All About That Base. . . and Height

3

Area of Triangles and Quadrilaterals

MATERIALS

Scissors
Glue sticks or tape
Patty paper

Lesson Overview

Students use previously known area formulas and the principle of area conservation to investigate the areas of parallelograms, triangles, and trapezoids. They use this knowledge to develop formulas for the areas of these shapes, practice calculating areas, and solving area-related problems. Students learn that the choice of base or height does not affect the area of the shape.

Grade 6
Expressions, Equations, and Relationships

(8) The student applies mathematical process standards to use geometry to represent relationships and solve problems. The student is expected to:

(B) model area formulas for parallelograms, trapezoids, and triangles by decomposing and rearranging parts of these shapes.

(C) write equations that represent problems related to the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers.

(D) determine solutions for problems involving the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- The formula for the area of a rectangle is $A = lw$, where A is the area of the rectangle, l is the length of the rectangle, and w is the width of the rectangle.
- The formula for the area of a parallelogram is $A = bh$, where A is the area of the parallelogram, b is the length of the base of the parallelogram, and h is the height of the parallelogram.
- The formula for the area of a triangle is $A = \frac{1}{2}bh$, where A is the area of the triangle, b is the length of the base of the triangle, and h is the height of the triangle.
- The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where A is the area of the trapezoid, h is the height of the trapezoid, and b_1 and b_2 are bases of the trapezoid.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: In the 20s

Students are given four different shapes drawn on a grid. They discuss the attributes of each shape and think about any attributes that are shared across the different shapes. Students are asked to justify that the area of each shape is exactly 20 square units.

Develop

Activity 3.1: Investigating the Area of a Parallelogram

Students decompose a parallelogram and then recompose the pieces into a rectangle to determine the area of the parallelogram. They learn that the choice of base and height does not change the area of the parallelogram.

Activity 3.2: Investigating the Area of a Triangle

Students use a parallelogram to determine the area of a triangle. They derive the formula for the area of a triangle.

Day 2

Activity 3.3: Investigating the Area of a Trapezoid

Students recreate different ways of decomposing trapezoids into shapes with known area formulas and derive a formula for the area of a trapezoid.

Activity 3.4: Calculating Areas of Figures

Students practice calculating areas of parallelograms, triangles, and trapezoids and solving area-related problems.

Demonstrate

Talk the Talk: Figure 'Em Out!

Students use pictures and words to summarize the formulas for the areas of parallelograms, triangles, and trapezoids. They also demonstrate that two triangles with congruent bases and congruent heights have the same area.

Facilitation Notes

In this activity, students use given figures drawn on a grid to describe the attributes of each shape. Each figure is a different polygon but has the same area.

Ask students to complete Question 1 individually and share responses as a class. Have students complete Question 2 with a partner or in groups. Share responses as a class.

Differentiation strategies

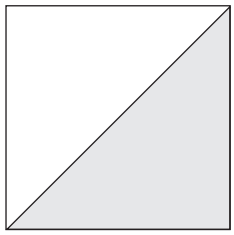
- To scaffold support, ask students what they can tell you about the sides (number of sides, parallel or not, same length) and angles of their figure.
- This may be a good opportunity to use the jigsaw strategy. Have students begin in groups of 4 (with team members A, B, C and D) responding to Questions 1 and 2 for one of the four figures. Then, have the students regroup by letter to share their responses for all of the figures.

As students share, look for

- The use of vocabulary, e.g., *polygon, 4-sided figure or quadrilateral, parallel sides, congruent sides, right angles, perpendicular sides, opposite sides,* or *adjacent sides*.
- The use of area formulas.
- Counting squares.
- Composition and decomposition to rearrange the figure into a familiar shape.

Misconceptions

- Students may include counting partial squares as whole squares or combining any two partial squares and assuming it makes a whole square.
- Students confuse length and area. For example, in the diagram shown, students may incorrectly state that the length of the square is $\frac{1}{2}$ unit, rather than 1 unit.



Questions to ask

- What mathematical terms did you use to describe the sides?
- What mathematical terms did you use to describe the angles?
- Did your group use the term *polygon* in your discussion? What is a polygon?
- What unit of measure is used to describe a side length?
- What unit of measure is used to describe area?

