Composing and Decomposing Numbers

3

Least Common Multiple and Greatest Common Factor

MATERIALS

None

Lesson Overview

Students continue to expand their understanding of factors, multiples, common factors, and common multiples as introduced in previous lessons. They use greatest common factors (GCF) and least common multiples (LCM) to solve problems.

Grade 6

Expressions, Equations, and Relationships

(7) The student applies mathematical process standards to develop concepts of expressions and equations. The student is expected to:

(A) generate equivalent numerical expressions using order of operations, including whole number exponents and prime factorization.

(D) generate equivalent expressions using the properties of operations: inverse, identity, commutative, associative, and distributive properties.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- Number relationships are useful in solving problems in context.
- Common factors help determine how to divide or share things equally.
- Common multiples help determine how things with different cycles can occur at the same time.

Lesson Structure and Pacing: 1 Day

Engage

Getting Started: We Have That in Common

Students analyze word problems to determine if they can be solved using a common factor or a common multiple.

Develop

Activity 3.1: Using GCF and LCM to Solve Problems

Students solve problems related to real-world situations using either the greatest common factor or the least common multiple.

Demonstrate

Talk the Talk: Finishing Where We Started

Students revisit the situations from the Getting Started. They apply the strategies to calculate the GCF or LCM to solve problems.

Getting Started: We Have That in Common

ENGAGE

Facilitation Notes

In this activity, students analyze word problems. They determine if they can be solved using the GCF or LCM.

Ask a student to read the introduction and directions aloud. Have students complete Questions 1 through 4 with a partner or group. Share responses as a class.

Questions to ask

- What is the problem asking you to solve?
- Explain how you determined whether to use common factors or common multiples.
- Would you solve the problem by dividing or equally sharing?
- Would you solve the problem by thinking about when things occur at the same time?
- If the problem is about dividing or equally sharing, would you use a common factor or a common multiple to solve it? Explain.
- If the problem is about determining when things occur at the same time, would you use a common factor or a common multiple to solve it? Explain.

As students work, look for

- Correct reasoning with incorrect use of terminology.
- Methods students use to make their determination, such as drawing diagrams and acting out the context.
- Attempts to solve the problems, which is not a requirement for this activity.

Summary

Number relationships are useful in solving problems in context.

DEVELOP

Activity 3.1
Using GCF and LCM to Solve Problems

Facilitation Notes

In this activity, students solve problems related to real-world situations. They apply the greatest common factor or the least common multiple.

Ask a student to read the introduction aloud. Have students complete Questions 1 and 2 with a partner or group. Share responses as a class.

Misconception

Students may incorrectly think that there needs to be the same number of spacers, round beads, and rectangular beads in each bag (for example, 4 spacers, 4 round beads, and 4 rectangular beads). Explain that because no beads are left over, that interpretation does not make sense.

Questions to ask

- How did you solve this problem?
- How did you know whether to use factors or multiples to solve this problem?
- Explain why your process made sense.
- Could Emily have assembled another number of packages rather than 8? Explain your thinking.
- How many times will each rider go around the track?

As students work, look for

- Whether or not students rely on diagrams to make sense of the context.
- How students transition into solving problems with three values rather than a pair of values.
- Language relating to sharing things equally and different cycles occurring at the same time.
- Whether students made lists or used a factor table to solve each problem.

Summary

When solving problems in context, common factors help determine how to divide or share things equally, while common multiples help determine how things with different cycles can occur at the same time.

Talk the Talk: Finishing Where We Started

DEMONSTRATE

Facilitation Notes

In this activity, students revisit the situations from the Getting Started. They apply the strategies to calculate the GCF or LCM to solve problems.

Ask a student to read the directions aloud. Have students complete Questions 1 through 4 with a partner or group. Share responses as a class.

Questions to ask

- How did you know that it makes sense to use the LCM in this situation?
- What strategy did you use to calculate the LCM?
- How did you know that it makes sense to use the GCF in this situation?
- What strategy did you use to calculate the GCF?

Summary

You can solve real-world problems that involve common factors or common multiples by thinking about the question you are trying to answer and using an efficient strategy.

NOTES

3 Composing and Decomposing Numbers

Least Common Multiple and Greatest Common Factor

WARM UP

Write the prime factorization of each number.

