Shapes and Solids

KEY TERMS

- Triangle Inequality Theorem
- Triangle Sum Theorem
- parallelogram
- variable
- straightedge
- trapezoid
- geometric solid
- polyhedron
- face
- edge
- vertex
- right rectangular prism
- cube
- volume
- unit cube

LESSON 1

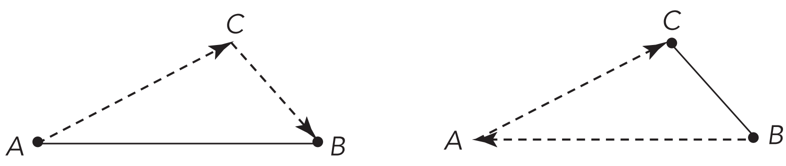
Consider Every Side

Triangles are congruent when all of their corresponding angle measures and corresponding side lengths are the same. When given information can be used to construct congruent triangles, the information is said to define a unique triangle.

The **Triangle Inequality Theorem** states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$AC + CB > AB$

$BA + AC > BC$



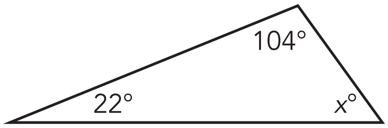
When given two line segments, it is possible to construct an infinite number of triangles. When given three line segments, it is possible to either construct a unique triangle or no triangle.

LESSON 2 Turning a One-Eighty!

The **Triangle Sum Theorem** states that the sum of the measures of the interior angles of a triangle is 180°. The longest side of a triangle is opposite the interior angle with the greatest measure, and the shortest side is opposite the interior angle with the least measure.

The Triangle Sum Theorem can be used to determine the measure of the third angle of a triangle when two angle measures of the same triangle are given.

$x + 22 + 104 = 180$
$x + 126 = 180$
$x = 54$



The measure of the third angle in this triangle is 54°.

$22° + 104° + 54° = 180°$

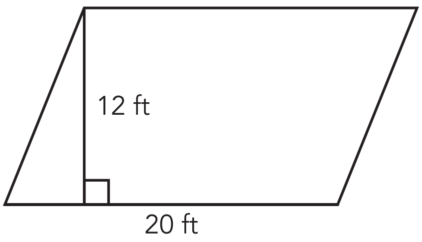
LESSON 3 All About That Base . . . and Height

A **parallelogram** is a four-sided figure with two pairs of parallel sides with each pair equal in length.

In a parallelogram, the height is the perpendicular distance from the base to the opposite side. The area of a parallelogram is equal to $b \cdot h$, where the variable b represents the base and h represents the height. A **variable** is a letter used to represent a number.

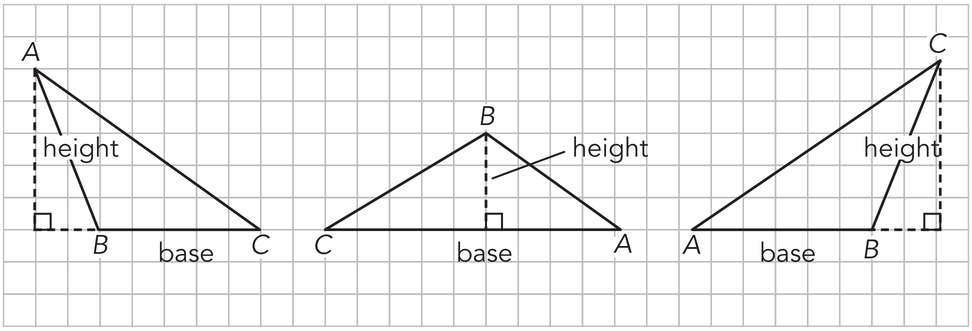
For example, in this parallelogram, the base, b, is 20 feet and the height, h, is 12 feet.

$$\begin{aligned}\text{Area of a parallelogram} &= bh \\ &= (20)(12) \\ &= 240 \text{ square feet}\end{aligned}$$



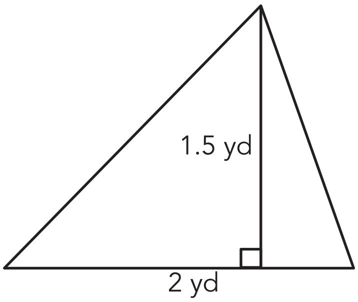
The area of a triangle is equal to $\frac{1}{2}bh$. The base of a triangle can be any of its sides.

The height of a triangle is the length of a line segment drawn from a vertex of the triangle to the opposite side so that it forms a right angle with the opposite side.



For example, in this triangle, the base, b, is equal to 2 yards, and the height, h, is equal to 1.5 yards.

