Dividend in the House

4

Dividing Whole Numbers and Decimals

MATERIALS

Calculator

Lesson Overview

In this lesson, students use the standard algorithm for long division with whole numbers. They demonstrate how the algorithm works for decimal dividends by relating it to a model and make sense of why the algorithm is modified to accommodate decimal divisors. Students solve area and volume problems requiring decimal division.

Grade 6

Number and Operations

(3) The student applies mathematical process standards to represent addition, subtraction, multiplication, and division while solving problems and justifying solutions. The student is expected to:

(E) multiply and divide positive rational numbers fluently.

Expressions, Equations, and Relationships

(8) The student applies mathematical process standards to use geometry to represent relationships and solve problems. The student is expected to:

(D) determine solutions for problems involving the area of rectangles, parallelograms, trapezoids, and triangles and volume of right rectangular prisms where dimensions are positive rational numbers.

ELPS

1.A, 1.B, 1.C, 1.D, 1.E, 1.H, 2.C, 2.D, 2.E, 2.G, 2.H, 2.I 3.A, 3.B, 3.C, 3.D, 3.F, 3.G, 3.J, 4.A, 4.B, 4.C, 4.F, 4.I, 4.K, 5.A, 5.E

Essential Ideas

- The long division algorithm is based on an organized estimation process to determine the quotient.
- When a quotient has a remainder, the situation informs how to interpret the remainder.
- When you have a decimal divisor, multiply it by a power of ten to convert it to a whole number. Then, multiply the dividend by the same power of ten. Because you multiplied

both the dividend and divisor by the same power of ten, the quotient will be the same as the quotient of the original problem.

- You can use the standard algorithms for whole number and decimal division to solve real-world problems.
- Use estimation to determine if the quotient of a division problem is reasonable.

Lesson Structure and Pacing: 3 Days

Day 1

Engage

Getting Started: Just the Facts

Students review the concept of division and the terms *dividend*, *divisor*, and *quotient*. They use a fact family to respond to a division problem and consider the inefficiency of relying on this strategy.

Develop

Activity 4.1: Whole Number Division

Students analyze three stages of peer work to make sense of and develop the standard long division algorithm with whole numbers. They address a common place-value error and solve problems in contexts where the dividend and divisor are whole numbers.

Activity 4.2: Interpreting Remainders in Solutions

Students solve real-world problems involving division where there is a remainder. In some problems, the answer to the question is the remainder itself, while in others, the correct answer is the whole number part of the quotient or the next whole number.

Day 2

Activity 4.3: Decimal Division

Students divide with decimal dividends and divisors. They compare a hundredths grid model and the standard algorithm to divide a decimal dividend by a whole number divisor. Then, students use the concept that multiplying the dividend and divisor by the same multiple of 10 will not affect the quotient to enhance the algorithm when the divisor is a decimal.

Activity 4.4: Fraction and Decimal Equivalents

Students convert fractions to decimals by using strip diagrams and interpreting the meaning of the fraction bar as division. They encounter both terminating and repeating decimals and explore patterns in repeating decimals.

Day 3

Activity 4.5: Solving Problems Using Decimal Division

Students solve problems in context requiring decimal division. The problems involve area, surface area, and volume.

Activity 4.6: Parts of a Dollar

Students write several money amounts as decimals and fractions of a dollar.

Demonstrate

Talk the Talk: It's Great to Estimate!

Students use estimation skills to make sense of the place value in quotients. They address the concept that division by an amount less than one results in a quotient larger than the dividend.

ENGAGE

Getting Started: Just the Facts

Facilitation Notes

In this activity, students review the concept of division and the terms *dividend*, *divisor*, and *quotient*. They use a fact family to respond to a division problem and consider the inefficiency of relying on this strategy.

Ask a student to read the introduction aloud, and discuss as a class. Have students work with a partner or in groups to complete Questions 1 and 2. Discuss responses as a class.

Questions to ask

- How do the division terms *dividend*, *divisor*, and *quotient* relate to the multiplication terms *factor* and *product*?
- What is meant by the statement that multiplication and division are inverse operations?
- How can you check John’s answer?

Summary

A fact family can be used to determine the quotient in a division problem if you already know the corresponding multiplication fact.

DEVELOP

Activity 4.1

Whole Number Division

Facilitation Notes

In this activity, students analyze three stages of peer work to make sense of and develop the standard long division algorithm with whole numbers. They address a common error involving place value and solve problems in context where the dividend and divisor are whole numbers.

Read the introduction aloud, and discuss Lori's strategy as a class. Complete Question 1 as a class. Have students work with a partner or in groups to complete Question 2. Discuss responses as a class.

Differentiation strategy

To assist all students, consider demonstrating Lori’s strategy in a step-by-step fashion. Use a different color to represent the values in each step.