- What strategy did you use to show the area was exactly 20 square units?
- Is there another way?

Summary

Different shapes may share common attributes such as area.

DEVELOP

Activity 3.1

Investigating the Area of a Parallelogram

Facilitation Notes

In this activity, students decompose a parallelogram to create rectangle and conclude that the two shapes have the same area. The same formula can be used to determine the area of either figure.

Ask a student to read the definition of *parallelogram* aloud. Have students complete Questions 1 through 5 with a partner or in groups. Share responses as a class.

Questions to ask

- What does it mean for sides of a figure to be parallel?
- Where are the two pairs of parallel sides located?
- Where are the opposite sides located?
- Is there a relationship between the adjacent sides of a parallelogram? Are the adjacent sides also equal in length?
- What shapes did you cut the parallelogram into to form the rectangle?
- How many cuts did you make?
- What do you notice about the area of the parallelogram and the area of the rectangle?
- Does the orientation of the parallelogram affect the area?
- Is there more than one way to label the base?
- What does *perpendicular* mean?
- Is there more than one way to label the height?
- Where else have you heard the term *base* being used? Did it have the same meaning as it does in math class?

Misconceptions

- Students may think that the base must be the bottom of the figure, given the way it is drawn on the page.
- Students may think that both the base and the height are always sides of the figure.

Differentiation strategies

- To scaffold support with base and height, it may be helpful to have students cut out figures. That way, they can turn the paper to see that different sides can be the base. They can then use folds to determine the height.
- To extend the activity, have students investigate these questions:
  - If two parallelograms have different heights and different base lengths, could the parallelograms have equal areas? Provide an example.
  - What happens to the height of a parallelogram as the length of the base changes but the area remains the same?
  - How is the area of a parallelogram affected if you double the length of the base? double the height? double both the base and height? Provide examples.

Read the information following Question 5. Have students complete Question 6 with their partner or in groups. Share responses as a class.

Questions to ask

- Why do you think the variable *b* is used for base and *h* is used for height?
- What variable should you use to represent area?
- How does the formula for the area of a parallelogram compare to the formula for the area of a rectangle?
- How does the model you cut out and rearranged support the fact that the area formulas for rectangles and parallelograms with the same base and height are equal?

Summary

A parallelogram can be decomposed into a rectangular shape. The area of a parallelogram and the area of a rectangle can be calculated using the same formula.

Activity 3.2
Investigating the Area of a Triangle

Facilitation Notes

In this activity, students compose two triangles to form a parallelogram. Therefore, the area of a triangle is one-half the area of a parallelogram sharing the same numeric values for height and base length. The discussions should focus on the justification of the triangle area formula, not the memorization of the formula.

Ask a student to read the introduction and Question 1 aloud, and examine the three triangles as a class.

Questions to ask

- How does the height of a triangle compare to the height of a parallelogram?
- What are the dimensions of each square on the grid?
- Is the height of a triangle always/sometimes/never a side of the triangle?
- Is the height of a triangle always/sometimes/never inside of the triangle?
- Is the height of a triangle always/sometimes/never perpendicular to the base of the triangle?

Differentiation strategy

To scaffold support, have students interact with the triangle figures by using colored pencils to trace the base and height as you explain their relationship.

Ask students to complete Questions 1 through 5 with a partner or in groups. Share responses as a class.

Differentiation strategies

- To scaffold support,
  - Provide the fourth vertex point to complete the parallelogram.
  - Have students write different forms of the formula (with a fraction, decimal, or division symbol), and have them select the form that makes the most sense to them.

Ask a student to read the Worked Example. Have students complete Question 6 with a partner or in groups. Share responses as a class.

Questions to ask

- Is there more than one way to label the base and height of a triangle?

- What is important to keep in mind when you are identifying the height of a triangle?
- What information is necessary to determine the area of a triangle?
- Think about how you calculate the area of any parallelogram. How does the area of a triangle with the same base and height compare?

Summary

Composing two of the same triangles forms a parallelogram; therefore, the area of a triangle is half the area of a parallelogram.

Activity 3.3

Investigating the Area of a Trapezoid

Facilitation Notes

In this activity, students use composition and decomposition to determine an area formula for trapezoids using the length of the two bases and the height.

