Going Strong 2

Comparing Ratios to Solve Problems

MATERIALS

Scissors
Poster paper

Lesson Overview

Students explore ratios in different real-world situations. They decide which of two or more ratios in each situation is greater using qualitative and quantitative reasoning. Students compare part-to-part and part-to-whole ratios represented pictorially, verbally, and numerically. The focus in this lesson is on reasoning rather than computation.

Grade 6
Proportionality

(4) The student applies mathematical process standards to develop an understanding of proportional relationships in problem situations. The student is expected to:

(B) apply qualitative and quantitative reasoning to solve prediction and comparison of real-world problems involving ratios and rates.

(C) give examples of ratios as multiplicative comparisons of two quantities describing the same attribute.

ELPS

1.A, 1.C, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 3.A, 3.B, 3.C, 3.D, 3.E, 3.F, 4.A, 4.B, 4.C, 4.F, 4.K, 5.E, 5.F

Essential Ideas

- A ratio is a comparison of two quantities.
- Qualitative comparisons are made in the absence of numeric values.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Lemony-er Lemonade

Students reason qualitatively about the ratio of lemon mix to water to determine which lemonade will have a stronger lemon flavor.

Develop

Activity 2.1: Qualitative Comparisons

Students continue to reason qualitatively about the ratios to determine which drinks have a stronger flavor or which person is faster.

Activity 2.2: Comparing Comparisons

Students use knowledge of ratios to determine cocoa recipes that contain the least or most chocolate flavor. They use only their own reasoning, including their knowledge of ratios and fractions. It is not expected for students to form equivalent ratios to compare the recipes.

Day 2

Activity 2.3: Ordering Part-to-Part and Part-to-Whole Ratios

Students use a card sort activity to compare and order part-to-part and part-to-whole ratios represented verbally, numerically, and/or pictorially. They sort the cards from least to greatest; however, some ratios are represented using more than one representation.

Demonstrate

Talk the Talk: Put Me In, Coach

Students use their knowledge of ratios to decide which soccer player should take a team penalty shot based on previous performance.

Getting Started: Lemony-er Lemonade

ENGAGE

Facilitation Notes

In this activity, students reason qualitatively about the ratio of lemon mix to water to determine which lemonade will have a stronger lemon flavor.

Ask a student to read the scenario aloud. Have them work with a partner or in groups to answer the question. Share responses as a class.

Questions to ask

- What does it mean for a glass of lemonade to have a weaker taste than another glass?
- How does the amount of water in a glass affect the lemonade flavor?
- How does the amount of lemon mix in a glass affect the lemonade flavor?
- Whose lemonade was initially weaker?
- What would have to happen to result in Tammy's lemonade being stronger?

Summarize

Two quantities can still be compared in the absence of numeric values.

Activity 2.1
Qualitative Comparisons

DEVELOP

Facilitation Notes

In this activity, students continue to reason qualitatively about the ratios to determine which drinks have the stronger flavors. Quantities are compared without knowing the amount of each quantity.

Have students work with a partner or in groups to complete Questions 1 and 2. Share the responses as a class.

Differentiation strategies

To scaffold support,

- Limit the number of questions and/or the number of choices per question.

- For Question 1, have students interact with the diagrams by labeling the glasses as being weaker or stronger and then using colored pencils to show additions.
- For Question 2, suggest students create their own diagrams.

As students work, look for

- Students who only refer to one of the quantities being compared in each question.
- Students who primarily refer to the orange or lemon flavor versus the amount of water in the glass.
- Students who do not provide a rationale when they state that there is not enough information to determine the comparison.
- Students who do not refer to the units in the speed of a car.

Questions to ask

- How does the amount of water in a glass affect the lemonade flavor?
- How does the amount of lemon mix in a glass affect the lemonade flavor?
- How will adding more water affect the taste of orange flavor?
- What are the units for the speed of a car?

Summary

Qualitative comparisons are made in the absence of numeric values.

Activity 2.2
Comparing Comparisons

Facilitation Notes

In this activity, students use knowledge of ratios to determine cocoa recipes that contain the least or most chocolate flavor.

Students should experience a productive struggle as they use only their own reasoning, including their knowledge of ratios and fractions, prior to the introduction to formal procedures. It is not expected for students to use a formal strategy to compare the recipes; nor is it expected for teachers to introduce equivalent ratios at this point.