Questions to ask

- Why do you think this is identified as an organized estimation strategy?
- What was Lori's first estimation? What fact family did she use to make that estimate?
- What did Lori do once she still had 4098 remaining in the dividend?
- What do the stacked values above the division problem represent?
- What does the 0 at the end of Lori's calculations mean?
- How is Rob's method different from Lori's method?
- Why did Morgan write a 5 above the 4 in 34,098?
- Why did Morgan write a 30 under the 34? What value does 30 really represent?
- Where does the 40 in Morgan's work come from?
- Whose method do you prefer? Why?

Have students work with a partner or in groups to complete Question 3. Discuss responses as a class.

Questions to ask

- Estimate the quotient of 26,112 ÷ 256.
- What advice would you give Dustin so that he avoids this common error in the future?

Have students work with a partner or in groups to complete Question 4. Discuss responses as a class.

Questions to ask

- How did you determine where to place the first digit of the quotient?
- How can you tell how many digits your quotient will have?
- What does the quotient represent in the situation?

Summary

The long division algorithm uses an organized estimation process to determine the quotient.

Activity 4.2
Interpreting Remainders in Solutions

Facilitation Notes

In this activity, students solve real-world problems involving division where there is a remainder. In some problems, the answer to the question is the remainder itself, while in others, the correct answer is the whole number part of the quotient or the next whole number.

Ask a student to read Question 1 aloud, and complete as a class. Ask a student to read the statement about division problems aloud. Have students work with a partner or in groups to complete Question 2. Discuss responses as a class.

Questions to ask

- What does the remainder mean in this situation?
- How do you suggest the Red Cross distribute all 3551 coats?
- Explain how you used the quotient and remainder to respond to this situation.
- How can you check your work in a division problem with a remainder?
- What does the whole number in the quotient represent?
- What does the numerator represent in the fraction part of the quotient?
- What does the denominator represent in the fraction part of the quotient?
- Will there be any leftover sandwiches?
- Why does it make sense to round up to the nearest whole number in this question?
- What would happen if the school purchased 15 sandwiches?

Differentiation strategies

- To scaffold support, allow students to use a calculator to determine the quotient, as the focus of this lesson is on interpreting remainders and not division itself. To interpret the decimal on the calculator as a remainder, multiply the decimal part of the answer by the divisor.
- To extend this activity, ask students to determine the smallest number of fifth graders that could attend the picnic such that there would be no leftover supplies nor an empty seat on the bus.

Misconception

Some students might think that the whole number portion of the quotient is always the correct answer. Have student identify the units on each quotient, and then discuss what the numerator and denominator in the fraction part of the quotient means in the context of the problem.

Have students work with a partner or in groups to complete Question 3. Discuss responses as a class.

Questions to ask

- What does the whole number in the quotient represent?
- What does the numerator represent in the fraction part of the quotient?
- What does the denominator represent in the fraction part of the quotient?
- How could you estimate the quotient of 28,654 ÷ 236?
- How do you know when to round up or down to the nearest whole number?

Summary

In division problems, the remainder can mean different things in different situations. Sometimes the remainder can be ignored, and sometimes the remainder is the answer to the problem. Sometimes the answer is the number without the remainder, and sometimes you need to use the next whole number up from the correct answer.

Activity 4.3

Decimal Division

Facilitation Notes

In this activity, students divide with decimal dividends and divisors. They compare a hundredths grid model and the standard algorithm to divide a decimal dividend by a whole number divisor. Then, students use the concept that multiplying the dividend and divisor by the same multiple of 10 will not affect the quotient to enhance the algorithm when the divisor is a decimal.

Read the introduction aloud, and discuss the Worked Examples as a class. Have students work with a partner or in groups to answer Question 1. Discuss responses as a class.

Questions to ask

- What extra step is required when the dividend has a decimal?
- Why isn't using a model an efficient method?

Discuss the Worked Example as a class.

Differentiation strategy

To assist all students, have them attempt the original division problem $3.6\overline{)7.56}$ so they experience the difficulty in estimating with a decimal divisor and understand the need to change the divisor to whole number.

Questions to ask

- Why were the dividend and divisor multiplied by 10 and not 100?

Have students work with a partner or in groups to answer Questions 2 through 5. Discuss responses as a class.

Questions to ask

- Why do you want the divisor to be a whole number?
- Why is it okay for the dividend to be a decimal value?
- What process did you use to multiply by a multiple of 10?
- How does realizing this pattern help you solve a problem with a decimal divisor?
- What extra step can you include in the long division algorithm to solve problems with a decimal divisor?
- Explain your steps to complete the division problem.

Differentiation strategies

To scaffold support, suggest that students write the division problems using long division notation to mimic how they would set up the problem and move the decimal point.

$$8.6\overline{)48} \longrightarrow 86\overline{)480}$$

Summary

When you have a decimal divisor, multiply it by a power of ten to convert it to a whole number. Then, multiply the dividend by the same power of ten. Because you multiplied both the dividend and divisor by the same power of ten, the quotient will be the same as the quotient of the original problem.

Activity 4.4
Fraction and Decimal Equivalents

Facilitation Notes

In this activity, students convert fractions to decimals by using strip diagrams and interpreting the meaning of the fraction bar as division. They encounter both terminating and repeating decimals and explore patterns in repeating decimals.

