Getting Closer

2

Benchmark Fractions

MATERIALS

None

Lesson Overview

Students translate their understanding of strip diagrams to number lines. They use the benchmark fractions 0, $\frac{1}{2}$, and 1 to estimate the value of fractions, write fractions that are close to these benchmarks, and estimate sums. Students solve a problem which involves comparing fractions that represent shaded parts of figures.

Grade 6

Number and Operations

(2) The student applies mathematical process standards to represent and use rational numbers in a variety of forms. The student is expected to:

(D) order a set of rational numbers arising from mathematical and real-world contexts.

Proportionality

(4) The student applies mathematical process standards to develop an understanding of proportional relationships in problem situations. The student is expected to:

(F) represent benchmark fractions and percents such as 1%, 10%, 25%, $33\frac{1}{3}$%, and multiples of these values using 10×10 grids, strip diagrams, number lines, and numbers.

ELPS

1.C, 1.E, 1.F, 1.H, 2.C, 2.E, 2.I, 3.E, 3.F, 4.F

Essential Ideas

- Benchmark fractions are common fractions used to estimate the value of fractions such as 0, $\frac{1}{2}$, and 1.
- A fraction is close to 0 when the numerator is very small compared to the denominator.
- A fraction is close to $\frac{1}{2}$ when the numerator is about half the size of the denominator.
- A fraction is close to 1 when the numerator is very close in size to the denominator.

Lesson Structure and Pacing: 1 Day

Engage

Getting Started: Shady Grids

Students use fractions to identify the shaded portions of grids.

Develop

Activity 2.1: Graphing Strip Diagrams

Students translate their understanding of strip diagrams as they label number lines to represent fractional parts.

Activity 2.2: Benchmark Fractions

Students use the benchmark fractions 0, $\frac{1}{2}$, and 1 to estimate the value of fractions and write fractions that are close to these benchmarks. They estimate sums and make generalizations about sums using their understanding of benchmark fractions.

Demonstrate

Talk the Talk: Copycat!

Students solve a problem which involves comparing fractions that represent shaded parts of figures.

Getting Started: Shady Grids

ENGAGE

Facilitation Notes

In this activity, students use fractions to identify the shaded portions of grids.

Have students complete Questions 1 and 2 individually. Share responses as a class.

As students work, look for

- Comparisons to the strip diagrams.
- Different fractions to represent the same shaded part, such as $\frac{25}{100}$ or $\frac{1}{4}$.

Questions to ask

- What did you consider as one whole when you wrote your fractions?
- What is a fraction with a denominator of 100 that represents the shaded part?
- What is another fraction that represents the shaded part?
- Rather than using folds, draw lines in the grids to represent the unit fraction represented by the shaded portions.

Summary

You can write different equivalent fractions to represent a part of a figure.

Activity 2.1
Graphing Strip Diagrams

DEVELOP

Facilitation Notes

In this activity, students translate their understanding of strip diagrams as they label number lines to represent fractional parts.

Ask a student to read the introduction. Complete Question 1 as a class.

Questions to ask

- Why is it important to scale the number line before plotting any points?
- How did you know the best way to divide up the number line?

- Did you divide the number line in half and in half again to split it more precisely? How is this strategy related to creating the strip diagrams?

Ask students to work with a partner or in groups to complete Question 2. Share responses as a class.

Questions to ask

- Explain your process to divide the number line into twelve parts of the same size.
- Did you have a sense of where this fraction would be placed on the number line before counting along the scale? If so, explain your thinking.
- Compare the fractions close to zero on all three number lines. What do you notice?
- What do the fractions close to one on all three number lines have in common?
- What is the relationship between the numerator and denominator of the fractions that are close to one? Why do you think that is the case?

Summary

You can plot fractions on a number line to get a sense of their value.