Have students read through the scenario and each recipe individually first to make a prediction of which recipe has the strongest taste of chocolate.

Have students work with a partner or in groups to complete Question 1.

Differentiation strategy

To scaffold support, the following questions may be useful.

- What type of ratios are provided in each recipe?
- How many total parts are in each homeroom's recipe?
- For each recipe, write a ratio that compares the number of parts of cocoa powder to the total number of parts in each recipe.
- For each recipe, write a ratio that compares the number of parts of milk to the total number of parts in each recipe.

As students work, look for

- Part-to-whole ratios.
- Ordering of fractions to compare.
- Students completing two comparisons of two fractions at a time, but not comparing the pairs relative to each other.
- Students considering a ratio with more milk to be stronger.

Questions to ask

- Are you using part-to-part or part-to-whole ratios to compare recipes?
- How did you determine the ratio of milk : cocoa powder?
- How did you determine which recipe uses the most chocolate?
- How did you determine which recipe uses the least chocolate?
- Which recipe uses the most milk?
- Which recipe uses the least milk?
- How can you use what you know about the relative size of fractions to determine the recipe with the strongest or weakest chocolate taste?

Misconceptions

Students may compare part-to-part ratios, which is an acceptable strategy; however, if they try to convert those to percents, remind them that percents are used to represent a part-to-whole relationship.

Ask each group to post their completed task in the classroom. Allow time for groups to review the strategies presented on each poster.

Summary

Ratios are compared by reasoning about the amount of each quantity.

Activity 2.3
Ordering Part-to-Part and Part-to-Whole Ratios

Facilitation Notes

In this activity, students sort punch ratio cards to compare and order part-to-part and part-to-whole ratios represented verbally, numerically, and/or pictorially. They sort the cards from least concentration to greatest; however, some ratios are represented using more than one representation.

Ask a student to read the introduction aloud. Note that every ratio will not be represented using all forms.

Have students cut out and sort the ratio cards with a partner or in groups. Remind students that they should discuss each card, determine the ratio represented, and specify if the ratios are part-to-part or part-to-whole.

Questions to ask

- How can you determine if the ratio is part-to-part or part-to-whole?
- How can you compare ratios in pictures with ratios written in words?
- How can you compare part-to-part and part-to-whole ratios?
- How can you determine which recipe has the greatest concentration of lemon-lime flavor?
- How can you determine which recipe has the least concentration of lemon-lime flavor?
- How can you order the ratios?
- How can you use what you know about the relative size of fractions to order the recipes?

As students work, look for

- Students who write all the ratios as part-to-part comparisons.
- Students who write all the ratios as part-to-whole comparisons.
- Students who mix up the lemon-lime soda and the pineapple juice.

Differentiation strategies

- To scaffold support, reduce the number of cards students must sort.

- To extend the activity, have them make new cards to complete the original set of cards so that each concentration has three cards: a part-to-whole ratio or comparison, a part-to-part ratio or comparison, and a picture.

Have students answer Question 1. As a class, have students share their sorts and their strategies for ordering the cards.

Questions to ask

- How are the strategies similar?
- How are the strategies different?
- Did everyone get the same answer even if they used different strategies?

Summarize

Ratios written in different forms are compared and ordered.

Talk the Talk: Put Me In, Coach

DEMONSTRATE

Facilitation Notes

In this activity, students use their knowledge of ratios to decide which soccer player should take a team penalty shot based on previous performance.

Ask a student to read the scenario aloud. Instruct students to work individually to complete this task. Share responses as a class.

Questions to ask

- Have you ever played soccer?
- What is a penalty shot?
- Are there penalty shots in every soccer game?
- What are the ratios provided in the scenario?
- Are the ratios part-to-part or part-to-whole?
- How can you use ratios to determine who should take the penalty shot?

Misconceptions

- Students may use additive reasoning to answer the question. For example, Amber and Li each missed only 1 shot, so either of them could be chosen.

- Students may consider a single quantity to compare rather than the relationship between the quantities. For example, Lindsay should take the shot because she has made 4 shots, which is more than the 3 shots made by Amber or the 2 shots made by Li.