Have students work individually to complete this activity.

Differentiation strategy

To scaffold support, have students work with a partner or in groups to complete this activity.

Questions to ask

- What operation does the fraction bar represent?
- How are the two Worked Examples different?
- How can you tell that a quotient is going to be a repeating decimal?
- What is another fraction that would result in a terminating decimal?
- What is another fraction that would result in a repeating decimal?
- When using a calculator, why might a decimal appear to be terminating when it really is repeating?

Summary

A fraction can be converted into a decimal by dividing the numerator by the denominator. The decimal equivalent is either a terminating or repeating decimal.

Activity 4.5
Solving Problems Using Decimal Division

Facilitation Notes

In this activity, students solve area, surface area, and volume problems requiring decimal division.

Ask a student to read the directions aloud, and discuss as a class. Have students work with a partner or in groups to answer Questions 1 through 4. Discuss responses as a class.

Differentiation strategies

- To scaffold support, provide a physical model to help students make sense of the situation.
- To extend this activity, ask students to estimate the dimensions of the cubes.

As students work, look for

- Students skimming the text and thinking that this is a perimeter— rather than an area— problem.
- Whether students draw a diagram for this situation.

Questions to ask

- Will there be any space remaining in the self-storage container after filling it with cubes?
- What are the characteristics of a cube?
- What is the difference between area and surface area?
- Why did you divide by 4?
- What does the phrase "within the perimeter" mean?
- How did you know that this is an area problem?
- How did you determine the base and height of the triangle?
- How is this different than the packing problem in Question 1?
- What type of the three-dimensional figure does the loaf pan resemble?
- What formula did you use for the volume of the pan?
- How close was your estimate to the actual volume?
- Why do you think Marjorie fills half the depth of the loaf pan?
- Explain how you derived your estimate.

Summary

You can use the standard algorithms for whole number and decimal division to solve real-world problems.

Activity 4.6

Parts of a Dollar

Facilitation Notes

In this activity, students write several money amounts as decimals and fractions of a dollar.

Ask a student to read the introduction aloud, and discuss as a class. Have students work with a partner or in groups to answer Questions 1 and 2. Discuss responses as a class.

Questions to ask

- What fraction of a dollar is one penny?
- What fraction of a dollar is one nickel?
- What fraction of a dollar is one dime?
- What fraction of a dollar is one quarter?
- How can you represent one penny, one nickel, one dime, and one quarter as decimals?

Summary

Many coins represent a part of a dollar. The value of a coin can be written as a fraction or a decimal.

Talk the Talk: It's Great to Estimate!

DEMONSTRATE

Facilitation Notes

In this activity, students use estimation skills to make sense of the place value in quotients. They address the concept that division by an amount less than one results in a quotient larger than the dividend.

Ask a student to read the introduction aloud, and discuss as a class. Have students complete Questions 1 through 4 with a partner or in groups. Discuss responses as a class.

Differentiation strategy

Explain that "use estimation with powers of 10" means to use a value such as 10, 1, 0.1, etc. as the divisor in order to get a quick idea of the place value of the quotient.

Questions to ask:

- Why is 10 divided by one-half equal to 20 rather than 5?
- Draw a diagram to show that 10 divided by one-half equals 20.
- Were any of your estimates off the mark? If so, explain the error in your thinking.
- If the directions say to "use estimation with the powers of 10," what are possible values of the divisor?
- What is the purpose of using only divisors such as 100, 10, 1 or 0.1?
- How did your estimated divisor guide you to place the decimal point in the quotient?
- Are the divisors smaller or larger than one?

- How does a divisor of more than one affect the quotient?
- How does a divisor of less than one affect the quotient?

Summary

Use estimation to determine if the quotient of a division problem is reasonable.

Dividend in the House

4

Dividing Whole Numbers and Decimals

WARM UP

Write a fact family for each division expression.

1. $72 \div 8 = 9$
2. $84 \div 6 = 14$
3. $2464 \div 308 = 8$

LEARNING GOALS

- Estimate quotients.
- Develop an algorithm for dividing whole numbers.
- Use the standard algorithm to divide decimals.
- Write fractions as decimals.

KEY TERMS

- terminating decimal
- repeating decimal

You know how to use place value to multiply a number by a power of 10. How can you use this knowledge to determine the quotients of decimals?

Warm Up Answers

1. $9 \times 8 = 72$
   $8 \times 9 = 72$
   $72 \div 8 = 9$
   $72 \div 9 = 8$
2. $14 \times 6 = 84$
   $6 \times 14 = 84$
   $84 \div 6 = 14$
   $84 \div 14 = 6$
3. $8 \times 308 = 2464$
   $308 \times 8 = 2464$
   $2464 \div 308 = 8$
   $2464 \div 8 = 308$

Answers

1a. John used a fact family to evaluate the expression since multiplication and division are inverse operations.

1b. dividend: 48; divisor: 6; quotient: 8

1c. You can switch the values 6 and 8 so that 8 is the divisor and 6 is the quotient because $48 \div 8 = 6$ is a part of the fact family.

