Product Placement

3

Multiplying Decimals

MATERIALS

None

Lesson Overview

In this lesson, students use an area model on a hundredths grid to represent the multiplication of two decimals less than one. They use estimation to reason about the placement of the decimal point in multiplication problems and then analyze patterns to develop the algorithm for multiplying decimals. Students solve area and volume problems that require multiplying, adding, and subtracting decimals.

Grade 6

Number and Operations

(3) The student applies mathematical process standards to represent addition, subtraction, multiplication, and division while solving problems and justifying solutions. The student is expected to:

(E) multiply and divide positive rational numbers fluently.

Expressions, Equations, and Relationships

(8) The student applies mathematical process standards to use geometry to represent relationships and solve problems. The student is expected to:

(D) determine solutions for problems involving the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- An area model using a hundredths grid can represent the product of two decimals less than one.
- Use estimation to determine if the product of two decimal factors is reasonable.
- When multiplying decimals, the number of decimal places in the product is equal to the sum of the decimal places in the factors.
- You can use the standard algorithms for decimal addition, subtraction, and multiplication to solve real-world problems.

Lesson Structure and Pacing: 1 Day

Engage

Getting Started: Doing Some Re-Modeling

Students use an area model on a hundredths grid to represent the multiplication of decimals. They make sense of the fact that the product of two decimals less than one is always less than one.

Develop

Activity 3.1: Decimal Products

Students use estimation to reason about the placement of the decimal point in multiplication problems. They analyze patterns to develop the algorithm to multiply decimals. Students address common misconceptions when multiplying decimals.

Activity 3.2: Solving Problems with Decimals

Students solve problems in context—including area and volume situations—that require multiplying, adding, and subtracting decimals.

Demonstrate

Talk the Talk: Disappearing Decimals

Students insert decimal points into factors or products to create correct multiplication statements.

Getting Started: Doing Some Re-Modeling

ENGAGE

Facilitation Notes

In this activity, students use an area model on a hundredths grid to model multiplication of decimals. They make sense of the fact that the product of two decimals less than one is always less than one.

Ask a student to read the introduction aloud and discuss as a class. Have students work with a partner or in groups to complete Questions 1 through 3. Discuss responses as a class.

Differentiation strategy

To assist all students, consider providing students two colored pencils to distinguish between the factors and their product.

Questions to ask

- Why do you think the area model is on a 10 by 10 grid?
- How do you know that the factors are decimal values less than one?
- Why do the factors have place values that are tenths?
- Why does the product have a place value in the hundredths?
- Does it make a difference whether columns or rows are used for each factor? Why not?
- Could you have predicted that product before using the grid? Explain your thinking.
- If you have an original value greater than one and multiply it by a value less than one, will the original value get larger or smaller? Why?
- If you have an original value less than one and multiply it by a value less than one, will the original value get larger or smaller? How does this reasoning coincide with your area model?

Summary

An area model using a hundredths grid can represent the product of two decimals less than one.

DEVELOP

Activity 3.1
Decimal Products

Facilitation Notes

In this activity, students use estimation to reason about the placement of the decimal point in multiplication problems. They analyze patterns to develop the algorithm to multiply decimals and address common misconceptions when multiplying decimals.

Ask a student to read the situation aloud, and discuss the Worked Example as a class. Have students work with a partner or in groups to complete Question 1. Discuss responses as a class.

Questions to ask

- How did Kenji modify the traditional multiplication algorithm to multiply decimals?
- Explain how your estimation helped you decide where to place the decimal point.
- Would the process work the same whether you rounded 0.6 to 0.5 or 1? Explain your thinking.

Misconception

Students may think that they must line up the decimal points to perform multiplication as they do with addition and subtraction. Explain that when multiplying, we address place value at the end of the process.

Have students work with a partner or in groups to complete Questions 2 through 4. Discuss responses as a class.

Differentiation strategies

To scaffold support,

- Consider allowing students to use calculators to complete this activity.
- Suggest that students count the number of decimal places starting at the end of the value. That way, if more decimal places are required, they will insert them immediately after the decimal point.

Questions to ask

- What pattern do you notice as you inspect each column?
- What pattern do you notice as you inspect each row?
- Why does the product of 3.2 and 100 have one zero instead of two zeros?
- Why does the product of 3.2 and 10 have no zeros?
- Use a pattern to explain why $0.32 \times 0.001 = 0.00032$.

- How is the number of decimal places in the factors related to the number of decimal places in the product?
- Why did you place the extra 0 before the 8 rather than after the 6?
- Provide an example to support your thinking.