Differentiation strategy

To extend the activity, provide students with hundredths grids and ask them to convert their answers to percents.

Summary

Ratios are useful when making decisions.

2

Going Strong

Comparing Ratios to Solve Problems

WARM UP

Use reasoning to compare each pair of fractions.

1. $\frac{6}{7}$ and $\frac{8}{9}$
2. $\frac{7}{13}$ and $\frac{5}{11}$
3. $\frac{4}{5}$ and $\frac{4}{3}$

LEARNING GOALS

- Apply qualitative ratio reasoning to compare ratios in real-world and mathematical problems.
- Apply quantitative ratio reasoning to compare ratios in real-world and mathematical problems.
- Compare and order part-to-part and part-to-whole ratios represented verbally, pictorially, and numerically.

You know how to write a ratio as a comparison of two quantities. How can you compare two ratios to make decisions in real-world situations?

Warm Up Answers

1. $\frac{6}{7} < \frac{8}{9}$
2. $\frac{7}{13} > \frac{5}{11}$
3. $\frac{4}{5} < \frac{4}{3}$

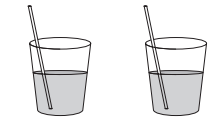
Answers

1. Jen's glass of lemonade will have the stronger tasting lemon flavor. Initially, Jen's glass of lemonade had the stronger tasting lemon flavor. Since both glasses have the same volume of lemonade and the same amount of lemon mix was added to both glasses, Jen's glass will still have the stronger tasting lemon flavor.

Getting Started

Lemony-er Lemonade

Tammy's glass of lemonade has a weaker tasting lemon flavor than Jen's glass of lemonade. The shaded portion in each glass represents an amount of lemonade.



Tammy's Glass Jen's Glass

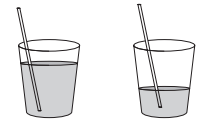
1. If one teaspoon of lemon mix is added to both Jen's and Tammy's glasses, which glass will contain the lemonade with the stronger lemon flavor? Explain your reasoning.

ACTIVITY
2.1 Qualitative Comparisons

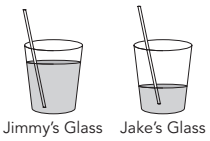
In this activity you will compare ratios without measuring or counting quantities. When you reason like this, it is called *qualitative reasoning*.

1. The shaded portion in each glass represents an amount of lemonade. Answer each question and explain your reasoning.

a. Beth's glass of lemonade has a weaker tasting lemon flavor than John's glass of lemonade. If two ounces of water are added to Beth's glass and one teaspoon of lemon mix is added to John's glass, which glass will contain the lemonade with the stronger lemon flavor?



b. Jimmy and Jake have glasses of lemonade that taste the same. If one teaspoon of lemon mix is added to each glass, which glass will contain the lemonade with the stronger lemon flavor?



c. Jack's glass of lemonade has a stronger tasting lemon flavor than Karen's glass of lemonade. If one teaspoon of lemon mix is added to Karen's glass and one ounce of water is added to Jack's glass, which glass will contain the lemonade with the stronger lemon flavor?

Answers

1a. John's glass will contain the lemonade with the stronger lemon flavor.
1b. Jake's glass will contain the lemonade with the stronger lemon flavor.
1c. There is not enough information to determine which glass will contain the lemonade with the stronger lemon flavor.

ELL Tip

Because of the quantity of words in these situations, English Language Learners may struggle to understand the situations. For the first problem, have students engage in a Think-Pair-Share activity, but with three students per group. Each student should be assigned one of the three parts. Make sure that each ELL student is grouped with native English speakers. Having them talk through their part will help them understand what is being asked, and will also allow them to practice their spoken language skills.

Answers

2a. Luke's orange drink today would have a stronger tasting orange flavor because the ratio of orange mix to water increased.

2b. Cannot be determined. There is not enough information given.

2c. The speed of the race car would be faster today.