2. Sample answer. No. I don't have every multiplication fact memorized, so I can't always use a fact family to determine a quotient.

Getting Started

Just the Facts

Take Note...
Quotients can be whole numbers, decimals, or fractions.

You previously learned that division helps you determine how many times one number contains another number. In other words, division is determining the quotient given a dividend and a divisor.

$$\text{divisor}\overline{)\text{dividend}}^{\ \text{quotient}}$$

or

$$(\text{quotient})(\text{divisor}) = (\text{dividend})$$

Remember...
Multiplication and division are inverse operations.

Consider the expression $48 \div 6$.

1. John evaluated the expression using the following strategy.

> John
> I know that $6 \times 8 = 48$,
> so $48 \div 6 = 8$.

a. Describe the strategy John used to evaluate the expression.

b. Identify the dividend, divisor, and quotient in the statement $48 \div 6 = 8$.

c. Which values in the statement $48 \div 6 = 8$ can you switch and the statement is still true? Explain your reasoning.

2. Is John's strategy reasonable for evaluating any division expression? Explain your reasoning.

ACTIVITY **4.1**

Whole Number Division

If you don't know a fact family for a division problem, you can use other strategies to determine the quotient.

Consider the expression 34,098 ÷ 6. Analyze Lori's strategy to determine the quotient.

Lori

I used an organized estimation strategy.

1. I estimated how many 6s are in 34,098. I used 6 × 5000 = 30,000 and then I subtracted.

2. I have 4098 left. Next, I tried 6 × 700 = 4200, which is too big. So, I used 6 × 600 = 3600.

3. I still had 498 left, so I tried 6 × 50 = 300. It's too small. So, I used 6 × 80 = 480.

4. I still had 18. 6 × 3 = 18

```
        3
       80
      600
     5000
 6)34,098
  −30,000
     4098
    −3600
      498
     −480
       18
      −18
        0
```

```
    3
   80
  600
+5000
 5683
```

So $6\overline{)34{,}098}$ = 5683

1. In each step, why did Lori subtract after she determined each estimate?

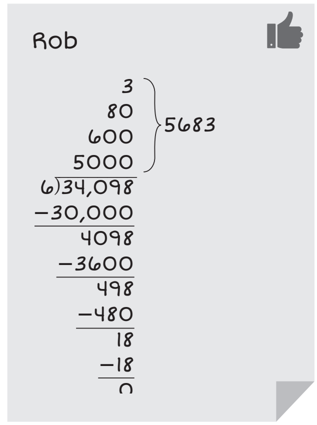
Answers

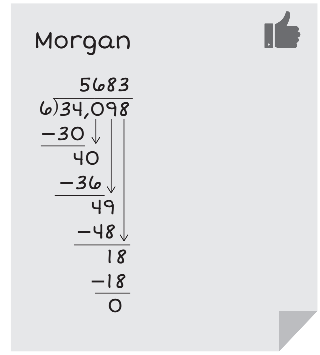
1. Lori needs to subtract to determine the next number in the quotient. By subtracting the product from the dividend, Lori then knows what her next estimate should be.

Answers

2. They both got the same quotient. Morgan is not showing place value.

Rob and Morgan agreed with Lori's logic, but shortened the process.





2. Compare Rob's and Morgan's strategies. What are the similarities and differences?

Carnegie Middle School conducted a month-long food and clothing drive to assist in disaster relief. They collected the following items for distribution.

- 13,312 cans of food
- 9472 blankets
- 19,456 batteries
- 26,112 bottles of water

If the students want to make 256 disaster-relief shipping crates, how many bottles of water will they put in each shipping crate?

3. Analyze each solution.

Morgan

I used my strategy from earlier.

```
      102
256)26,112
   −256
      512
     −512
```

They must load 102 bottles of water into each crate.

Dustin

I think there should be 12 bottles of water in each crate.

```
       12
256)26,112
   −256
      512
     −512
```

They must load 12 bottles of water into each crate.

a. What did Dustin do incorrectly?

b. How could Dustin have checked his work to know that his answer was incorrect?

Take Note...

You can use double lines at the end of a long division problem when the last difference is 0.

c. There should have been 3 digits in Dustin's quotient. How could he have determined that before he started dividing?

Answers

3a. When 256 would not go into 51, Dustin did not show a 0 in his quotient.

3b. Dustin could have multiplied his answer by the divisor and noticed that the product did not equal the dividend.

3c. As you think about how many times 256 goes into 26,112, you know that 256 goes into 261 one time, so you would write the value of 1 over the 261 in the hundreds position. Therefore, the quotient has to have 3 digits.

Answers

4a.

```
       52
256)13,312
  − 1280↓
      512
    − 512
        0
```

Each shipping crate should have 52 cans of food.

4b.

```
       37
256) 9472
   − 768↓
     1792
   − 1792
        0
```

Each shipping crate should have 37 blankets.