Have students work with a partner or in groups to complete Questions 5 and 6. Discuss responses as a class.

Question to ask

- How would you suggest Selena modify her rule to include decimals?

Summary

Use estimation to determine if the product of two decimal factors is reasonable. When multiplying decimals, the number of decimal places in the product is equal to the sum of the decimal places in the factors.

Activity 3.2
Solving Problems with Decimals

Facilitation Notes

In this activity, students solve problems in context—including area and volume situations—that require multiplying, adding, and subtracting decimals.

Ask a student to read the directions aloud, and discuss as a class. Have students answer Questions 1 through 6 with a partner or in groups. Discuss responses as a class.

Differentiation strategy

As an alternative grouping strategy, use the jigsaw method. Divide the class into groups of four; assign half of the groups the even-numbered problems and the other half the odd-numbered problems. Then, regroup the class so that each group of four has a pair of students who solved different problems. Have the student pairs explain their problems to the other group members.

As students work, look for

Correct use of units in the final answers.

Questions to ask

- How can you use estimation to check the reasonableness of your answer?
- How did you determine where to place the decimal point in your final answer?
- What are the names of the three-dimensional figures in this problem?
- What formula did you use for the cardboard box?
- Explain your strategy to solve this problem.
- What is another strategy to solve this problem?
- What is the most efficient strategy? Why?
- What is another way to decompose this figure?
- How did you determine the dimensions that were not provided?

Summary

You can use the standard algorithms for decimal addition, subtraction, and multiplication to solve real-world problems.

DEMONSTRATE

Talk the Talk: Disappearing Decimals

Facilitation Notes

In this activity, students insert decimal points into factors and products to create correct multiplication statements.

Ask a student to read the directions aloud, and discuss as a class. Have students work individually to complete Question 1. Discuss responses as a class.

Questions to ask

- What are other possible placements of the decimal points?
- What rule did you use to determine where to place the decimal point(s)?

Summary

When multiplying decimals, the number of decimal places in the product is equal to the sum of the decimal places in the factors.

3 Product Placement

Multiplying Decimals

WARM UP

Round each decimal to the nearest ten and then to nearest tenth.

1. 19.54
2. 11.96
3. 124.35
4. 245.339
5. 10.888
6. 1258.445

LEARNING GOALS

- Estimate the product of decimals.
- Analyze patterns of products of decimals.
- Multiply decimals by whole numbers.
- Use the standard algorithm to multiply two decimals.
- Solve real-world and mathematical problems involving area and volume.

KEY TERMS

- kite
- composite solid

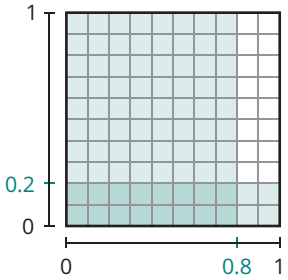
You know how to multiply multi-digit whole numbers. How can you use place value to determine the position of a decimal point in the product of decimals?

Warm Up Answers

1. 20; 19.5
2. 10; 12.0
3. 120; 124.4
4. 250; 245.3
5. 10; 10.9
6. 1260; 1258.4

Answers

1. $0.5 \times 0.7 = 0.35$
2. $0.2 \times 0.8 = 0.16$



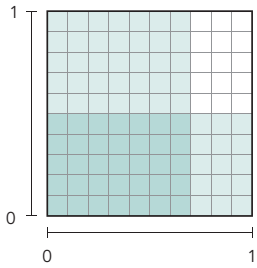
3. Benny is correct. If the factors are decimals smaller than one, the squares on the area model that have overlapping shading will always be contained in one unit square, so the product is always less than one.

Getting Started

Doing Some Re-Modeling

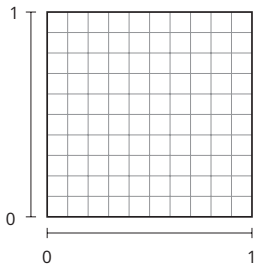
You previously used an area model to show the product of two fractions. An area model can also represent the product of two decimals.

Consider the area model shown.



1. Write the multiplication problem represented by the area model.

2. Use the hundredths grid shown to model the expression 0.2×0.8 and determine the product.



3. Benny says that the product of any two decimals that are less than one will always be less than one. Angel says that the product can be greater than one. Who's correct? Use the area model to explain your reasoning.

ACTIVITY
3.1 Decimal Products

Kenji needs to calculate the area of the floor in a hallway of his house so he can buy new carpeting. The floor is 32.75 feet long and 7.5 feet wide.

