



**TEXAS MATH
SOLUTION**

Accelerated Grade 7

Teacher's Implementation Guide

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Teacher's Implementation Guide

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Our Manifesto

WE BELIEVE that quality math education is important for all students, to help them develop into creative problem solvers, critical thinkers, life-long learners and more capable adults.

WE BELIEVE that math education is about more than memorizing equations or performing on tests—it's about delivering the deep conceptual learning that supports ongoing growth and future development.

WE BELIEVE all students learn math best when teachers believe in them, expect them to participate, and encourage them to own their learning.

WE BELIEVE teachers are fundamental to student success and need powerful, flexible resources and support to build dynamic cultures of collaborative learning.

WE BELIEVE our learning solutions and services can help accomplish this, and that by working together with educators and communities we serve, we guide the way to better math learning.

LONG + LIVE + MATH



At Carnegie Learning, we choose the path that has been proven most effective by research and classroom experience. We call that path the Carnegie Learning Way. Follow this code to take a look inside.

Acknowledgments

Middle School Math Solution Authors

- Sandy Bartle Finocchi, Chief Mathematics Officer
- Amy Jones Lewis, Senior Director of Instructional Design
- Kelly Edenfield, Instructional Designer
- Josh Fisher, Instructional Designer

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“Mathematics is so much more than rules and algorithms. It is learning to reason, to make connections, and to make sense of the world. We believe in Learning by Doing™—students need to actively engage with the content if they are to benefit from it. Your classroom environment will determine what type of discourse, questioning, and sharing will take place. Students deserve a safe place to talk, to make mistakes, and to build deep understanding of mathematics. My hope is that these instructional materials help you shift the mathematical authority in your class to your students. Be mindful to facilitate conversations that enhance trust and reduce fear.”

Sandy Bartle Finocchi, Chief Mathematics Officer

“My hope is that you know that your students are capable of thinking like mathematicians. This book is designed to give them the opportunity to struggle with challenging tasks, to talk about math with their classmates, and to make and fix mistakes. I hope that you use this book to build this capacity in your students—to ask the necessary questions to uncover what students already know and connect it to what they are learning, to encourage creative thinking, and to give just enough support to keep students on the right path.”

Amy Jones Lewis, Senior Director of Instructional Design

“At Carnegie Learning we have created an organization whose mission and culture is defined by student success. Our passion is creating products that make sense of the world of mathematics and ignite a passion in students. Our hope is that students will enjoy our resources as much as we enjoyed creating them.”

Barry Malkin, CEO

The Carnegie Learning Way

At Carnegie Learning, we choose the path that has been proven most effective by research and classroom experience. We call that path the **Carnegie Learning Way**.

Our Instructional Approach

Carnegie Learning’s instructional approach is a culmination of the collective knowledge of our researchers, instructional designers, cognitive learning scientists, and master practitioners. It’s based on both a scientific understanding of how people learn and a real-world understanding of how to apply that science to mathematics instructional materials. At its core, our instructional approach is based on three simple, key components:



ENGAGE

Activate student thinking by tapping into prior knowledge and real-world experiences. Provide an introduction that generates curiosity and plants the seeds for deeper learning.



DEVELOP

Build a deep understanding of mathematics through a variety of activities—real-world problems, sorting activities, Worked Examples, and peer analysis—in an environment where collaboration, conversations, and questioning are routine practices.



DEMONSTRATE

Reflect on and evaluate what was learned. Ongoing formative assessment underlies the entire learning experience, driving real-time adjustments, next steps, insights, and measurements.



Our Research

Carnegie Learning has been deeply immersed in research ever since it was founded by cognitive and computer scientists from Carnegie Mellon University. Our research extends far beyond our own walls, playing an active role in the constantly evolving field of cognitive and learning science. Our internal researchers collaborate with a variety of independent research organizations, tirelessly working to understand more about how people learn, and how learning is best

facilitated. We supplement this information with feedback and data from our own products, teachers, and students, to continuously evaluate and elevate our instructional approach and its delivery.

Our Support

We're all in. In addition to our books and software, implementing Carnegie Learning in your classroom means you get access to an entire ecosystem of ongoing classroom support, including:

Professional Learning: Our team of Master Math Practitioners is always there for you, from implementation to math academies to a variety of other options to help you hone your teaching practice.