4c.

```
       76
256)19,456
  − 1792↓
     1536
   − 1536
        0
```

Each shipping crate should have 76 batteries.

4. Use Morgan's method of long division to determine how many of each item the students must load into each of the 256 shipping crates.

Think About...

Determining the number of digits in your answer first helps you to know whether your quotient is correct.

a. cans of food

b. blankets

c. batteries

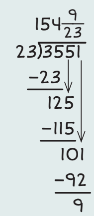
ACTIVITY **4.2**

Interpreting Remainders in Solutions

In division problems, the remainder can mean different things in different situations. Sometimes the remainder can be ignored, and sometimes the remainder is the answer to the problem. Sometimes the answer is the number without the remainder, and sometimes you need to use the next whole number up from the correct answer.

1. The Red Cross disaster relief fund collected 3551 winter coats to distribute to flood victims. If there are 23 distribution centers, how many coats can be sent to each center? Marla's calculations are shown.

Marla said, "The Red Cross can send $154\frac{9}{23}$ coats to each center." Madison replied, "You cannot have a fraction of a coat. So, each center will receive 154 coats and there will be 9 coats left over." Who's correct and why?



2. The Carnegie Middle School is hosting a picnic for any fifth grader who will be attending school next year as a sixth grader. The hospitality committee is planning the picnic for 125 students. Each fifth grader will get a sandwich, a drink, and a dessert.

   a. The hospitality committee is ordering large sandwiches that each serve 8 people. If 125 fifth graders are coming to the picnic, how many sandwiches should the committee buy?

   b. The committee is planning to have frozen fruit bars for dessert. If frozen fruit bars come in boxes of 12, how many boxes of frozen fruit bars should they order?

Think About...

In other words, you can round down if you don't need to use the remainder, and you can round up if you need the next whole number larger than your answer.

Answers

1. Madison is correct. You cannot have a fraction of a coat.

2a. They should buy 16 sandwiches.

2b. They should order 11 boxes of frozen fruit bars.

Answers

2c. They should order 6 cases of water, and they will have 19 extra bottles.

2d. It will take 4 buses to transport the 5th graders. There will be 3 empty seats.

3. Each organization will receive 121 pairs of eyeglasses. 98 pairs will be left over.

c. They will be serving bottles of water. Bottled water comes in cases of 24. How many cases of water will they need? Will there be any extra bottles of water? If so, how many?

d. The fifth graders will take a bus from the elementary school to the middle school on the afternoon of the picnic. If each bus seats 32 passengers, how many buses will be needed to transport the students? How many seats will be empty?

3. Throughout the year, local businesses collected 28,654 pairs of eyeglasses for disaster victims. If they have requests from 236 relief organizations, how many pairs of eyeglasses can each organization receive? How many pairs, if any, will be left over?

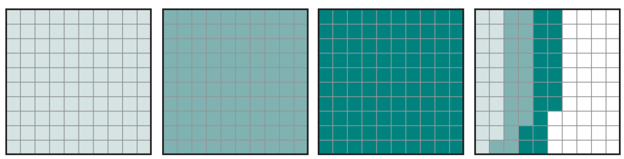
ACTIVITY
4.3 Decimal Division

You can use hundredths grids to model dividing decimals.

Consider the quotient 3.57 ÷ 3.

WORKED EXAMPLE

Step 1 Represent 3.57 using hundredths grids.

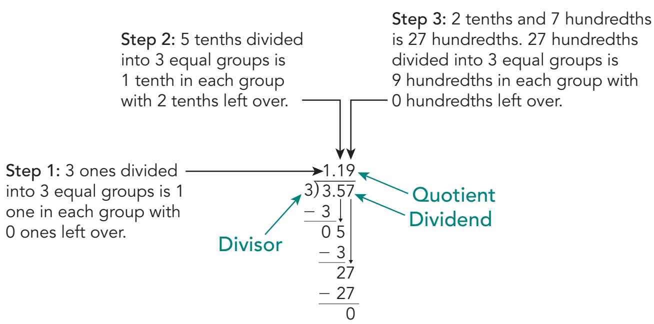


Step 2 Divide the shaded model into three equal groups.

One whole grid and 19 small squares are in each group.
So, 3.57 ÷ 3 = 1.19.

You can also use a standard algorithm to divide 3.57 ÷ 3.

WORKED EXAMPLE



Answers

1a. It represents the dividend by all the shaded squares of the hundredths grids. It represents the divisor by the three colors of shading. It represents the quotient by the number of shaded squares in each group.

1b. 3 ones are divided into 3 equal groups, which is one 1 in each group. To determine the leftover ones, you have to subtract 1×3, or 3.

1c. The 05 represents 5 tenths that are divided into 3 equal groups.

1d. 27 hundredths divided into 3 equal groups is 9 hundredths. Then, you subtract 3×9 hundredths, or 27 hundredths, from 27 hundredths.

1. Compare the two Worked Examples.

a. Describe how the hundredths grid model represents different parts of the standard algorithm.

b. Why does the standard algorithm show subtracting 3 from the 3 ones in the dividend?

c. What does the 05 represent in the standard algorithm?

d. What does 27 – 27 represent in the standard algorithm? Use the hundredths grid model to help you explain.