Ask a student to read the introduction and definition of *trapezoid* aloud. Have students complete Questions 1 and 2. Share responses as a class.

Questions to ask

- How do you know which sides should be labeled as the bases?
- Why can you draw in the height anywhere between the bases of the trapezoid and still get the same measure?
- What was the area of the trapezoid?
- What was the numerical expression you used to determine the area?
- What expression best represents the base of the parallelogram?
- What is the height of the parallelogram? Is the height of the parallelogram you composed the same as the height of the original trapezoid?
- How many trapezoids are in the parallelogram?

Differentiation strategies

- To scaffold support, have students label the vertices inside the trapezoid before they cut it out, so that they can follow oral directions if they need assistance.
- To extend the activity, ask students to explain how calculating the area of a trapezoid is related to calculating the average area of two rectangles.

Complete Question 2 as a class and discuss.

Summary

The area of any trapezoid is half the product of its height times the quantity of the sum of the two bases.

Activity 3.4
Calculating Areas of Figures

Facilitation Notes

In this activity, students use the formulas for the area of parallelograms, triangles, and trapezoids in mathematical and contextual problems. They calculate the area of these polygons and determine an unknown dimension of a polygon given its area and a known dimension.

Have students complete Questions 1 through 4 with a partner or in groups. Share responses as a class.

Questions to ask

- Could the dimensions of the figures been labeled in a different way?
- What formula did you use for each figure?
- Why do some of the figures have dashed lines?
- What are the units for each of the answers?

Have students complete Questions 5 and 6 with a partner or in groups. Share responses as a class.

Questions to ask

- How would the question change if you were given the opposite dimension?

- Why does one given number have "square units," while the other has "units"?

Differentiation strategy

To scaffold support, have students draw and label a picture of each figure, so that they can visualize what they are looking for.

Ask a student to read the directions above Question 7 aloud. Have students complete Questions 7 through 9 with a partner or in groups. Share responses as a class.

Questions to ask

- What shape is the stamp?
- What shape is the Yield sign?
- What are the units for your answer?

Summary

If given the area of a parallelogram, triangle, or trapezoid and one dimension, the area formulas can be used to solve for the missing dimension. The areas of these three shapes can be used to solve real-world problems.

Talk the Talk: Figure 'Em Out!

DEMONSTRATE

Facilitation Notes

In this activity, students draw, label, and write the area formula for a parallelogram, triangle, and trapezoid.

Ask students to complete Questions 1 and 2 in pairs or groups. Share responses as a class.

As students work, look for

- Parallelograms and trapezoids that are drawn so no side lengths are parallel.
- Different orientations of the shapes.
- Different ways the base and height are labeled in each shape.
- Shapes drawn so no side lengths are horizontal or vertical.

Questions to ask

- What determines which side length is the base in a parallelogram?
- What determines how you label the height in a parallelogram?
- What determines which side length is the base of a triangle?
- What determines how you label the height of a triangle?

- What determines which side lengths are the bases of a trapezoid?
- What determines how you label the height of a trapezoid?
- How do the area of a parallelogram and the area of a triangle with the same base and height compare?

Differentiation strategy

To extend the activity,

- For Question 1, ask students to include examples in their explanations.
- For Question 2, ask students to generalize by answering the question without calculating the area.

Summary

The area of any parallelogram is the length of the base times the height. The area of any triangle is half the product of the length of the base times the height. The area of any trapezoid is half the product of the height times the sum of the two bases.

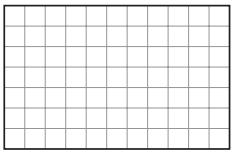
3

All About That Base. . . and Height

Area of Triangles and Quadrilaterals

WARM UP

Write 3 different expressions to describe the total area of the rectangle shown.



LEARNING GOALS

- State and compare the attributes of different figures.
- Explain that the area of a parallelogram is the same as the area of a rectangle with the same base length and height.
- Derive the formulas for the areas of triangles, parallelograms, and trapezoids by composing or decomposing the various figures into rectangles and triangles.
- Solve real-world and mathematical problems by composing and decomposing figures.

KEY TERMS

- parallelogram
- variable
- straightedge
- trapezoid

You can take a figure apart and put it back together in a different way without changing its area. How can you compose and decompose rectangles to reason about the areas of common figures?