Texas Support Center: We've customized a Support Center just for you and your students. The Texas Support Center provides articles and videos to help you implement the Texas Math Solution, from the basics to get you started to more targeted support to guide you as you scaffold instruction for all learners in your classroom. Visit www.CarnegieLearning.com/texas-help to explore online and to access content that you can also share with your students and their caregivers.

MyCL: This is the central hub that gives you access to all of the products and resources that you and your students will need. Visit MyCL at www.CarnegieLearning.com/login.

LONG + LIVE + MATH: When you join this community of like-minded math educators, suddenly you're not alone. You're part of a collective, with access to special content, events, meetups, book clubs, and more. Because it's a community, it's constantly evolving! Visit www.longlivemath.com to get started.

Scan this code to visit the Texas Support Center and look for references throughout the Front Matter to learn more about the robust resources you will find in the Support Center.



Our Blend of Learning

Carnegie Learning combines consumable textbooks, MATHia® (our intelligent 1-on-1 math tutoring software), and transformative professional learning and data analysis services into a comprehensive and cohesive learning solution.

A key aspect of this blend is its combination of two forms of learning:

Learning Together: With our consumable textbooks, students work in groups, not only to develop math skills, but to learn how to collaborate, create, communicate and problem-solve.



Learning Individually: Through MATHia, students receive 1-to-1 adaptive math coaching, providing a personalized learning path and ongoing formative assessment.



Carnegie Learning’s blend also strikes the right balance in other ways:

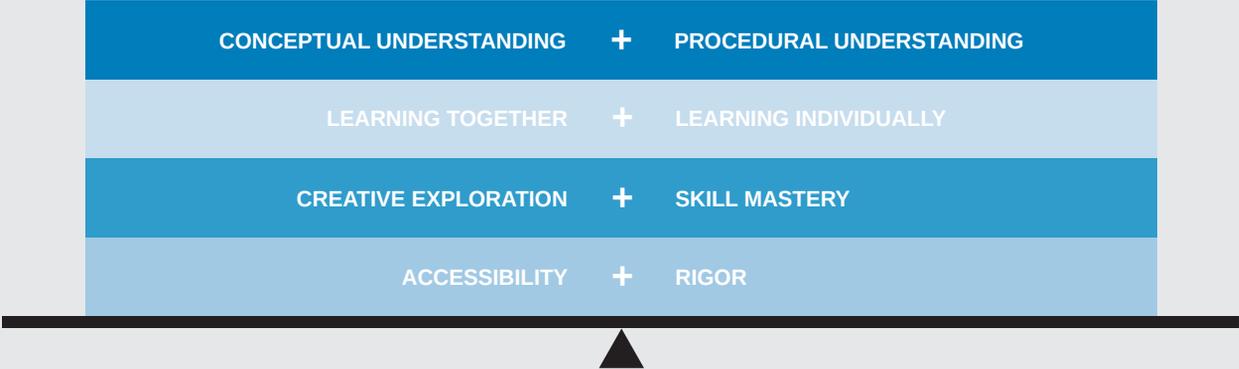


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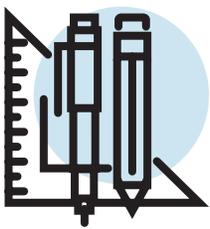
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Glossary

Instructional Design

In a word, every single piece of Carnegie Learning’s Texas Math Solution is **intentional**. Our instructional designers work alongside our master math practitioners, cognitive scientists, and researchers to intentionally design, draft, debate, test, and revise every piece, incorporating the latest in learning science.



Intentional Mathematics Design

Carnegie Learning’s Texas Math Solution is thoroughly and thoughtfully designed to ensure students build the foundation they’ll need to experience ongoing growth in mathematics.

Mathematical Coherence: The MSMS arc of mathematics develops coherently, building understanding by linking together within and across grades, so students can learn concepts more deeply and apply what they’ve learned to more complex problems going forward.

Mathematical Process Standards: Carnegie Learning is organized around the Mathematical Process Standards to encourage experimentation, creativity, and false starts, which is critical if we expect students to tackle difficult problems in the real world, and persevere when they struggle.

Multiple Representations: Carnegie Learning recognizes the importance of connecting multiple representations of mathematical concepts. Lessons present content visually, algebraically, numerically, and verbally.

Transfer: Carnegie Learning focuses on developing transfer. Doing A and moving on isn’t the goal; being able to do A and then do B, C, and D, transferring what you know from A, is the goal.