Take Note...

The quotient remains the same because you are dividing by a form of 1.

If you multiply both the dividend and the divisor by the same number, the quotient remains unchanged.

$12 \div 3 = 4$

$(12 \times 10) \div (3 \times 10) = 4$

$(12 \times 100) \div (3 \times 100) = 4$

You can use this information to change any divisor into a whole number to make the division of a decimal easier to solve.

Consider 7.56 ÷ 3.6.

WORKED EXAMPLE

Multiply both numbers by the least power of 10 that makes the divisor into a whole number.	(7.56 × 10) ÷ (3.6 × 10) = 75.6 ÷ 36
Then, divide with whole numbers.	$\begin{array}{r} 2.1 \\ 36\overline{)75.6} \\ -\,72\downarrow \\ \hline 36 \\ -\,36 \\ \hline 0 \end{array}$

2. Rewrite each division problem so the divisor is a whole number. Explain how you determined your answer.

a. 48 ÷ 8.6

b. 59.5 ÷ 0.17

c. 6.2 ÷ 0.02

You have seen how to divide decimals by whole numbers. Let's think about how to divide decimals by decimals.

3. Look at these division problems.

$7\overline{)56}$ $70\overline{)560}$ $700\overline{)5600}$ $7000\overline{)56,000}$

a. How are the divisors and dividends in the last three problems related to the first problem?

Answers

2a. 480 ÷ 86; I multiplied both the dividend and divisor by 10.

2b. 5950 ÷ 17; I multiplied both the dividend and divisor by 100.

2c. 620 ÷ 2; I multiplied both the dividend and divisor by 100.

3a. The divisor and dividends have each been multiplied by 10, 100, and 1000.

Answers

3b. All quotients are 8.

3c. The quotient remains unchanged.

4a. Same quotient. Both dividend and divisor were divided by 100.

4b. Different quotient. The dividend was divided by 10 and the divisor was divided by 1000.

4c. Different quotient. The dividend was divided by 1000 and the divisor was divided by 100.

4d. Same quotient. Both dividend and divisor were divided by 10,000.

5a. 620

5b. 30.1

b. Calculate all four quotients. What do you notice about them?

c. What happens to the quotient when you multiply the dividend and divisor by the same number?

4. Which of the division expressions shown have the same quotient as 475 ÷ 25? How do you know?

Remember…

The definition of division is $a \div b = \frac{a}{b}$. Therefore, you can use what you already know about equivalent fractions to determine which expressions have the same quotient as 475 ÷ 25.

a. 4.75 ÷ 0.25

b. 47.5 ÷ 0.025

c. 0.475 ÷ 0.25

d. 0.0475 ÷ 0.0025

5. Calculate each quotient.

a. 74.4 ÷ 0.12

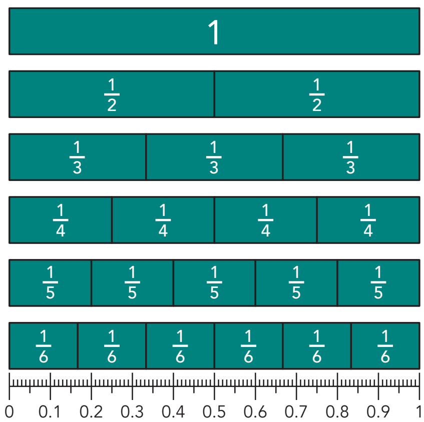
b. 66.22 ÷ 2.2

ACTIVITY **4.4**

Fraction and Decimal Equivalents

You can use strip diagrams to determine equivalent decimals.

1. **Use a straightedge and the chart to determine the decimal that is equal to or approximately equal to the fractions given. Write the decimal to the nearest hundredth.**



a. $\frac{1}{3}$ = ________ **b.** $\frac{3}{5}$ = ________ **c.** $\frac{1}{4}$ = ________

d. $\frac{1}{5}$ = ________ **e.** $\frac{4}{6}$ = ________ **f.** $\frac{1}{6}$ = ________

Answers

1a. 0.33
1b. 0.60
1c. 0.25
1d. 0.20
1e. 0.67
1f. 0.17

Answers

2a. 0.125
terminating

2b. 0.625
terminating

2c. 0.667
repeating

2d. 0.520
terminating

2e. 0.167
repeating

2f. 0.350
terminating

You can also change a fraction to a decimal by using the meaning of the fraction bar.

WORKED EXAMPLE

$\frac{3}{8}$ means 3 divided by 8.

```
   0.375
8)3.000
 − 24↓
    60
  − 56↓
     40
   − 40
      0
```

so $\frac{3}{8} = 0.375$

WORKED EXAMPLE

$\frac{2}{3}$ means 2 divided by 3.

```
   0.666
3)2.000
 − 18↓
    20
  − 18↓
     20
   − 18
      2
```

so $\frac{2}{3} = 0.6\overline{6}$

This decimal, 0.375, is called a **terminating decimal** because there is a remainder of 0. So, the denominator divides evenly into the numerator.