Warm Up Answers

Sample answers.

77

7×11

$7(6 + 5)$

Answers

1a. Rectangle; 4 sides and 4 right angles

1b. Triangle; 3 sides and 1 right angle

1c. Parallelogram; 4 sides and opposite sides parallel

1d. Trapezoid; 4 sides and one pair of opposite sides parallel

2. Sample answers.
   a. The rectangle has 5 rows and 4 columns of unit squares.
   b. The triangle is half of a rectangle that has 10 rows and 4 columns.
   c. The parallelogram has a "middle" rectangle of 15 square units and two side triangles that together create a rectangle with an area of 5 square units.
   d. The trapezoid has a "middle" rectangle of 16 square units and two side triangles that together create a rectangle with an area of 4 square units.

Getting Started

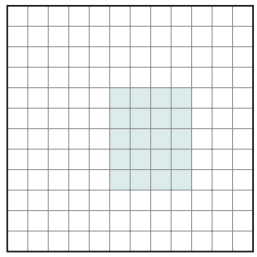
In the 20s

Consider each two-dimensional figure.

Take Note...
An attribute is a characteristic to describe a figure.

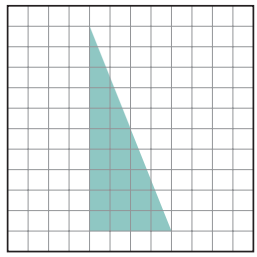
1. Name each figure and describe the attributes.

a.



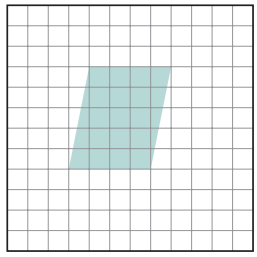
Name: _____
Attributes:

b.



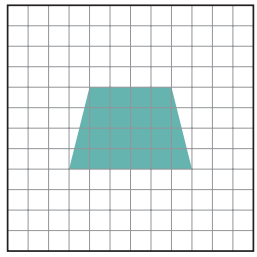
Name: _____
Attributes:

c.



Name: _____
Attributes:

d.



Name: _____
Attributes:

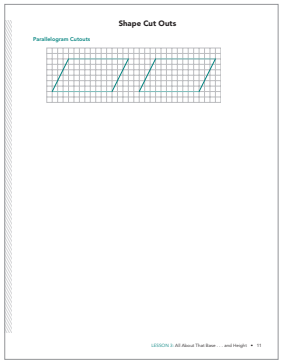
2. Each shaded figure shown has an area of exactly 20 square units. Show how you know.

ACTIVITY
3.1

Investigating the Area of a Parallelogram

In this activity, you will investigate the area of a *parallelogram* using what you know about the area of a rectangle. A **parallelogram** is a four-sided figure with two pairs of parallel sides and opposite sides that are equal in length.

Cut out a parallelogram from the grid located on page 11.



Remember...
A rectangle is a special type of parallelogram.

1. Cut your parallelogram into pieces so that you can reassemble it to form a rectangle. Tape your rectangle in the space provided.

Parallelogram **Rectangle**

In a parallelogram, you can label any of the four sides as the base. The height, represented by a line segment, is the perpendicular distance from a base to its opposite side.

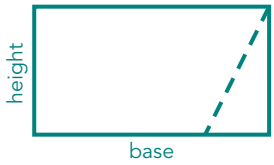
2. Label the base and height of the parallelogram and rectangle.

Take Note...
The right angle symbol indicates that the lines are perpendicular.

Answers

1. **Parallelogram**

Rectangle



2. See diagram above.

Answers

3. The heights of both the parallelogram and the rectangle have the same measure. The bases of both the parallelogram and the rectangle have the same measure.
4. The areas of the parallelogram and rectangle are equal.
5. To calculate the area of a parallelogram, you multiply the length of the base times the height.
6. The formulas are the same. $A = bh$

Compare the attributes of the parallelogram and the rectangle you composed.

Ask Yourself...

When you write a sentence to explain your reasoning, be sure to express a complete idea. If you cover up the question, does your sentence make sense?

3. How does the height of the parallelogram relate to the height of the rectangle? How does the length of the base of the parallelogram relate to the length of the base of the rectangle? Explain your reasoning.