Texas Math Solution Year at a Glance

This Year at a Glance highlights the sequence of topics and the number of blended instructional days (1 day is 45 minutes) allocated for Accelerated Grade 7 in the Texas Math Solution. The pacing information also includes time for assessments, providing you with an instructional map that covers 180 days of the school year. As you set out at the beginning of the year, we encourage you to still modify this plan as necessary.

Want More Support Designing Your Long Term Plan?

You can find this Year at a Glance and additional guidance on planning intentionally and flexibly on the Texas Support Center at www.CarnegieLearning.com/texas-help.



Texas Accelerated Grade 7: Year at a Glance

*1 Day Pacing = 45 min. Session

Module	Topic	Pacing	TEKS
Process Standards are embedded in every module: 8.1A, 8.1B, 8.1C, 8.1D, 8.1E, 8.1F, 8.1G			
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	2: Compound Probability	9	7.6A, 7.6B, 7.6C, 7.6D, 7.6I,
	3: Drawing Inferences	14	7.6F, 7.6G, 7.12A, 7.12B, 7.12C, 8.11B, 8.11C
	4: Financial Literacy: Your Financial Future	6	8.12A, 8.12B, 8.12C, 8.12D, 8.12E, 8.12F, 8.12G
		39	
Total Days:		180	

Connecting Content and Practice

Lesson Structure

Each lesson of the Texas Math Solution has the same structure. This consistency allows both you and your students to track your progress through each lesson. Key features of each lesson are noted.

ENGAGE

Establishing Mathematical Goals to Focus Learning

Create a classroom climate of collaboration and establish the learning process as a partnership between you and students.

Communicate continuously with students about the learning goals of the lesson to encourage self-monitoring of their learning.

Visit the Texas Support Center for additional guidance on how to foster a classroom environment that promotes collaboration and communication.



Lesson Structure

Patty Paper, Patty Paper 1
Introduction to Congruent Figures

WARM UP
Draw an example of each shape.

1. parallelogram
2. trapezoid
3. pentagon
4. regular hexagon

LEARNING GOALS 1

- Define congruent figures.
- Use patty paper to verify experimentally that two figures are congruent by obtaining the second figure from the first using a sequence of slides, flips, and/or turns.
- Use patty paper to determine if two figures are congruent.

KEY TERMS

- congruent figures
- corresponding sides
- corresponding angles

2 You have studied figures that have the same shape or measure. How do you determine if two figures have the same size and the same shape?

LESSON 1: Patty Paper, Patty Paper • 1

1. Learning Goals
Learning goals are stated for each lesson to help you take ownership of the learning objectives.

2. Connection
Each lesson begins with a statement connecting what you have learned with a question to ponder.

Return to this question at the end of this lesson to gauge your understanding.

“ Mathematics is the science of patterns. So, we encourage students throughout this course to notice, test, and interpret patterns in a variety of ways—to put their “mental tentacles” to work in every lesson, every activity. Our hope is that this book encourages you to do the same for your students, and create an environment in your math classroom where productive and persistent learners develop and thrive. ”

Josh Fisher, Instructional Designer

Activating Student Thinking

Your students enter each class with varying degrees of experience and mathematical success. The focus of the Getting Started is to tap into prior knowledge and real-world experiences, to generate curiosity, and to plant seeds for deeper learning. Pay particular attention to the strategies students use, for these strategies reveal underlying thought processes and present opportunities for connections as students proceed through the lesson.

Supporting English Language Learners

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they complete the Getting Started activities in each lesson.



3. Getting Started

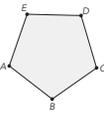
Each lesson begins with a Getting Started. When working on the Getting Started, use what you know about the world, what you have learned previously, or your intuition. The goal is just to get you thinking and ready for what's to come.

3
Getting Started

It's Transparent!

Let's use patty paper to investigate the figure shown.

Patty paper is great paper to investigate geometric properties. You can write on it, trace with it, and see creases when you fold it.



1. List everything you know about the shape.
2. Use patty paper to compare the sizes of the sides and angles in the figure.
 - a. What do you notice about the side lengths?
 - b. What do you notice about the angle measures?
 - c. What can you say about the figure based on this investigation?
3. Use five folds of your patty paper to determine the center of each side of the shape. What do you notice about where the folds intersect?

Trace the polygon onto a sheet of patty paper.

Patty paper was originally created for separating patties of meat! Little did the inventors know that it could also serve as a powerful geometric tool.