Take Note...

The bar over the 6 means that the 6 repeats without ending.

This decimal, $0.6\overline{6}$, is called a *repeating decimal*. A **repeating decimal** is a decimal in which a digit or a group of digits repeats without end. So, the denominator does not divide evenly into the numerator.

2. Convert each fraction to a decimal written to the nearest thousandths place. Identify whether the decimal is terminating or repeating.

a. $\frac{1}{8}$ **b.** $\frac{5}{8}$ **c.** $\frac{2}{3}$

d. $\frac{13}{25}$ **e.** $\frac{1}{6}$ **f.** $\frac{7}{20}$

3. Use a calculator to write the first 10 digits of the decimal for each fraction. Do not round the answers.

a. $\frac{1}{7}$

b. $\frac{2}{7}$

c. $\frac{3}{7}$

d. $\frac{4}{7}$

4. Consider the decimals you wrote in Question 3.

a. What pattern do you notice in the decimals' values?

b. Use the pattern to write the first ten digits of the decimal equivalent of $\frac{5}{7}$. Explain your reasoning.

5. Write the first 4 digits of the decimal for each fraction. Do not round the answers.

a. $\frac{1}{9}$

b. $\frac{2}{9}$

c. $\frac{3}{9}$

d. $\frac{4}{9}$

e. $\frac{5}{9}$

f. $\frac{6}{9}$

6. Consider the decimals you wrote in Question 5.

a. What pattern do you notice in the decimal values?

b. Use the pattern to write the first 4 digits of the decimal equivalents of $\frac{7}{9}$ and $\frac{8}{9}$.

Answers

3a. 0.1428571428

3b. 0.2857142857

3c. 0.4285714285

3d. 0.5714285714

4a. The decimals are all repeating decimals. The digits 142857 repeat in each successive fraction, but each begins with a different digit in the pattern.

4b. 0.7124857142
Sample answer.
I know that $\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$, so I added their decimal equivalents.

5a. 0.1111

5b. 0.2222

5c. 0.3333

5d. 0.4444

5e. 0.5555

5f. 0.6666

6a. The decimals are all repeating decimals. The numerator of the fraction is the repeated digit in the decimal value.

6b. $\frac{7}{9} = 0.7777$, $\frac{8}{9} = 0.8888$

Answers

1. 7.5 ÷ 2.5 = 3
   17.5 ÷ 2.5 = 7
   3 × 3 × 7 = 63
   The greatest number of boxes that Nia can fit into the storage container is 63.

2a. 36.45 ÷ 6 = 6.075
    6.075 square inches

2b. 768 ÷ 6 = 192
    128 square feet

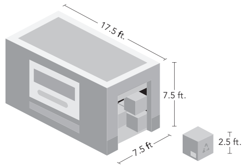
2c. 59.94 ÷ 6 = 9.99
    9.99 square centimeters

ACTIVITY 4.5

Solving Problems Using Decimal Division

Solve each problem without the use of a calculator.

1. A portable self-storage container is in the shape of a rectangular prism. Nia is packing it with cube-shaped boxes that each have a side length of 2.5 feet.



What is the greatest number of boxes that Nia can fit into the storage container?

2. Calculate the side length of a square with the given perimeter.

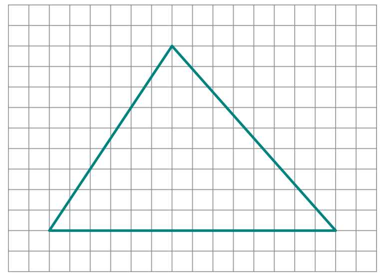
a. 36.45 inches

b. 768 feet

c. 59.94 centimeters

3. The triangle shown on the grid represents a sailboat racecourse. Each square on the grid represents 0.1 mile by 0.1 mile.

The race organizers want any boats anchored within the perimeter of the course to have at least 0.002 square mile of space to prevent overcrowding. What is the maximum number of boats that can anchor within the perimeter of the racecourse?



4. Marjorie uses a loaf pan to make cornbread. The pan is 8.5 inches long, 4.5 inches wide, and 2.5 inches deep.

   a. The pan has a volume of approximately 6.6 cups. What is the approximate volume of each cup in cubic inches? Estimate and then calculate your answer. Show your work.

   b. The cornbread Marjorie makes fills only half the depth of the loaf pan. How much cornbread does Marjorie make? Give your answer in cups and cubic inches.

Answers

3. Length of triangle:
   $14 \times 0.1 = 1.4$ miles
   Height of triangle:
   $9 \times 0.1 = 0.9$ mile
   Area of triangle:
   $\frac{1}{2}(1.4)(0.9) = 0.63$ square mile
   $0.63 \div 0.002 = 315$
   The maximum number of boats that can anchor within the perimeter of the racecourse is 315.

