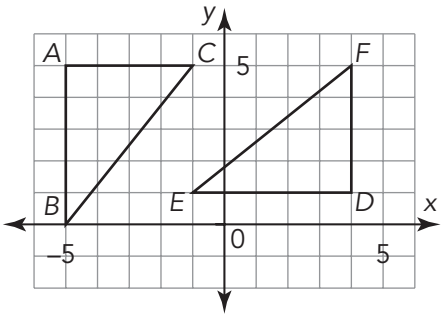
1

Consider Every Side

Constructing Triangles Given Sides

WARM UP

Use the coordinate plane to determine each distance. Show your work.



1. What is the distance from point *F* to point *D*?
2. What is the distance from point *A* to point *B*?
3. What is the distance from point *C* to point *A*?
4. What is the distance from point *E* to point *D*?

LEARNING GOALS

- Use patty paper to investigate triangles.
- Construct triangles from three angle measures or side lengths, identifying when the conditions determine a unique triangle, more than one triangle, or no triangle.

KEY TERM

- Triangle Inequality Theorem

You know how to draw a triangle. Can you construct a specific triangle if you are given only two or three possible side lengths? Is there more than one possible triangle that you can construct?

Tri-, Tri-, and Tri- Again

Classify each statement as *always* or *sometimes* true about triangles.

- For each *always* true statement, explain your reasoning.
- For each *sometimes* true statement, provide an example and a counterexample.

1. The angles of a triangle have the same measure.

2. A triangle has three angles.

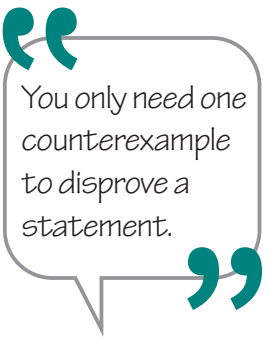
3. Two sides of a triangle have the same measure.

4. One angle of a triangle measures 90 degrees.

ACTIVITY
1.1 Pasta Triangles

Let's investigate the conditions necessary for forming a triangle with different side lengths.

1. **Sarah says that when you know 2 segment lengths, you can form many different triangles. She claims she can also use any 3 segment lengths to form a triangle. Sam does not agree. He thinks some combinations will not work. Who is correct? Use a counterexample to disprove the incorrect statement.**



Sam then claims that he can just look at the three lengths and know immediately if they will work. Sarah is unsure. She decides to explore this for herself.

Help Sarah by working through the following investigation.

To begin, you will need a piece of strand pasta. Break the pasta at two random points so the strand is divided into three pieces.

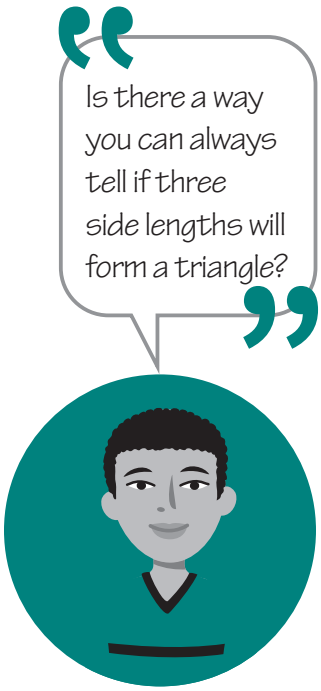
- Try to form a triangle from your three pieces of pasta.
- Measure each of your three pieces of pasta in centimeters.
- Repeat the experiment with a new piece of pasta.

2. **Record your measurements and the measurements of your group members in the table provided.**

3. **Now, add to your table. Collect and record your classmates' measurements.**

Piece 1 (cm)	Piece 2 (cm)	Piece 3 (cm)	Forms a Triangle? (yes or no)

4. **Examine the lengths of the pasta pieces that did form a triangle. Compare them with the lengths of the pasta pieces that did not form a triangle. Make a conjecture about the conditions under which it is possible to form a triangle.**



ACTIVITY
1.2
A Triangle Given Three Segments

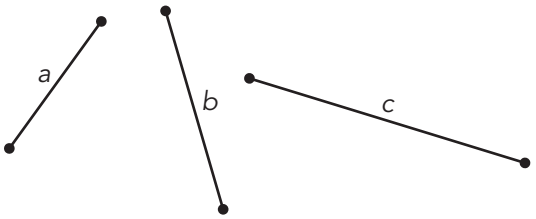
Let's continue to investigate Sarah's question and your conjecture with patty paper.

1. **Trace each of the three segments onto its own sheet of patty paper.**

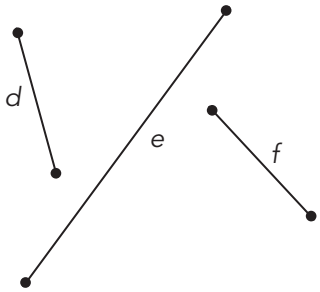
   a. **Overlay the sheets to determine if you can create a triangle. If you can, record the triangle on its own sheet of patty paper.**

You and your partner should use different sets of segments for this investigation.

Set 1:



Set 2:



b. Now create as many different triangles as you can, using the given segments as sides of a possible triangle. Use a different sheet of patty paper to record each unique triangle.

c. What do you notice? How many different triangles were you able to create?

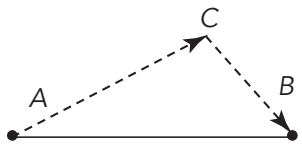
Triangles are *congruent* when all of their corresponding angle measures and corresponding side lengths are the same.

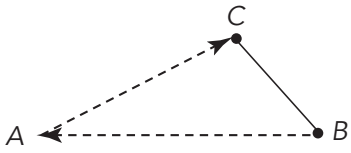
2. Use the patty paper examples from Set 1 and Set 2 to make a conjecture about when three segments can be used to create a triangle. Test your conjecture by creating additional triangles.

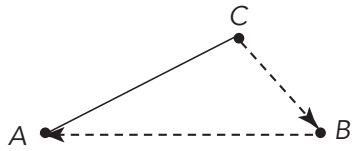
ACTIVITY
1.3 Triangle Inequality Theorem

In Euclidean geometry—the geometry of straight lines on flat planes—the shortest distance between two points is a straight line.

- any distance $AC + CB$ will be greater than the distance AB
- any distance $BA + AC$ will be greater than or equal to the distance BC
- any distance $CB + BA$ will be greater than the distance AC







1. How could you use this fact to test whether three line segments can form a triangle? Explain your reasoning.

The measure of $\overline{AB}$ can be expressed in two different ways. AB is read as "the distance from point A to point B." $m\overline{AB}$ is read as "the measure of line segment AB."

2. Provide examples of line segments that cannot possibly form a triangle.

3. **What would it mean for the distance *AC* + *CB* to be equal to the distance *AB*? Would these three segments form a triangle?**

4. **Based on your observations, determine if it is possible to form a triangle using segments with the given measurements. Explain your reasoning.**

   a. 2 cm, 5.1 cm, 2.4 cm

   b. 9.2 cm, 7 cm, 1.9 cm

A theorem is a mathematical rule that can be formally proven.

The rule that you have been using is known as the *Triangle Inequality Theorem*. The **Triangle Inequality Theorem** states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

NOTES

TALK the TALK

None, One, or Many?

Determine if the given information could be used to form a unique triangle, many different triangles, or no triangles. Explain your reasoning.

1. 3 in., 2.9 in., 5 in.

2. 112 mm, 300 mm

3. 5 yd, 10 yd, 21 yd

4. 8 ft, 9 ft, 11 ft

5. 13.8 km, 6.3 km, 7.5 km