4 Dividend in the House

Dividing Whole Numbers and Decimals

WARM UP

Write a fact family for each division expression.

1. $72 \div 8 = 9$

2. $84 \div 6 = 14$

3. $2464 \div 308 = 8$

LEARNING GOALS

- Estimate quotients.
- Develop an algorithm for dividing whole numbers.
- Use the standard algorithm to divide decimals.
- Write fractions as decimals.

KEY TERMS

- terminating decimal
- repeating decimal

You know how to use place value to multiply a number by a power of 10. How can you use this knowledge to determine the quotients of decimals?

Just the Facts

Take Note...
Quotients can be whole numbers, decimals, or fractions.

You previously learned that division helps you determine how many times one number contains another number. In other words, division is determining the quotient given a dividend and a divisor.

$$\text{divisor})\overline{\text{dividend}}\quad \text{quotient}$$

or

$$(\text{quotient})(\text{divisor}) = (\text{dividend})$$

Remember...
Multiplication and division are inverse operations.

Consider the expression 48 ÷ 6.

1. John evaluated the expression using the following strategy.

John
I know that 6 × 8 = 48,
so 48 ÷ 6 = 8.

a. Describe the strategy John used to evaluate the expression.

b. Identify the dividend, divisor, and quotient in the statement 48 ÷ 6 = 8.

c. Which values in the statement 48 ÷ 6 = 8 can you switch and the statement is still true? Explain your reasoning.

2. Is John's strategy reasonable for evaluating any division expression? Explain your reasoning.

ACTIVITY
4.1 Whole Number Division

If you don't know a fact family for a division problem, you can use other strategies to determine the quotient.

Consider the expression 34,098 ÷ 6. Analyze Lori's strategy to determine the quotient.

Lori

I used an organized estimation strategy.

1. I estimated how many 6s are in 34,098. I used 6 × 5000 = 30,000 and then I subtracted.

3. I still had 498 left, so I tried 6 × 50 = 300. It's too small. So, I used 6 × 80 = 480.

2. I have 4098 left. Next, I tried 6 × 700 = 4200, which is too big. So, I used 6 × 600 = 3600.

4. I still had 18. 6 × 3 = 18

```
   3
  80
 600
5000
6)34,098
-30,000
   4098
  -3600
    498
   -480
     18
    -18
      0
```

```
   3
  80
 600
+5000
 5683
```

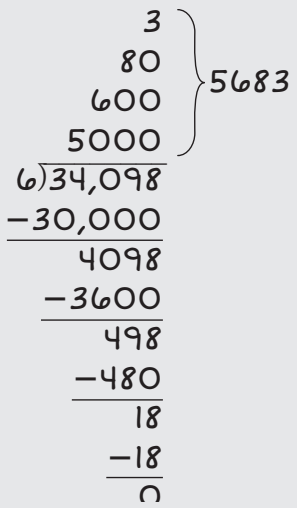
So

```
   5683
6)34,098
```

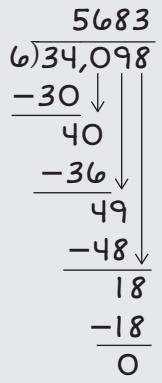
1. **In each step, why did Lori subtract after she determined each estimate?**

Rob and Morgan agreed with Lori's logic, but shortened the process.

Rob



Morgan



2. Compare Rob's and Morgan's strategies. What are the similarities and differences?

Carnegie Middle School conducted a month-long food and clothing drive to assist in disaster relief. They collected the following items for distribution.

- 13,312 cans of food
- 9472 blankets
- 19,456 batteries
- 26,112 bottles of water

If the students want to make 256 disaster-relief shipping crates, how many bottles of water will they put in each shipping crate?

3. **Analyze each solution.**

Morgan

I used my strategy from earlier.

$$\begin{array}{r} 102 \\ 256\overline{)26{,}112} \\ -256 \\ \hline 512 \\ -512 \\ \hline \end{array}$$

They must load 102 bottles of water into each crate.