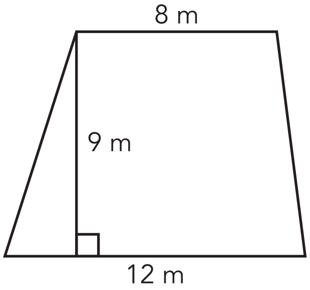
$$\begin{aligned}\text{Area of a triangle} &= \frac{1}{2}bh \\ &= \frac{1}{2}(2)(1.5) \\ &= 1.5 \text{ square yards}\end{aligned}$$



A **trapezoid** is a quadrilateral with two bases, often labeled b_1 and b_2. The bases are parallel to each other. The height is the perpendicular distance between the bases. The area of a trapezoid is equal to $\frac{1}{2}(b_1 + b_2)h$.

For example, in this trapezoid, one of the bases is 8 meters and the other base is 12 meters. The height, h, of the trapezoid is 9 meters.

$$\begin{aligned}\text{Area of trapezoid} &= \frac{1}{2}(b_1 + b_2)h \\ &= \frac{1}{2}(8 + 12)(9) \\ &= \frac{1}{2}(20)(9) \\ &= 90 \text{ square meters}\end{aligned}$$



LESSON 4

Length, Width, and Depth

The mathematical definition of *point* is a location in space, often represented using a dot and named by a capital letter. A *line segment* is a portion of a line that includes two points and the points between those two points.

A *polygon* is a closed figure formed by three or more line segments. A **geometric solid** is a bounded three-dimensional geometric figure. A **polyhedron** is a three-dimensional solid figure made up of polygons; each of these polygons is called a **face**. An **edge** is the intersection of two faces, and a **vertex** is the point where the edges meet.

For example, Figure A is a **right rectangular prism**, which is a polyhedron with three pairs of congruent and parallel rectangular faces.

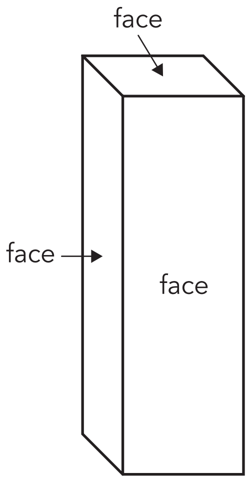


Figure A

Figure B is a **cube**, which is a polyhedron that has six congruent squares as faces.

Volume is the amount of space occupied by an object. The volume of an object is measured in cubic units. A **unit cube** is a cube whose sides are all 1 unit long.

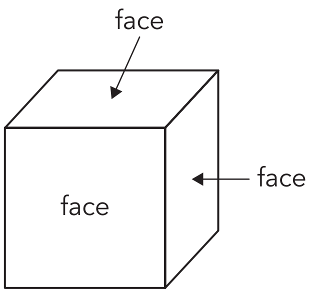
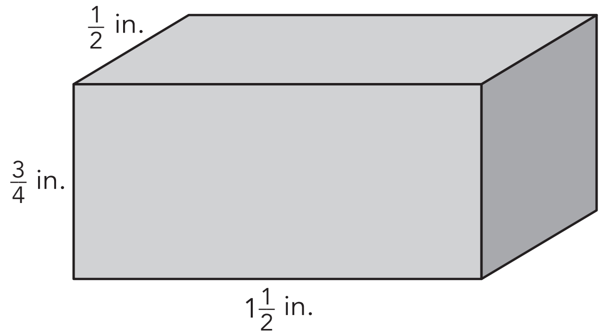


Figure B

The volume of a rectangular prism is the product of its length, width, and height: $V = l \cdot w \cdot h$.

For example, to determine the volume of the right rectangular prism shown with the given dimensions, you can fill the prism with cubes, but smaller unit cubes with fractional side lengths are required.



Assign a unit fraction to the dimensions of each cube. Use the least common multiple (LCM) of the fraction denominators to determine the unit fraction.	LCM(2, 4) = 4 So, each cube will measure $\frac{1}{4}$ in. $\times$ $\frac{1}{4}$ in. $\times$ $\frac{1}{4}$ in. The volume of each unit cube is $\frac{1}{64}$ cubic inches.
Determine the number of cubes needed to pack the prism in each dimension.	length: $1\frac{1}{2} \div \frac{1}{4} = 6$ width: $\frac{1}{2} \div \frac{1}{4} = 2$ height: $\frac{3}{4} \div \frac{1}{4} = 3$
Determine the number of cubes that make up the right rectangular prism.	$6 \times 2 \times 3 = 36$
Multiply the number of cubes by the volume of each cube to determine the volume of the right rectangular prism.	$36 \times \frac{1}{64} = \frac{36}{64}$ $= \frac{9}{16}$

You can use the formula $V = Bh$ to calculate the volume of any prism. However, the formula for calculating the value of B will change depending on the shape of the base. In a rectangular prism, $B = l \cdot w$.

$V = Bh = (l \cdot w) \cdot h$

$1\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{3}{4} = \frac{9}{16}$

The volume of the right rectangular prism is $\frac{9}{16}$ cubic inches.