Activity 2.2
Benchmark Fractions

Facilitation Notes

In this activity, students use the benchmark fractions 0, $\frac{1}{2}$, and 1 to estimate the value of fractions and write fractions that are close to these benchmarks. They estimate sums and make generalizations about sums using their understanding of benchmark fractions.

Ask a student to read the introduction aloud. Discuss the number line diagram as a class.

Questions to ask

- Why do you think 0, $\frac{1}{2}$, and 1 are called common benchmark fractions?
- What might be considered another common benchmark fraction?

- What is an example of a fraction that fits the criteria noted for each benchmark fraction?

Have students complete Questions 1 and 2 with a partner or in a group. Share responses as a class.

Questions to ask

- How can you tell if a fraction is a little smaller or larger than $\frac{1}{2}$?
- Go back to the meaning of a fraction. What does $\frac{4}{9}$ mean? How does that meaning relate to comparing numerators and denominators to estimate the value of fractions?

Have students complete Questions 3 through 5 with a partner or in a group. Share responses as a class.

Questions to ask

- Explain your strategy to determine the numerator when the fraction is close to but less than $\frac{1}{2}$.
- If you are determining the denominator when the fraction is close to, but less than, $\frac{1}{2}$, could you just double the numerator? If not, what adjustment should you make?
- What is another possible answer?
- What strategy did you use to write fractions close to, but less than, one?
- How did you use benchmark fractions to support your reasoning?

Have students complete Questions 6 through 8 with a partner or in a group. Share responses as a class.

Questions to ask

- What information do you know for sure about the sum?
- How does using benchmarks support using mental math?

Summary

Three common benchmark fractions are 0, $\frac{1}{2}$, and 1. A fraction is close to 0 when the numerator is very small compared to the denominator. A fraction is close to $\frac{1}{2}$ when the numerator is about half the size of the denominator. A fraction is close to 1 when the numerator is very close in size to the denominator.

DEMONSTRATE

Talk the Talk: Copycat!

Facilitation Notes

In this activity, students solve a problem which involves comparing fractions that represent shaded parts of figures.

Ask a student to read the introduction aloud. Have students work with a partner or in groups to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What information in the table was Lily considering?
- What information is Lily overlooking?
- Explain how Molly misinterpreted the phrase “largest portion of parts.”
- What would you say to Molly to help her correct the error in her thinking?

Summary

When making comparisons about size, it makes more sense to consider ratios rather than the magnitude of a single number.

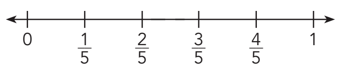
Getting Closer

2

Benchmark Fractions

WARM UP

1. Locate $\frac{1}{2}$ on this number line.



2. How did you determine the location of $\frac{1}{2}$ on the number line?

LEARNING GOALS

- Identify fractions and their equivalents on a 10 X 10 grid.
- Represent fractions on a number line.
- Estimate fractions by using benchmark fractions.
- Estimate sums of fractions using benchmark fractions.

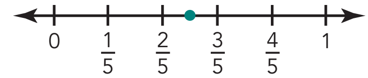
KEY TERM

- benchmark fractions

You have created strip diagrams and used them to compare numbers. How can you use grids and number lines of common fractions like $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ to estimate the value of other fractions?

Warm Up Answers

1.



2. Sample answer.
I knew that $\frac{1}{2}$ was halfway between $\frac{2}{5}$ and $\frac{3}{5}$ because $\frac{2}{5}$ is equal to $\frac{4}{10}$ and $\frac{3}{5}$ is equal to $\frac{6}{10}$, so $\frac{5}{10}$ is exactly halfway between $\frac{4}{10}$ and $\frac{6}{10}$.

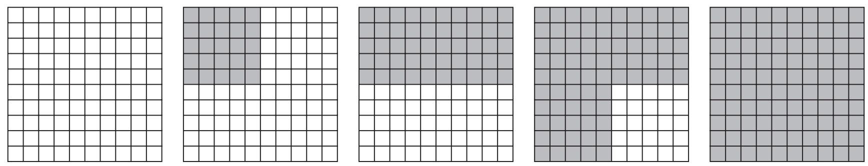
Answers

1. See grids.
   $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$
2. The shading on each grid increases by $\frac{1}{4}$ each time from left to right.
   The shading on each grid increases by 25 squares each time from left to right.

