Positive Rational Numbers

Topic 2 Overview

How is *Positive Rational Numbers* organized?

This topic highlights the on-grade level standard of multiplication and division of fractions by fractions. The topic begins with students creating and using physical models to represent and compare fractions, as well as to determine equivalent fractions. Students begin moving from concrete models to abstract thinking when they connect strip diagrams to a number line to represent and compare fractions. Students reason about the relative size of a fraction by comparing it to a benchmark fraction and investigating the relationship between the numerator and denominator. Students use an area model to represent multiplication of fractions before moving to the algorithm. Students engage with fraction division by reasoning with visual models, dividing across, and then using the standard algorithm. The design of the topic highlights the inverse relationship between multiplication and division.

The focus standard of the topic is fraction multiplication and division. Students engage in reasoning about numbers and strategies before learning the algorithm. By learning multiple division strategies, students can make decisions about when specific strategies are most efficient. The focus on reasoning also prevents the rush to rules that results in surface mastery of fluency skills, which quickly fades. Although algorithms for fraction multiplication and division are discussed in this topic, students may not achieve fluency within the timeline allowed for this topic. Fluency requires time and practice, and students will continue to develop fluency with fraction operations throughout the course.

What is the entry point for students?

Students began their formal study of fractions in the elementary grades. They understand fractions as numbers and can reason about relative sizes of fractions. Students have learned to compare and order fractions, determine equivalent fractions, and add and subtract fractions with like and unlike denominators. Students also know how to multiply whole numbers by fractions, divide whole numbers by unit fractions (e.g., $a \div \frac{1}{b}$), and divide unit fractions by whole numbers (e.g., $\frac{1}{b} \div a$).

This topic draws on students' foundational knowledge of fractions and the meanings of multiplication and division. Students will apply their prior experience with area models, number lines, and fact families to develop visual models for fraction multiplication and division.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic if they can:

- Determine and use equivalent fractions to show equal parts of the same whole.
- Represent and compare benchmark fractions using models, including number lines.
- Order positive rational numbers in mathematical and real-world contexts.
- Compute products of fractions multiplied by whole numbers and fractions (including mixed numbers).
- Determine whether a quantity is increased or decreased when multiplied by a fraction greater than one or less than one.
- Compute quotients of fractions divided by whole numbers and fractions (including mixed numbers).
- Interpret quotients of fractions in real-world and mathematical problems.
- Solve real-world problems involving multiplication and division of fractions and mixed numbers using visual models.
- Recognize that dividing by a rational number and multiplying by its reciprocal result in equivalent values.
- Use an algorithm to fluently solve multiplication and division problems with fractions.

Why is *Positive Rational Numbers* important?

This topic represents the culmination of students' learning about operations with fractions. In future modules, students will use their knowledge of fractions as they determine equivalent ratios, percents, and unit rates. Students will be introduced to the full set of rational numbers, and they will order and compare positive and negative rational numbers written in a variety of forms. Beyond ordering the full set of rational numbers, they also operate with integers. Students will be prepared to plot and order the full set of rational numbers on a number line and as pairs on a coordinate plane. Students will be expected to operate with rational numbers as they rewrite algebraic expressions and solve algebraic equations.

How do the activities in *Positive Rational Numbers* promote student expertise in the mathematical process standards?

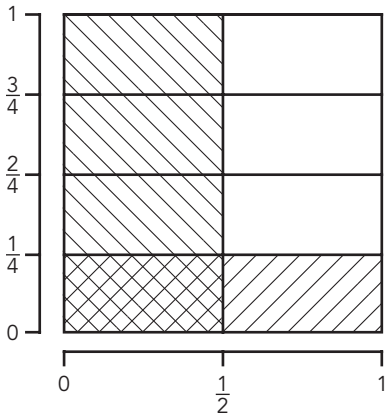
All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the mathematical process standards should be evident in all lessons. Students are expected to make

sense of problems and work towards solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

This topic highlights the need for precision in language in explanations about and computation of fraction multiplication and division. Students are also expected to reason about the relative sizes of numbers and determine whether their answers make sense. They should use a variety of strategies and tools to build understanding of fraction division, and they are expected to know when each strategy is most efficient. Throughout the topic, students should be looking for patterns to discern their own shortcuts before being provided with a standard algorithm.