Dustin

I think there should be 12 bottles of water in each crate.

$$\begin{array}{r} 12 \\ 256\overline{)26{,}112} \\ -256 \\ \hline 512 \\ -512 \\ \hline \end{array}$$

They must load 12 bottles of water into each crate.

a. **What did Dustin do incorrectly?**

b. **How could Dustin have checked his work to know that his answer was incorrect?**

Take Note...

You can use double lines at the end of a long division problem when the last difference is 0.

c. **There should have been 3 digits in Dustin's quotient. How could he have determined that before he started dividing?**

4. **Use Morgan's method of long division to determine how many of each item the students must load into each of the 256 shipping crates.**

Think About...

Determining the number of digits in your answer first helps you to know whether your quotient is correct.

a. cans of food

b. blankets

c. batteries

ACTIVITY
4.2

Interpreting Remainders in Solutions

In division problems, the remainder can mean different things in different situations. Sometimes the remainder can be ignored, and sometimes the remainder is the answer to the problem. Sometimes the answer is the number without the remainder, and sometimes you need to use the next whole number up from the correct answer.

1. **The Red Cross disaster relief fund collected 3551 winter coats to distribute to flood victims. If there are 23 distribution centers, how many coats can be sent to each center? Marla's calculations are shown.**

Marla said, "The Red Cross can send $154\frac{9}{23}$ coats to each center." Madison replied, "You cannot have a fraction of a coat. So, each center will receive 154 coats and there will be 9 coats left over." Who's correct and why?

$$
\begin{array}{r}
154\frac{9}{23} \\
23\overline{)3551} \\
-23 \\
\hline
125 \\
-115 \\
\hline
101 \\
-92 \\
\hline
9
\end{array}
$$

2. **The Carnegie Middle School is hosting a picnic for any fifth grader who will be attending school next year as a sixth grader. The hospitality committee is planning the picnic for 125 students. Each fifth grader will get a sandwich, a drink, and a dessert.**

   a. **The hospitality committee is ordering large sandwiches that each serve 8 people. If 125 fifth graders are coming to the picnic, how many sandwiches should the committee buy?**

   b. **The committee is planning to have frozen fruit bars for dessert. If frozen fruit bars come in boxes of 12, how many boxes of frozen fruit bars should they order?**

Think About...

In other words, you can round down if you don't need to use the remainder, and you can round up if you need the next whole number larger than your answer.

c. **They will be serving bottles of water. Bottled water comes in cases of 24. How many cases of water will they need? Will there be any extra bottles of water? If so, how many?**

d. **The fifth graders will take a bus from the elementary school to the middle school on the afternoon of the picnic. If each bus seats 32 passengers, how many buses will be needed to transport the students? How many seats will be empty?**

3. **Throughout the year, local businesses collected 28,654 pairs of eyeglasses for disaster victims. If they have requests from 236 relief organizations, how many pairs of eyeglasses can each organization receive? How many pairs, if any, will be left over?**

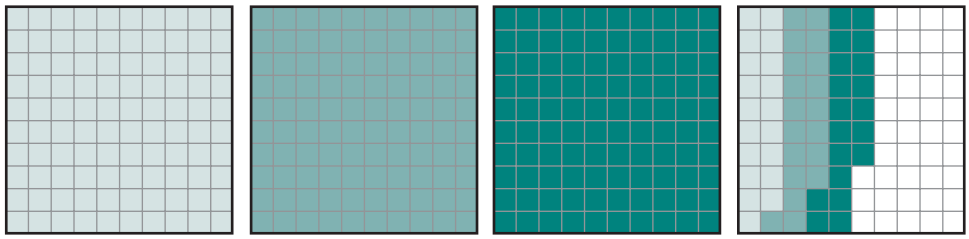
ACTIVITY
4.3 Decimal Division

You can use hundredths grids to model dividing decimals.

Consider the quotient 3.57 ÷ 3.

WORKED EXAMPLE

Step 1 Represent 3.57 using hundredths grids.



Step 2 Divide the shaded model into three equal groups.

One whole grid and 19 small squares are in each group.
So, 3.57 ÷ 3 = 1.19.

You can also use a standard algorithm to divide 3.57 ÷ 3.