Getting Started

Shady Grids

Consider the grids shown.



1. What fraction does each grid represent? Write a fraction under each grid.

2. How does each 10 × 10 grid representation change as you move from left to right? Explain your reasoning.

ACTIVITY 2.1

Graphing Strip Diagrams

In the previous lesson, you folded strips of paper to represent fractions. Let's consider how to translate those strip diagrams to number lines.

1. Label the number line to represent fourths.

a. How does this number line connect with the grid representations in the Getting Started?

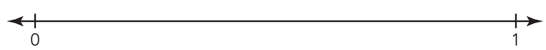
b. Plot the fractions $\frac{1}{4}$ and $\frac{3}{4}$.

2. Label each number line to represent the fractional part provided and plot the given fractions.

a. twelfths

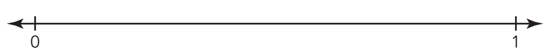
- $\frac{5}{12}$
- $\frac{11}{12}$
- $\frac{1}{12}$
- $\frac{7}{12}$

b. sixteenths



- $\frac{10}{16}$
- $\frac{1}{16}$
- $\frac{7}{16}$
- $\frac{15}{16}$

c. eighths

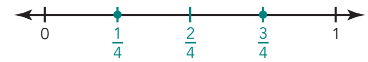


- $\frac{5}{8}$
- $\frac{7}{8}$
- $\frac{1}{8}$
- $\frac{3}{8}$

> *As you get ready to label each number line, think about how you folded your strips in the Rocket Strips lesson. This will help you get the number line evenly spaced. To get fourths, you folded the strip in half first, so mark $\frac{2}{4}$ on the number line first. Then, mark $\frac{1}{4}$, and finally mark $\frac{3}{4}$.*

Answers

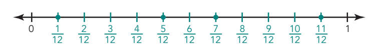
1.



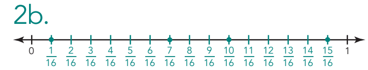
1a. The pattern is the same. Each of the numbers on the number line are separated by an interval of $\frac{1}{4}$. The shading on the grids increases by $\frac{1}{4}$ each time from left to right.

1b. See number line above.

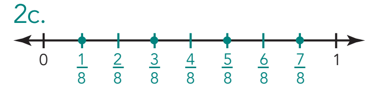
2a.



2b.



2c.



Answers

1a. Answers may vary.
$\frac{1}{4}, \frac{3}{8}, \frac{3}{12}, \frac{3}{16}$

1b. $\frac{2}{4}, \frac{4}{8}, \frac{6}{12}, \frac{8}{16}$

1c. Answers may vary.
$\frac{3}{4}, \frac{5}{8}, \frac{7}{12}, \frac{9}{16}$

1d. Answers may vary.
$\frac{1}{4}, \frac{1}{8}, \frac{1}{12}, \frac{1}{16}$

1e. $\frac{4}{4}, \frac{8}{8}, \frac{12}{12}, \frac{16}{16}$

1f. Answers may vary.
$\frac{3}{4}, \frac{7}{8}, \frac{11}{12}, \frac{15}{16}$

1g. $\frac{0}{4}, \frac{0}{8}, \frac{0}{12}, \frac{0}{16}$

2a. $\frac{1}{2}$

2b. 1

2c. 0

2d. 0

2e. $\frac{1}{2}$

2f. $\frac{1}{2}$

2g. 1

2h. $\frac{1}{2}$

2i. 1

2j. 0

2k. $\frac{1}{2}$

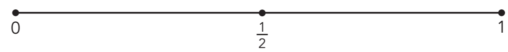
2l. $\frac{1}{2}$

ACTIVITY 2.2

Benchmark Fractions

Benchmark fractions are common fractions you can use to estimate the value of fractions.