4. Describe the relationship between the areas of a parallelogram and rectangle that have the same base and height.

5. Use the terms *base* and *height* to describe how to calculate the area of a parallelogram.

Take Note...

A **variable** is a letter used to represent a number.

When you want to represent a quantity that varies or changes, you can use a *variable*. The use of variables helps you write formulas to express relationships.

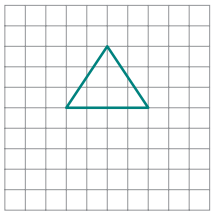
6. Write the formulas to calculate the areas of a parallelogram and a rectangle. Use *b* to represent the length of the base and *h* to represent the height.

ACTIVITY
3.2 Investigating the Area of a Triangle

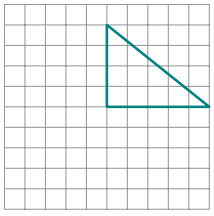
In this activity, you will investigate the area of a triangle using what you know about the area of a parallelogram.

Consider each triangle shown.

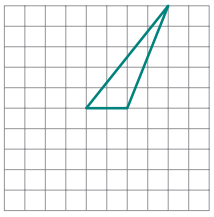
Triangle 1



Triangle 2



Triangle 3



1. **Use a separate piece of patty paper to trace each triangle.**
   a. **Rotate the patty paper to create a parallelogram composed of two identical triangles.**
   b. **Draw the parallelogram you created on your patty paper and label its base and height.**

Take Note...

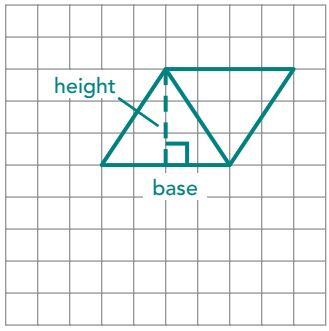
When you trace or draw a figure, use a *straightedge*. A **straightedge** is a tool to draw straight lines.

2. **For each triangle, compose a second parallelogram using a different side of the triangle.**

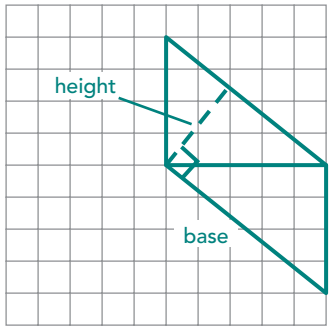
Answers

1a. and b.
Sample parallelograms.

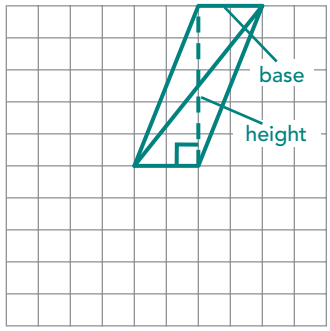
Triangle 1



Triangle 2



Triangle 3



2. Check students' drawings.

Answers

3. Parallelogram created from Triangle 1: 12 square units
   Parallelogram created from Triangle 2: 20 square units
   Parallelogram created from Triangle 3: 10 square units
4. The area of each triangle is half the area of the parallelogram.
5. $A = \frac{1}{2}bh$
6. To determine the area of a triangle, you can use any side length for the base and the height of the triangle formed from that base. Triangle *ABC* has the same area regardless of its orientation. When the base changes, then the height changes, but the area stays the same.

3. Determine the area of each parallelogram you created.

4. How does the area of each triangle relate to the area of the parallelogram?

Remember...

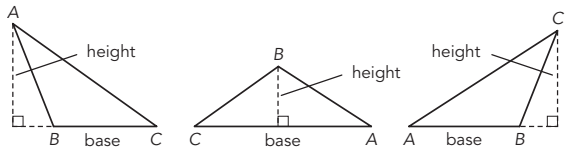
The formula for the area of a parallelogram is $A = bh$.

5. Write a formula to calculate the area of a triangle using the formula for the area of a parallelogram. Use *b* to represent the length of the base and *h* to represent the height.

WORKED EXAMPLE

As with the base of a parallelogram, the base of a triangle can be any of its sides. The height of a triangle, represented by a line segment, is the perpendicular distance from a vertex to the line containing the base.

Triangle *ABC* is shown in three different positions.