WORKED EXAMPLE

Step 2: 5 tenths divided into 3 equal groups is 1 tenth in each group with 2 tenths left over.

Step 3: 2 tenths and 7 hundredths is 27 hundredths. 27 hundredths divided into 3 equal groups is 9 hundredths in each group with 0 hundredths left over.

Step 1: 3 ones divided into 3 equal groups is 1 one in each group with 0 ones left over.

```
      1.19
   ┌──────
3) 3.57
  − 3
    0 5
  − 3
      27
    − 27
      0
```

Quotient

Dividend

Divisor

1. **Compare the two Worked Examples.**

   a. **Describe how the hundredths grid model represents different parts of the standard algorithm.**

   b. **Why does the standard algorithm show subtracting 3 from the 3 ones in the dividend?**

   c. **What does the 05 represent in the standard algorithm?**

   d. **What does 27 − 27 represent in the standard algorithm? Use the hundredths grid model to help you explain.**

Take Note...

The quotient remains the same because you are dividing by a form of 1.

If you multiply both the dividend and the divisor by the same number, the quotient remains unchanged.

$12 \div 3 = 4$

$(12 \times 10) \div (3 \times 10) = 4$

$(12 \times 100) \div (3 \times 100) = 4$

You can use this information to change any divisor into a whole number to make the division of a decimal easier to solve.

Consider 7.56 ÷ 3.6.

WORKED EXAMPLE

Multiply both numbers by the least power of 10 that makes the divisor into a whole number.

$(7.56 \times 10) \div (3.6 \times 10)$
$= 75.6 \div 36$

Then, divide with whole numbers.

$$\begin{array}{r} 2.1 \\ 36\overline{)75.6} \\ -\,72\downarrow \\ \hline 36 \\ -\,36 \\ \hline 0 \end{array}$$

2. Rewrite each division problem so the divisor is a whole number. Explain how you determined your answer.

a. 48 ÷ 8.6

b. 59.5 ÷ 0.17

c. 6.2 ÷ 0.02

You have seen how to divide decimals by whole numbers. Let's think about how to divide decimals by decimals.

3. Look at these division problems.

$7\overline{)56}$ $70\overline{)560}$ $700\overline{)5600}$ $7000\overline{)56{,}000}$

a. How are the divisors and dividends in the last three problems related to the first problem?

b. Calculate all four quotients. What do you notice about them?

c. What happens to the quotient when you multiply the dividend and divisor by the same number?

4. Which of the division expressions shown have the same quotient as 475 ÷ 25? How do you know?

a. 4.75 ÷ 0.25

b. 47.5 ÷ 0.025

c. 0.475 ÷ 0.25

d. 0.0475 ÷ 0.0025

5. Calculate each quotient.

a. 74.4 ÷ 0.12

b. 66.22 ÷ 2.2

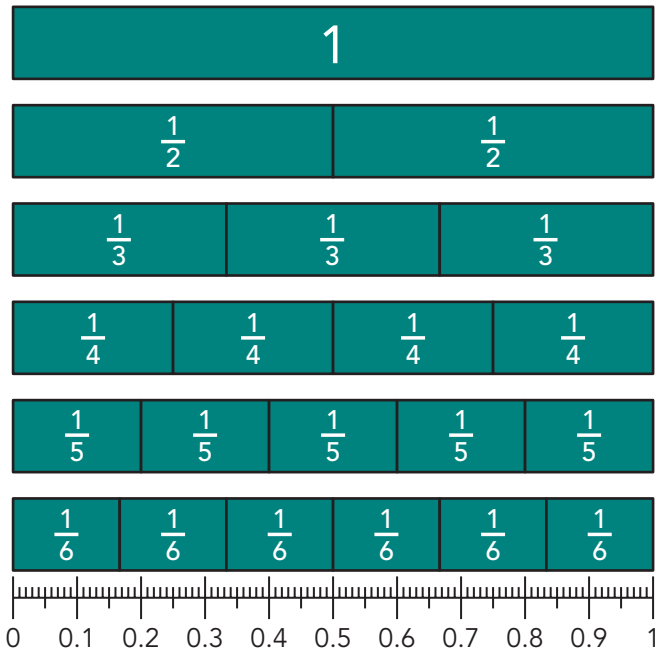
Remember...

