1: Using a Rectangular Coordinate System

## YOU MIGHT BE WONDERING . . .

### Why do we believe in our brand of blended: Learning Together and Learning Individually?

There has been lots of research on the benefits of learning collaboratively. Independent practice is necessary for students to become fluent and automatic in a skill. A balance of these two pieces provides students with the opportunity to develop a deep conceptual understanding through collaboration with their peers, while demonstrating their understanding independently.

### Why don't we have a Worked Example at the start of every lesson?

In all aspects of the Texas Math Solution, we provide worked examples. Sweller and Cooper (1985) argue that worked examples are educationally efficient because they reduce working memory load. Ward and Sweller (1990) found that alternating between problem solving and viewing worked examples led to the best learning. Students often read worked examples with the intent to confirm that they understand the individual steps. However, the educational value of the worked example often lies in thinking about how the steps connect to each other and how particular steps might be added, omitted, or changed, depending on context.

### Where are the colorful graphics to get students' attention?

Our instructional materials have little extraneous material; we do not use illustrations unless they are essential to helping students understand the material. This approach follows from research showing that “seductive details” used to spice up the presentation of material often have a negative effect on student learning (Mayer et al., 2001; Harp & Meyer, 1998). Students may not know which elements of an instructional presentation are essential and which are intended simply to provide visual interest. So, we focus on the essential material. While we strive to make our educational materials attractive and engaging to students, research shows that only engagement based on the mathematical content leads to learning.

### Why is the book so big?

The student textbook contains all of the resources students need to complete the Learning Together component of the course. Students are to actively engage in this textbook, topic-by-topic, creating a record of their learning as they go. There is room to record answers, take notes, draw diagrams, and fix mistakes. Visit the Texas Support Center at <https://www.CarnegieLearning.com/texas-help/> for tips on managing your textbooks.

### CUSTOMER SUPPORT

The Carnegie Learning Texas Support Team is available to help with any issue at [help@carnegielearning.com](mailto:help@carnegielearning.com).

**Monday–Friday  
8:00 am–8:00 pm CT**  
via email, phone, or  
live chat

Our expert team provides support for installations, networking, and technical issues, and can also help with general questions related to pedagogy, classroom management, content, and curricula.

# Notes

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“ If you have questions, reach out to us for support. Our team of master practitioners have been where you are. We made mistakes and we learned from them. We want to help you. We have many professional development options. Whether we come to your school for a workshop, join you in your classroom for modeling or coaching, or you join us online for a webinar or an entire course, our goal is to make sure you feel supported and prepared to use the tasks you'll find in this book to their fullest!

”

Kasey Bratcher, Senior VP of Professional Learning