New Tools and Notation

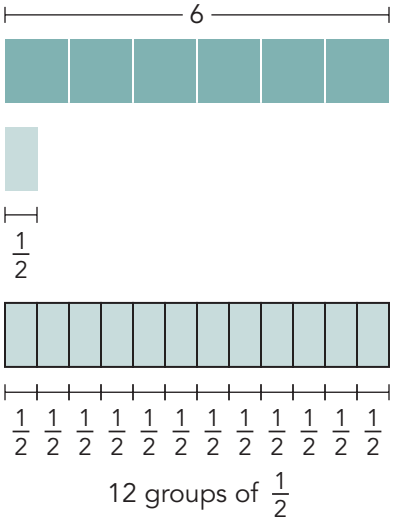
Area Models to Multiply Fractions:
$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$



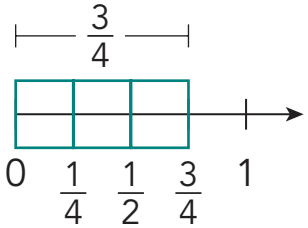
Materials Needed

- Scissors

Strip Diagrams to Divide with Fractions
$6 \div \frac{1}{2} = 12$



Strip Diagrams/Number Lines to Divide Fractions
$\frac{3}{4} \div \frac{1}{4} = 3$



Learning Together

ELPS: 1.A, 1.C, 1.E, 1.F, 1.H, 2.C, 2.D, 2.E, 2.I, 3.D, 3.E, 3.F, 3.G, 3.H, 4.F, 4.G, 5.F

Lesson	Lesson Name	TEKS	Days	Highlights
1	Rocket Strips: Dividing a Whole into Fractional Parts	6.4F 6.5C	1	Students create strip diagrams for unit fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$, and $\frac{1}{16}$. They identify equivalent fractions by aligning the strip diagrams on the fold lines, and then complete a graphic organizer to represent all the equivalent fractions represented by the strip diagrams. Students conclude that the numberator and denominator of equivalent fractions are multiples of the original unit fractions.
2	Getting Closer: Benchmark Fractions	6.2D 6.4F	1	Students translate their understanding of strip diagrams to number lines. They use the benchmark fractions 0, $\frac{1}{2}$, and 1 to estimate the value of fractions, write fractions that are close to these benchmarks, estimate sums, and solve problems by comparing fractions that represent shaded parts of figures.
3	Did You Get the Part?: Multiplying Fractions	6.3B 6.3E	2	Students review the area model for multiplication and apply it to multiplying mixed numbers. They analyze two methods for multiplying mixed numbers and then use these methods to answer questions in the context of a real-world scenario.
4	Yours IS to Reason Why!: Fraction by Fraction Division	6.2E 6.3A 6.3E	3	Students connect multiplication to division by writing fraction fact families for area models. They then use strip diagram and number line models to investigate the division of fractions by fractions. Students use these models to develop an algorithm for rewriting division sentences as multiplication sentences. They apply the procedure to solve problems involving fractions and mixed numbers.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 6.4F, 6.5C **LESSON 1** **Rocket Strips** GETTING STARTED ACTIVITY 1 TALK THE TALK	TEKS: 6.2D, 6.4F **LESSON 2** **Getting Closer** GETTING STARTED ACTIVITY 1 ACTIVITY 2 TALK THE TALK	TEKS: 6.3B, 6.3E **LESSON 3** **Did You Get the Part?** GETTING STARTED ACTIVITY 1	**LESSON 3** continued ACTIVITY 2 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
TEKS: 6.2E, 6.3A, 6.3E **LESSON 4** **Yours IS to Reason Why** GETTING STARTED ACTIVITY 1	**LESSON 4** continued ACTIVITY 2 ACTIVITY 3	**LESSON 4** continued ACTIVITY 4 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	END OF TOPIC ASSESSMENT

Assessments

There is one assessment aligned to this topic: End of Topic Assessment.