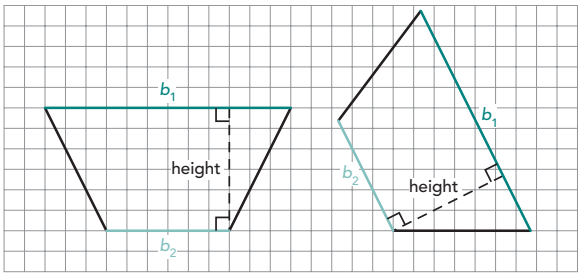
6. Analyze the Worked Example. What general statement can you make about determining the area of a triangle? Explain your reasoning.

ACTIVITY
3.3

Investigating the Area of a Trapezoid

You have seen that decomposing and composing can help you think about shapes differently to determine their areas. In this activity, you will use the same strategy to determine the formula for calculating the area of a *trapezoid*.

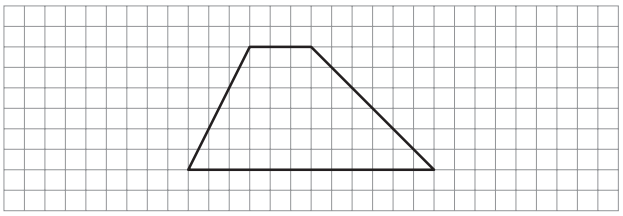
A **trapezoid** is a quadrilateral with two bases that are parallel to each other. The other two sides of a trapezoid are called the *legs* of the trapezoid. A height of a trapezoid is the length of a line segment drawn perpendicular from one base to the other.



Take Note...

The variable b represents a base, but a trapezoid has two bases. So, we use subscripts to distinguish between the two different bases: b_1 and b_2 are not equal in length.

1. To figure out the exact area of the trapezoid shown, compose two trapezoids into a parallelogram. Show how you can determine the area of the trapezoid.

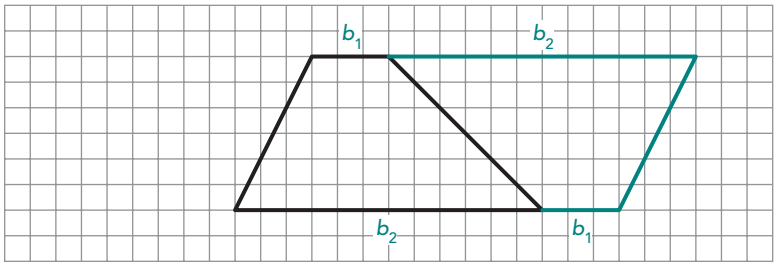


2. Describe how to calculate the area of any trapezoid in terms of the two bases and the height.

Answers

1. See sample parallelogram below. You can determine the area of the trapezoid by calculating the area of the composed parallelogram and then dividing it in half.
2. The area of any trapezoid is half the product of the height times the quantity of the sum of the two bases.

1.



Answers

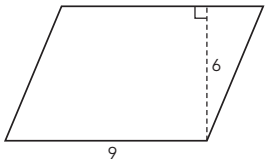
1. $(6)(9) = 54$
   54 square units
2. $\frac{1}{2}(24)(8) = 96$
   96 square units
3. $\frac{1}{2}(9)(11 + 5) = 72$
   72 square units
4. $\frac{1}{2}(7)(16) = 56$
   56 square units
5. $63 \div 7 = 9$
   9 units

ACTIVITY
3.4

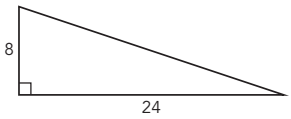
Calculating Areas of Figures

Calculate the area of each figure.

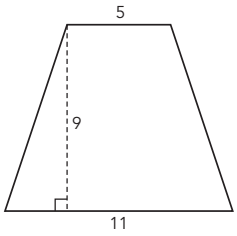
1.



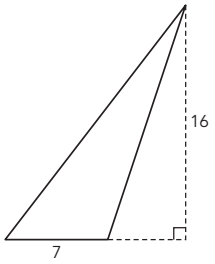
2.



3.



4.



Determine the unknown length in each figure.

5. A parallelogram has an area of 63 square units. The height of the parallelogram is 7 units. What is the base length of the parallelogram?