1. 21

2. 30

3. 42

4. 19

LEARNING GOALS

- Determine the greatest common factor of two whole numbers less than or equal to 100.
- Use greatest common factors and the Distributive Property to rewrite the sum of whole numbers 1 – 100.
- Determine the least common multiple of two whole numbers.

Number relationships are useful in solving everyday problems and in mental arithmetic. Understanding these relationships will deepen your knowledge of the structure of the number system. How can you use LCM and GCF to compose and decompose numbers?

Warm Up Answers

1. $3 \cdot 7$
2. $2 \cdot 3 \cdot 5$
3. $2 \cdot 3 \cdot 7$
4. $1 \cdot 19$

Answers

1. I would use a common multiple since the problem is about when the two different quantities of items will have the same amount.
2. I would use a common factor since the problem is about sharing the quantities of each item in the same number of packages.
3. I would use a common factor since the problem is about sharing the number of students in each grade in the same number of groups.
4. I would use a common multiple since the problem is about when the total amount that each person makes is the same.

Getting Started

We Have That in Common

You can solve real-world problems that involve common factors or common multiples by thinking about the question you are trying to answer.

Consider each scenario. Determine whether you would use either a common factor or a common multiple to solve the problem. Explain your reasoning.

1. **Hot dogs come in packs of 8 and hot dog buns come in packs of 6. What is the least number of hot dog packs you can buy if you want to have the same number of hot dogs and buns?**

2. **Zev has 36 pencils and 45 erasers. He wants to use all of the pencils and erasers to make identical packages for his friends. What is the greatest number of packages Zev can make?**

3. **There are 40 sixth graders and 24 seventh graders in an afterschool program. The director wants to create groups where each group has the same number of sixth graders, and each group has the same number of seventh graders. What is the greatest number of groups she can make?**

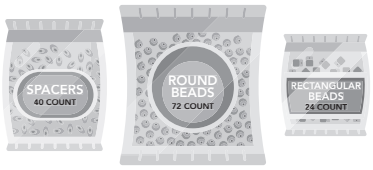
4. **Every time Sariyah babysits, she saves $12 of her earnings. Every time Aaron babysits, he saves $9 of his earnings. After babysitting a number of times, Sariyah and Aaron have saved the exact same amount of earnings. What is the least possible amount of savings they could each have?**

ACTIVITY
3.1

Using GCF and LCM to Solve Problems

Read and solve each problem using either the greatest common factor or the least common multiple.

Emily has three bags of different types of beads. She wants to split up the beads into mixed packages to share with her friends. She wants each package to have exactly the same number of each type of bead with no beads left over.



1. **What is the greatest number of packages that Emily can assemble? Describe the collection of beads in each package.**

Common factors help you think about how to divide, or share things equally. Common multiples help you think about how things with different cycles can occur at the same time.

A cyclist completes a lap around a track in 12 minutes. A second cyclist completes a lap around the same track in 18 minutes.

2. **If both riders begin at the starting line at the same time and maintain their speed, after how many minutes will they meet again at the starting line? Explain your reasoning.**

Answers

1. Emily can make 8 equal bags of beads. Each one will contain 5 spacers, 9 round beads, and 3 rectangular beads.
2. The two riders will meet in 36 minutes. The first rider will have gone around the track three times, the second one twice.

ELL Tip

The word *lap* has multiple meanings. In this situation, a lap refers to the length around the track one time.

Answers

1. The LCM of 8 and 6 is 24. So, you want to buy 24 hot dogs in 3 packs of 8 and 24 hot dog buns in 4 packs of 6.
2. The GCF of 36 and 45 is 9. So, Zev can make 9 packages, each with 4 pencils and 5 erasers.
3. The GCF of 40 and 24 is 8. So, the director can make 8 groups, each with 5 sixth graders and 3 seventh graders.
4. The LCM of 12 and 9 is 36. So, they each saved $36 minimum. Sariyah babysat 3 times to get that amount. Aaron babysat 4 times to get that amount.

NOTES

TALK the TALK

Finishing Where We Started

You have used LCM and GCF to solve real-world problems. Now you can go back and solve the problems from the Started.

1. **What is the least number of hot dog packs you can buy if you want to have the same number of hot dogs and buns?**

2. **What is the greatest number of packages Zev can make?**

3. **What is the greatest number of groups the director can make?**

4. **What is the least possible amount of savings Sariyah and Aaron could each have?**