The definition of division is $a \div b = \frac{a}{b}$. Therefore, you can use what you already know about equivalent fractions to determine which expressions have the same quotient as 475 ÷ 25.

ACTIVITY
4.4 Fraction and Decimal Equivalents

You can use strip diagrams to determine equivalent decimals.

1. Use a straightedge and the chart to determine the decimal that is equal to or approximately equal to the fractions given. Write the decimal to the nearest hundredth.



a. $\frac{1}{3}$ = _____ b. $\frac{3}{5}$ = _____ c. $\frac{1}{4}$ = _____

d. $\frac{1}{5}$ = _____ e. $\frac{4}{6}$ = _____ f. $\frac{1}{6}$ = _____

You can also change a fraction to a decimal by using the meaning of the fraction bar.

WORKED EXAMPLE

$\frac{3}{8}$ means 3 divided by 8.

$$\begin{array}{r} 0.375 \\ 8\overline{)3.000} \\ -\,24\downarrow \\ \hline 60 \\ -\,56\downarrow \\ \hline 40 \\ -\,40 \\ \hline 0 \end{array}$$

so $\frac{3}{8} = 0.375$

WORKED EXAMPLE

$\frac{2}{3}$ means 2 divided by 3.

$$\begin{array}{r} 0.666 \\ 3\overline{)2.000} \\ -\,18\downarrow \\ \hline 20 \\ -\,18\downarrow \\ \hline 20 \\ -\,18 \\ \hline 2 \end{array}$$

so $\frac{2}{3} = 0.6\overline{6}$

This decimal, 0.375, is called a **terminating decimal** because there is a remainder of 0. So, the denominator divides evenly into the numerator.

Take Note...

The bar over the 6 means that the 6 repeats without ending.

This decimal, $0.6\overline{6}$, is called a *repeating decimal*. A **repeating decimal** is a decimal in which a digit or a group of digits repeats without end. So, the denominator does not divide evenly into the numerator.

2. Convert each fraction to a decimal written to the nearest thousandths place. Identify whether the decimal is terminating or repeating.

a. $\frac{1}{8}$ b. $\frac{5}{8}$ c. $\frac{2}{3}$

d. $\frac{13}{25}$ e. $\frac{1}{6}$ f. $\frac{7}{20}$

3. Use a calculator to write the first 10 digits of the decimal for each fraction. Do not round the answers.

   a. $\frac{1}{7}$

   b. $\frac{2}{7}$

   c. $\frac{3}{7}$

   d. $\frac{4}{7}$

4. Consider the decimals you wrote in Question 3.

   a. What pattern do you notice in the decimals' values?

   b. Use the pattern to write the first ten digits of the decimal equivalent of $\frac{5}{7}$. Explain your reasoning.

5. Write the first 4 digits of the decimal for each fraction. Do not round the answers.

   a. $\frac{1}{9}$

   b. $\frac{2}{9}$

   c. $\frac{3}{9}$

   d. $\frac{4}{9}$

   e. $\frac{5}{9}$

   f. $\frac{6}{9}$

6. Consider the decimals you wrote in Question 5.

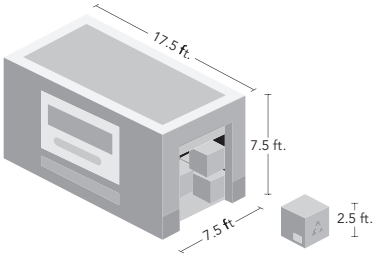
   a. What pattern do you notice in the decimal values?

   b. Use the pattern to write the first 4 digits of the decimal equivalents of $\frac{7}{9}$ and $\frac{8}{9}$.

ACTIVITY
4.5 Solving Problems Using Decimal Division

Solve each problem without the use of a calculator.

1. A portable self-storage container is in the shape of a rectangular prism. Nia is packing it with cube-shaped boxes that each have a side length of 2.5 feet.