Kenji said, "I use estimation to help place the decimal point correctly in the product."

WORKED EXAMPLE

The area of the floor is 32.75 feet × 7.5 feet.

He estimates his two numbers.

- 32.75 is close to 30
- 7.5 is close to 7
- 30 × 7 = 210

So he knows his product is close to 210, but larger since he rounded down. Next, he calculates the product of 32.75 × 7.5.

$$\begin{array}{r} 32.75 \\ \times \quad 7.5 \\ \hline 16375 \\ +\ 229250 \\ \hline 245.625 \end{array}$$

Kenji knows the product will be close to, but greater than, 210, so he must place the decimal point after the 5. The area of the floor is 245.625 square feet.

1. Try Kenji's estimation strategy to determine the correct decimal place in each multiplication problem.

a. 52.6 × 0.83 = 43658

b. 7.9 × 0.6 = 474

c. 0.94 × 24.9 = 23406

Take Note...

Multiply decimals as you would with whole numbers. Then, place the decimal point in the product.

Answers

1a. 50 × 1 = 50
43.658

1b. 8 × 0.5 = 4
4.74

1c. 1 × 25 = 25
23.406

Answers

2a. See table below.

2b. The decimal point moves to the right when I multiply by a power of 10. The decimal point moves to the left when I multiply by a power of 10 that is less than 1.

3a. 80.6

3b. 8.06

3c. 0.806

3d. 0.806

3e. 8.06

3f. 0.0806

3g. 0.00806

3h. 0.0806

4. I can look at the sum of the decimal places of both factors. Expressions that have the same total number of decimal places will have the same product.

2. Casey thought that using a pattern would help her understand how to calculate the product in a decimal multiplication problem.

a. Complete the table.

Problem	Product	Problem	Product	Problem	Product
32 × 100		3.2 × 100		0.32 × 100	
32 × 10		3.2 × 10		0.32 × 10	
32 × 1		3.2 × 1		0.32 × 1	
32 × 0.1		3.2 × 0.1		0.32 × 0.1	
32 × 0.01		3.2 × 0.01		0.32 × 0.01	
32 × 0.001		3.2 × 0.001		0.32 × 0.001	

b. Describe any patterns that you notice.

3. Use the fact that 26 × 31 = 806 to calculate each product. Do not use your calculator.

a. 2.6 × 31 **b. 2.6 × 3.1** **c. 0.26 × 3.1** **d. 2.6 × 0.31**

e. 0.26 × 31 **f. 2.6 × 0.031** **g. 0.026 × 0.31** **h. 0.26 × 0.31**

4. Look at the pattern in Question 3. How can you tell without multiplying which expressions will have the same product?

2a.

Problem	Product	Problem	Product	Problem	Product
32 × 100	3200	3.2 × 100	320	0.32 × 100	32
32 × 10	320	3.2 × 10	32	0.32 × 10	3.2
32 × 1	32	3.2 × 1	3.2	0.32 × 1	0.32
32 × 0.1	3.2	3.2 × 0.1	0.32	0.32 × 0.1	0.032
32 × 0.01	0.32	3.2 × 0.01	0.032	0.32 × 0.01	0.0032
32 × 0.001	0.032	3.2 × 0.001	0.0032	0.32 × 0.001	0.00032

5. Consider Selena's statement.

> **Selena**
>
> Write a zero at the end of a number to multiply it by 10, and two zeros at the end to multiply it by 100. So, 3.2 × 10 = 3.20 and 3.2 × 100 = 3.200

a. Explain to Selena why her reasoning is incorrect.

b. Explain to Selena the similarities and differences between these number sentences.

32 × 10 = 320 **3.2 × 10 = 32** **32 × 100 = 3200** **3.2 × 100 = 320**

When multiplying decimals, the number of decimal places in the product is equal to the sum of the decimal places in the factors.

6. Consider Camila's statement.

> **Camila**
>
> I multiplied 3.65 × 4.22 on my calculator and got 15.403. There should be four decimal places in the product, so it must actually be 1.5403

a. Explain to Camila why the calculator is correct.

b. How could the estimate of 3.65 × 4.22 have helped Camila understand?

Answers

5a. The value of the decimal didn't change by adding the 0 to the end. When Selena multiplies a decimal by 10, the decimal point needs to move one place to the right to show the value of the number increasing by the multiple of 10.

5b. In the first two, the decimal point moved right one place to show multiplying by 10. In the next two, the decimal point moved right two places to show multiplying by 100.

6a. The calculator doesn't show the 0 in the ten-thousandths place because 15.4030 is equal to 15.403.