2. Choose the correct statement to complete each sentence and explain your reasoning. If the answer cannot be determined, explain why not.

a. If Luke plans to use four more tablespoons of orange mix today than he used yesterday to make the same amount of orange drink, his orange drink today would have:

- **a stronger tasting orange flavor.**
- **a weaker tasting orange flavor.**
- **a mix that has the same strength of orange taste as yesterday.**

b. Dave and Sandy each made a pitcher of orange drink. Sandy's pitcher is larger than Dave's pitcher. Sandy used more orange mix than Dave. Dave's orange drink has:

- **a stronger tasting orange flavor.**
- **a weaker tasting orange flavor.**
- **a mix that has the same strength of orange taste as Sandy's drink.**

c. If a race car travels more laps in less time than it did yesterday, its speed would be:

- **slower.**
- **exactly the same.**
- **faster.**

ACTIVITY
2.2 Comparing Comparisons

The 6th grade students are making hot chocolate to sell at the Winter Carnival. Each homeroom suggested a different recipe.

HR 6A
2 cups milk
3 T cocoa powder

HR 6B
5 cups milk
8 T cocoa powder

HR 6C
3 cups milk
4 T cocoa powder

HR 6D
4 cups milk
7 T cocoa powder

1. **Consider the given recipes to answer each question.**
   a. **Use reasoning to determine which recipe has the strongest chocolate taste and which recipe has the weakest chocolate taste.**
   b. **Show how you used ratio reasoning to order the recipes. Identify the ratios that you used as part-to-part or part-to-whole.**
   c. **Create a poster to explain your answer and strategy to the class. Prepare to share!**

Answers

1a. HR 6D has the strongest chocolate taste. HR 6C has the weakest chocolate taste.
1b. Answers will vary.
1c. Answers will vary.

ELL Tip

Even though the margin note points out that "T" stands for *tablespoon*, it is possible that English Language Learners may not be familiar with standard units of measurement.

Answers

1. See table below.

ACTIVITY 2.3

Ordering Part-to-Part and Part-to-Whole Ratios

Suppose your class is in charge of providing punch at the upcoming open house. The Parent-Teacher Association bought lemon-lime soda and pineapple juice to combine for the punch, but they did not tell your class how much of each to use. Your classmates submitted suggestions for how to make the tastiest punch.

Cut out the punch ratio cards at the end of the lesson. Order the cards from the least lemon-lime concentration to the most lemon-lime concentration. If you think more than one card describes the same ratio of lemon-lime soda and pineapple juice, group those cards together.

The shading, or lack of shading, of each cup represents the difference in the type of concentration.

1. Describe the strategies you used to sort and order the cards.

1.

Order from Least to Greatest	Part-to-Part	Part-to-Whole	Pictorial Representation
1 : 3	K	C	
1 : 2	I		B
2 : 3	J	L	F
4 : 5	G		D
1 : 1	A	E	H

NOTES

TALK the TALK

Put Me In, Coach

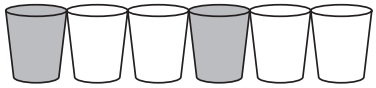
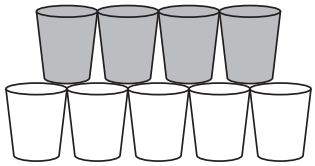
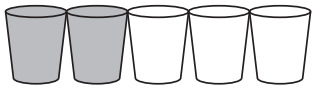
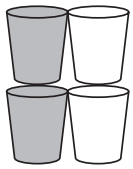
A soccer team has been awarded a penalty shot at the end of a tie game. If they make the penalty shot, they will win the league championship. The coach is considering three players to take the penalty. Amber has taken 4 penalty shots this season and has made 3 of them. Lindsay has taken 6 penalty shots and made 4. Li has taken 3 penalty shots and made 2.

1. **Which player would you recommend take the penalty shot? Why?**

Answers

1. Amber should take the penalty shot because she has the greatest ratio of shots made to shots attempted.

Punch Ratio Cards

A For every lemon-lime soda, there is a pineapple juice.	**B** 
C One-fourth of the punch is lemon-lime soda.	**D** 
E Half of the mixture is pineapple juice.	**F** 
G Lemon-lime soda : Pineapple juice = 4 : 5	**H** 
I For every lemon-lime soda, there are two pineapple juices.	**J** For every lemon-lime soda, there are $1\frac{1}{2}$ pineapple juices.
K Pineapple juice : lemon-lime soda = 3 : 1	**L** Three-fifths of the punch is pineapple juice.

Why is this page blank?

So you can cut out the Punch Ratio Cards on the other side.