What is the greatest number of boxes that Nia can fit into the storage container?

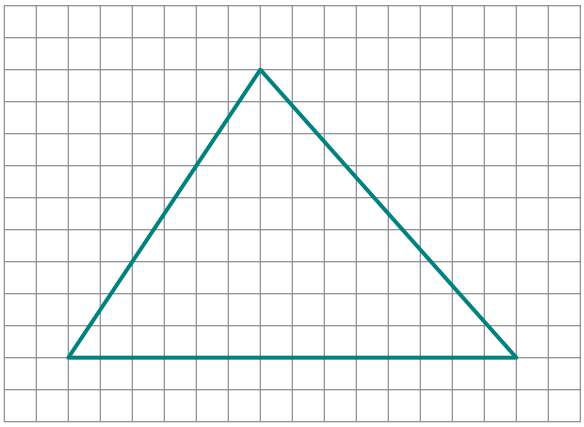
2. Calculate the side length of a square with the given perimeter.

a. 36.45 inches

b. 768 feet

c. 59.94 centimeters

3. The triangle shown on the grid represents a sailboat racecourse. Each square on the grid represents 0.1 mile by 0.1 mile.



The race organizers want any boats anchored within the perimeter of the course to have at least 0.002 square mile of space to prevent overcrowding. What is the maximum number of boats that can anchor within the perimeter of the racecourse?

4. Marjorie uses a loaf pan to make cornbread. The pan is 8.5 inches long, 4.5 inches wide, and 2.5 inches deep.

   a. The pan has a volume of approximately 6.6 cups. What is the approximate volume of each cup in cubic inches? Estimate and then calculate your answer. Show your work.

   b. The cornbread Marjorie makes fills only half the depth of the loaf pan. How much cornbread does Marjorie make? Give your answer in cups and cubic inches.

ACTIVITY
4.6 Parts of a Dollar

Miguel has just pulled his coat out of the closet for the first time since last winter. When he puts his hands in the pockets, he discovers some spare change.

1. In the right-hand coat pocket, Miguel finds 9 pennies, 8 nickels, and 2 dimes. Write the answer to each question in both fraction and decimal form. Show all your work.

a. What fraction of a dollar does Miguel have in pennies?

b. What fraction of a dollar does Miguel have in nickels?

c. What fraction of a dollar does Miguel have in dimes?

d. What is the total amount of money that Miguel found in his right-hand pocket?

2. **In the left-hand pocket of his coat, Miguel finds nickels, dimes, and quarters. He calculates the fraction of a dollar he has in terms of each coin.**

   Write the answer to each question in decimal form. Show all your work.

   a. **Miguel has $\frac{3}{20}$ of a dollar in nickels. How much money does he have in nickels?**

   b. **Miguel has $\frac{2}{5}$ of a dollar in dimes. How much money does he have in dimes?**

   c. **Miguel has $\frac{1}{4}$ of a dollar in quarters. How much money does he have in quarters?**

   d. **What is the total amount of money Miguel found in his left-hand pocket?**

NOTES

TALK the TALK

It's Great to Estimate!

Recall that estimation is a helpful strategy when operating with decimals to make sense of your solutions.

1. **Estimate the quotients for each expression shown. Make sure to show your work.**

   a. $7.5 \div 0.8$ b. $98.3 \div 23$ c. $99.2 \div 1.6$

   d. $10.35 \div 0.45$ e. $24.6 \div 0.6$ f. $7.4 \div 25$

2. **Divide each problem in Question 1 using your calculator. Round your answers to the nearest thousandth. Compare your estimates to the actual calculations.**

3. **Place the decimal point in each quotient to make the division sentence true. Use estimation with powers of 10.**

   a. $23.4 \div 0.9 = 260$ b. $5.51 \div 0.16 = 344375$

   c. $10.25 \div 8.2 = 125$

4. **Jared started his homework and did the first two problems**

   $1.3 \div 0.25 = 0.52$

   $39.6 \div 0.11 = 36$

   Adam said immediately that his answers were wrong. How did Adam know there was a mistake just by looking at the two problems and not doing any calculations?