Three common benchmark fractions are 0, $\frac{1}{2}$, and 1.



A fraction is close to 0 when the numerator is very small compared to the denominator.

A fraction is close to $\frac{1}{2}$ when the numerator is about half the size of the denominator.

A fraction is close to 1 when the numerator is very close in size to the denominator.

1. Use each number line you completed in Activity 2.1 to write a fraction that is:

a. less than $\frac{1}{2}$

b. exactly $\frac{1}{2}$

c. greater than but not equal to $\frac{1}{2}$

d. close to but not equal to 0

e. exactly 1

f. close to but not equal to 1

g. exactly 0

2. Name the closest benchmark fraction for each fraction given.

a. $\frac{4}{9}$	b. $\frac{8}{9}$	c. $\frac{6}{100}$
d. $\frac{5}{67}$	e. $\frac{7}{15}$	f. $\frac{7}{12}$
g. $\frac{5}{6}$	h. $\frac{14}{27}$	i. $\frac{12}{13}$
j. $\frac{1}{17}$	k. $\frac{5}{11}$	l. $\frac{3}{7}$

ELL Tip

Sometimes having background knowledge of the origin of a word can help students understand the meaning in a mathematical context. Begin by explaining that a *benchmark* is *a point of reference used for comparing*. In the 1800s, surveyors would chisel a "mark" into stone and use it as a "bench" for a leveling rod. Thus, the term *benchmark* was originally coined as a reference for height. Similarly, *benchmark* fractions are used as a reference for length. They are fractions we use often, and learning them helps to estimate fractional amounts.

3. Write the unknown numerator or denominator so that each fraction is close to but less than $\frac{1}{2}$.

a. $\frac{(\quad)}{12}$ b. $\frac{(\quad)}{27}$

c. $\frac{8}{(\quad)}$ d. $\frac{7}{(\quad)}$

e. $\frac{(\quad)}{13}$ f. $\frac{9}{(\quad)}$

4. Write the unknown numerator or denominator so that each fraction is close to but less than 1.

a. $\frac{(\quad)}{17}$ b. $\frac{11}{(\quad)}$

c. $\frac{(\quad)}{8}$ d. $\frac{(\quad)}{18}$

e. $\frac{13}{(\quad)}$ f. $\frac{(\quad)}{10}$

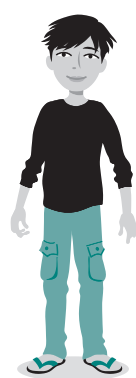
5. Rewrite each expression using benchmark fractions. Then, estimate the sum. Explain your reasoning.

a. $\frac{8}{9} + \frac{6}{7}$

b. $\frac{1}{11} + \frac{8}{17}$

c. $\frac{10}{11} + \frac{11}{12} + \frac{12}{13} + \frac{13}{14} + \frac{14}{15}$

"Remember, to estimate means to give an educated guess."



Answers

3a. $\frac{(5)}{12}$

3b. $\frac{(13)}{27}$

3c. $\frac{8}{(17)}$

3d. $\frac{7}{(15)}$

3e. $\frac{(6)}{13}$

3f. $\frac{9}{(19)}$

4a. $\frac{(16)}{17}$

4b. $\frac{11}{(12)}$

4c. $\frac{(7)}{8}$

4d. $\frac{(17)}{18}$

4e. $\frac{13}{(14)}$

4f. $\frac{(9)}{10}$

5a. $1 + 1 = 2$

I know that both fractions are close to 1. So, when I estimate the sum, it is close to, but less than, 2.

5b. $0 + \frac{1}{2} = \frac{1}{2}$

I know that the first fraction is close to 0, and the second fraction is close to $\frac{1}{2}$. So when I estimate the sum, it is close to $\frac{1}{2}$.