6b. The estimate is $4 \times 4 = 16$.
If Camila has a sense of the final product, she has a good idea where the decimal point should be placed.

Answers

1. $16.9 \times 8 = 135.2$
   Jonah drinks 135.2 fluid ounces of water each day.
2. $6.72 \times 2.5 = 16.8$
   Holly will pay $16.80 for the sliced turkey.
3. $7.625 \times 12 = 91.5$, so the rope is 91.5 inches long.
   $91.5 \times 2.54 = 232.41$
   The rope is about 232.41 centimeters long.
4. Area of base: $(2)(3.46) = 6.92$ square inches
   Volume of prism:
   $6.92 \times 36 = 249.12$
   $249.12 - 113.04 = 136.08$
   The amount of space left in the cardboard box is 136.08 cubic inches.

ACTIVITY 3.2

Solving Problems with Decimals

Use the algorithms for operating with decimals to solve each problem.

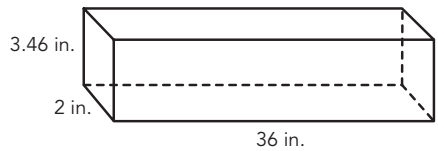
1. A water bottle holds 16.9 fluid ounces of water.

Jonah drinks 8 bottles of water each day. How many fluid ounces of water does Jonah drink each day?

2. Holly is buying 2.5 pounds of sliced turkey from the deli. The sliced turkey sells for $6.72 a pound. How much will Holly pay for the sliced turkey?

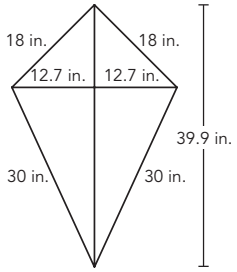
3. One inch is equal to about 2.54 centimeters. About how many centimeters long is a rope that is 7.625 feet long?

4. A poster is rolled up and mailed in a cardboard box in the shape of a rectangular prism.



The rolled-up poster takes up 113.04 cubic inches of space. Calculate the amount of space left in the cardboard box after the poster is placed inside it.

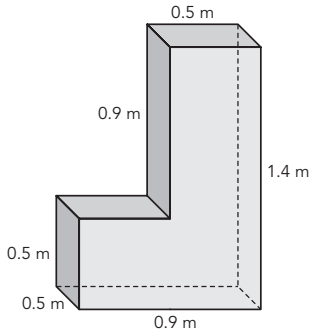
5. **Reene is designing her own kite. She uses two wooden sticks as a frame to hold the material. The longer stick measures 39.9 inches and divides the shorter stick into equal lengths of 12.7 inches where they cross. Determine the area of the material on Reene's kites.**



Take Note...

In mathematics, a **kite** is a quadrilateral with two pairs of consecutive congruent sides.

6. **Your class is building a concrete chair for a community garden. A drawing of the chair design is shown. Calculate the amount of concrete needed to build the chair.**



Take Note...

The chair is a *composite solid*. A **composite solid** is made up of more than one geometric solid.

Answers

5. Area of triangle: $\frac{1}{2}(39.9)(12.7) = 253.365$
   Area of kite:
   $2 \times 253.365 = 506.73$ square inches
6. Volume of prism with dimensions 0.5 m $\times$ 1.4 m $\times$ 0.5 m: 0.35 cubic meters.
   Volume of prism with dimensions 0.5 m $\times$ 0.4 m $\times$ 0.5 m: 0.1 cubic meters. $0.35 + 0.1 = 0.45$ cubic meters of concrete

Answers

1a. Sample answer.
$3.68 \times 5.26 = 19.3568$

1b. Sample answer.
$0.8962 \times 91.21 = 81.742402$

1c. $75.6 \times 98.75 = 7465.500$

1d. Sample answer.
$1.52 \times 152 = 231.04$

1e. Sample answer.
$0.5875 \times 0.2569 = 0.15092875$

1f. $94.05 \times 6.27 = 589.6935$

NOTES

TALK the TALK

Disappearing Decimals

Use the patterns of decimal products learned in this lesson to complete this activity.

You did your homework on multiplying decimals, but when you were walking to school, it started raining and some of the decimal points got washed away.

1. Insert decimal points to make each multiplication sentence correct.

a. 368 × 526 = 19.3568

b. 8962 × 9121 = 81.742402

c. 75.6 × 98.75 = 7465500

d. 152 × 152 = 231.04

e. 5875 × 2569 = 0.15092875

f. 94.05 × 6.27 = 5896935