6. **A triangle has an area of 24 square units. The base length of the triangle is 4 units. What is the height of the triangle?**

Solve each area problem.

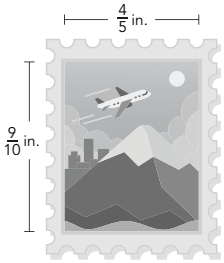
7. **Sanjay has enough material to build a rectangular dance floor with an area of 200 square feet. The greatest width the dance floor can be is $12\frac{1}{2}$ feet. What would be the length of this dance floor?**

Remember...

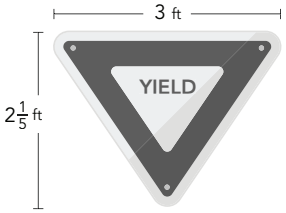
Use a straightedge to draw your figures.

8. **Determine the area of each object.**

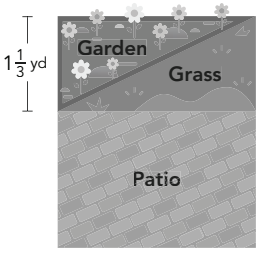
a.



b.



9. **Tara has enough garden soil to make a garden bed with an area of 6 square yards. She wants to build the garden bed in the corner of her yard so it is the shape of a right triangle. Because of the location of her patio, one leg of the triangle must be $1\frac{1}{3}$ yards long. Determine the length of the other leg of the triangle.**



Answers

6. 12 units
$24 \div (4 \div 2) = 12$

7. 16 feet
$200 \div 12\frac{1}{2} = 16$

8a. $\frac{18}{25}$ square inch
$\frac{4}{5} \times \frac{9}{10} = \frac{18}{25}$

8b. $\frac{33}{10}$, or $3\frac{3}{10}$, square feet
$\frac{1}{2}\left(\frac{11}{5} \times \frac{3}{1}\right) = \frac{33}{10}$

9. 9 yards
$\frac{1}{2} \cdot b \cdot 1\frac{1}{3} = 6$
$\frac{1}{2} \cdot \frac{4}{3} \cdot b = 6$
$\frac{4}{6} \cdot b = 6$
$b = \frac{6}{1} \div \frac{4}{6}$
$b = \frac{6}{1} \cdot \frac{6}{4}$
$b = \frac{36}{4}$
$b = 9$

Answers

1a. Check students' drawings.

$A = bh$

1b. Check students' drawings.

$A = \frac{1}{2}bh$

1c. Check students' drawings.

$A = \frac{1}{2}h(b_1 + b_2)$

2. Triangle *RGM* and Triangle *PGM* share the same base, and their heights are equal length. So, the areas of the two triangles are the same.

NOTES

TALK the TALK

Figure 'Em Out!

You have decomposed and composed parallelograms in this lesson to derive the formulas for the area of a parallelogram, triangle, and trapezoid.

1. Draw each figure and then label a base and height. Next, write the formula to calculate the area of each. Use *A* for the area, *b* for the length of the base, and *h* for the height.

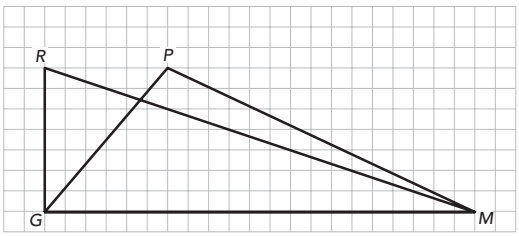
a. parallelogram

b. triangle

c. trapezoid

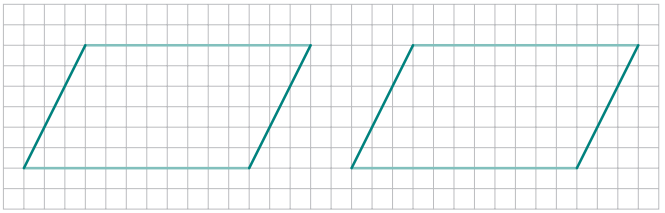
Consider △*RGM* and △*PGM*.

2. Without performing any calculations, determine which triangle has the greater area. Write a sentence to explain your reasoning.



Shape Cut Outs

Parallelogram Cutouts



Why is this page blank?

So you can cut out the shapes on the other side.