5c. $1 + 1 + 1 + 1 + 1 = 5$

I know that all the fractions in the expression are close to 1. So when I estimate the sum, it is close to, but less than, 5.

Answers

6. The sum of the three fractions would be between $1\frac{1}{2}$ and 3.

   I know this because the minimum value of each fraction is greater than $\frac{1}{2}$, so the minimum sum could be close to, but greater than, $1\frac{1}{2}$. I also know that the maximum value of each fraction might be close to 1, so the maximum sum could be close to, but less than, 3. Therefore, the sum of the three fractions would be between $1\frac{1}{2}$ and 3.

7. The sum of the fractions would be between 0 and 1.

   I know that the fractions cannot equal 1, and they must be less than 1. Therefore, I know that the sum must be between 0 and 1.

8. The approximate sum would be 7.

   I know that each fraction is close to, but not equal to, 1. So, I know that the sum must also be close to, but not equal to, 7.

NOTES

6. If three fractions that are greater than $\frac{1}{2}$ but less than 1 are added together, what can you say about their sum? Explain your reasoning.

7. If two fractions that are less than $\frac{1}{2}$ but greater than 0 are added together, what can you say about their sum? Explain your reasoning.

8. If seven fractions that are slightly less than 1 are added together, what can you say about their sum? Explain your reasoning.

6 • TOPIC 2: Positive Rational Numbers

NOTES

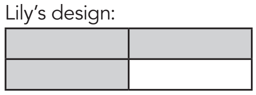
TALK the TALK

Copycat!

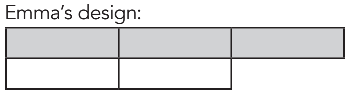
The teacher asked each of her students to create a rectangular design and shade only a portion of their design. The table describes three students' designs.

Student	Unshaded portion	Shaded portion
Lily	$\frac{1}{4}$	$\frac{3}{4}$
Emma	$\frac{2}{5}$	$\frac{3}{5}$
Molly	$\frac{3}{12}$	$\frac{9}{12}$

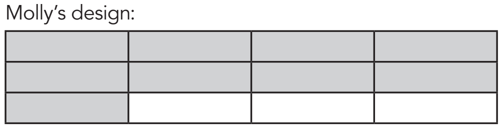
Lily's design:



Emma's design:



Molly's design:



1. Sample answer. No, I don't think Molly copied Lily's design. The designs are not exactly the same. There are 12 parts in Molly's design and there are only 4 parts in Lily's design. However, Lily thinks that Molly copied her idea because both designs have $\frac{3}{4}$ of the parts shaded and $\frac{1}{4}$ of the parts not shaded. $\frac{9}{12}$ and $\frac{3}{4}$ are equivalent fractions just as $\frac{1}{4}$ and $\frac{3}{12}$ are also equivalent fractions.
2. Molly thinks this way because she has the most number of unshaded parts. She has 3 unshaded parts, Emma has 2 unshaded parts, and Lily has 1 unshaded part. Emma's design had the largest unshaded portion because $\frac{2}{5}$ of her design was unshaded whereas only $\frac{1}{4}$ of both Lily and Molly's designs were unshaded. $\frac{2}{5}$ is equal to $\frac{8}{20}$ and $\frac{1}{4}$ is equal to $\frac{5}{20}$, so $\frac{2}{5}$ is larger than $\frac{1}{4}$.

When the students saw each other's designs Lily immediately accused Molly of being a copycat! Molly replied that she did no such thing and Lily must be confused, as usual. Molly said her design was much bigger than Lily's and not even the same shape. Emma said she was staying out of this argument and would not take Lily or Molly's side of the disagreement.

1. Do you think Molly copied Lily's design? Why would Lily accuse Molly of copying her design? Explain your reasoning.

2. Molly thought her design contained the largest portion of parts not shaded. Why does Molly think this way? Do you agree with her? Explain your reasoning.