4a. Volume of pan in cubic inches: $8.5 \times 4.5 \times 2.5 = 95.625$
   $95.625 \div 6.6 = 14.48\overline{63}$
   The volume of each cup is approximately 14.5 cubic inches.

4b. Volume of cornbread in cups: $6.6 \div 2 = 3.3$
   Volume of cornbread in cubic inches: $95.625 \div 2 = 47.8125$
   Marjorie makes 3.3 cups, or 47.8125 cubic inches, of cornbread.

Answers

1a. One penny is $\frac{1}{100}$, or 0.01, of a dollar. So, 9 pennies is $\frac{9}{100}$, or 0.09, of a dollar.

1b. One nickel is $\frac{5}{100}$, or 0.05, of a dollar. So, 8 nickels is $\frac{40}{100} = \frac{2}{5}$, or 0.40, of a dollar.

1c. One dime is $\frac{1}{10}$, or 0.10, of a dollar. So, 2 dimes is $\frac{2}{10} = \frac{1}{5}$, or 0.20, of a dollar.

1d. $\frac{9}{100} + \frac{40}{100} + \frac{2}{10} = \frac{9}{100} + \frac{40}{100} + \frac{20}{100} = \frac{69}{100}$
Miguel has a total of $\frac{69}{100}$, or 0.69, of a dollar in his right-hand pocket. So, he has \$0.69 in his right-hand pocket.

ACTIVITY
4.6

Parts of a Dollar

Miguel has just pulled his coat out of the closet for the first time since last winter. When he puts his hands in the pockets, he discovers some spare change.

1. In the right-hand coat pocket, Miguel finds 9 pennies, 8 nickels, and 2 dimes. Write the answer to each question in both fraction and decimal form. Show all your work.

a. What fraction of a dollar does Miguel have in pennies?

b. What fraction of a dollar does Miguel have in nickels?

c. What fraction of a dollar does Miguel have in dimes?

d. What is the total amount of money that Miguel found in his right-hand pocket?

2. In the left-hand pocket of his coat, Miguel finds nickels, dimes, and quarters. He calculates the fraction of a dollar he has in terms of each coin.

Write the answer to each question in decimal form. Show all your work.

a. Miguel has $\frac{3}{20}$ of a dollar in nickels. How much money does he have in nickels?

b. Miguel has $\frac{2}{5}$ of a dollar in dimes. How much money does he have in dimes?

c. Miguel has $\frac{1}{4}$ of a dollar in quarters. How much money does he have in quarters?

d. What is the total amount of money Miguel found in his left-hand pocket?

Answers

2a. I can write an equivalent fraction with a denominator of 100 to determine the decimal form.

$\frac{3}{20} = \frac{3 \times 5}{20 \times 5} = \frac{15}{100} = 0.15$

He has $0.15 in nickels.

2b. I can write an equivalent fraction with a denominator of 100 to determine the decimal form.

$\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100} = 0.40$

He has $0.40 in dimes.

2c. I can write an equivalent fraction with a denominator of 100 to determine the decimal form.

$\frac{1}{4} = \frac{1 \times 25}{4 \times 25} = \frac{25}{100} = 0.25$

He has $0.25 in quarters.

2d. $\frac{15}{100} + \frac{40}{100} + \frac{25}{100} = \frac{80}{100}$

Miguel has a total of $\frac{80}{100}$, or 0.80, of a dollar in his left-hand pocket. So, he has $0.80 in his left-hand pocket.

Answers

1a. 8 ÷ 1 = 8
9.375

1b. 100 ÷ 25 = 4
4.274

1c. 100 ÷ 2 = 50
62

1d. 10 ÷ 0.5 = 20
23

1e. 25 ÷ 0.5 = 50
41

1f. 7.5 ÷ 25 = 0.3
0.296

2. See 1a–f.

3a. 26.0

3b. 34.4375

3c. 1.25

4. Adam knows there was a mistake by estimating the quotients first. The divisors in both are less than the dividends, which means the quotient will be greater than the dividend. Jared's quotients are both less than the dividend.

NOTES

TALK the TALK

It's Great to Estimate!

Recall that estimation is a helpful strategy when operating with decimals to make sense of your solutions.

1. Estimate the quotients for each expression shown. Make sure to show your work.

a. 7.5 ÷ 0.8 **b. 98.3 ÷ 23** **c. 99.2 ÷ 1.6**

d. 10.35 ÷ 0.45 **e. 24.6 ÷ 0.6** **f. 7.4 ÷ 25**

2. Divide each problem in Question 1 using your calculator. Round your answers to the nearest thousandth. Compare your estimates to the actual calculations.

3. Place the decimal point in each quotient to make the division sentence true. Use estimation with powers of 10.

a. 23.4 ÷ 0.9 = 260 **b. 5.51 ÷ 0.16 = 344375**

c. 10.25 ÷ 8.2 = 125

4. Jared started his homework and did the first two problems

1.3 ÷ 0.25 = 0.52

39.6 ÷ 0.11 = 36

Adam said immediately that his answers were wrong. How did Adam know there was a mistake just by looking at the two problems and not doing any calculations?