2 • TOPIC 1: Rigid Motion Transformations

Aligning Teaching to Learning

Students learn when they are actively engaged in a task: reasoning about the math, writing their solutions, justifying their strategies, and sharing their knowledge with peers.

Support productive struggle by allowing students time to engage with, and persevere through, the mathematics.

Support student-to-student discourse as well as whole-class conversations that elicit and use evidence of student thinking.

4
ACTIVITY 1.1 Analyzing Size and Shape
©

A conjecture is a hypothesis or educated guess that is consistent with what you know but hasn't yet been verified. Reasoning through multiple conjectures and investigations is an important part of learning in mathematics.

ACTIVITY 1.2
Congruent or Not?
⚙️

Throughout the study of geometry, as you reason about relationships, study how figures change under specific conditions, and generalize patterns, you will engage in the geometric process of

- making a conjecture about what you think is true,
- investigating to confirm or refute your conjecture, and
- justifying the geometric idea.

In many cases, you will need to make and investigate conjectures a few times before reaching a true result that can be justified. Let's use this process to investigate congruent figures.

If two figures are congruent, you can slide, flip, and spin one figure until it lies on the other figure.

1. Consider the flowers shown following the table. For each flower, make a conjecture about which are congruent to the original flower, which is shaded in the center. Then, use party paper to investigate your conjecture. Finally, justify your conjecture by stating how you can move from the shaded flower to each congruent flower by sliding, flipping, or spinning the original flower.

Flower	Congruent to Original Flower?	How Do You Move the Original Flower onto the Congruent Flower?
A		
B		
C		
D		
E		
F		
G		
H		

4 • TOPIC 1: Rigid Motion Transformations

4. Activities

You are going to build a deep understanding of mathematics through a variety of activities in an environment where collaboration and conversations are important and expected.

You will learn how to solve new problems, but you will also learn why those strategies work and how they are connected to other strategies you already know.

Remember:

- It's not just about answer-getting. The process is important.
- Making mistakes is a critical part of learning, so take risks.
- There is often more than one way to solve a problem.

Activities may include real-world problems, sorting activities, Worked Examples, or analyzing sample student work.

Be prepared to share your solutions and methods with your classmates.

Lesson Structure • 17



Supporting English Language Learners

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they engage in mathematical discourse throughout each lesson.

5. Talk the Talk

Talk the Talk gives you an opportunity to reflect on the main ideas of the lesson.

- Be honest with yourself.
- Ask questions to clarify anything you don't understand.
- Show what you know!

Don't forget to revisit the question posed on the lesson opening page to gauge your understanding.

NOTES

5 TALK the TALK

The Core of Congruent Figures

Recall that if two figures are congruent, all corresponding sides and all corresponding angles have the same measure.

1. Use patty paper to determine which sides of the congruent figures are corresponding and which angles are corresponding.

2. How to c

A	B	C
D	E	F
G	H	I
J	K	L

6 • TOPIC 1: Rigid Motion Transformations

LESSON 1: Patty Paper Patty Paper • 7

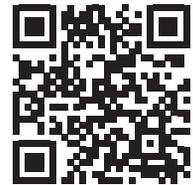
Ongoing Formative Assessment Drives Instruction

For students to take responsibility for their own learning, they need to be encouraged to self-assess. Students can use the Talk the Talk to monitor their own progress towards mastering the learning goals. Listen and review their answers and explanations and provide feedback to help them improve their understanding.

As you plan the next lesson, consider the connections you can make to build off the strengths or fill any gaps identified from this formative assessment.

Supporting English Language Learners

Visit the Texas Support Center for facilitation strategies to support students at varying levels of language proficiency as they demonstrate their understanding in the Talk the Talk activities in each lesson.



Assignment

An intentionally designed Assignment follows each lesson.

There is one Assignment per lesson. Lessons often span multiple days. Be thoughtful about which portion of the Assignment students can complete based on that day's progress.

The **Stretch** section is not necessarily appropriate for all learners. Assign this to students who are ready for more advanced concepts.

The **Review** section provides spaced practice of concepts from the previous lesson and topic and of the fluency skills important for the course.

Assignment

Assignment LESSON 1: Patty Paper, Patty Paper

6 Write
Explain what a conjecture is and how it is used in math.

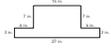
7 Remember
If two figures are congruent, all corresponding sides and all corresponding angles have the same measure.

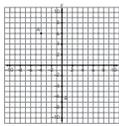
8 Practice
1. Determine which figures are congruent to Figure A. Follow the steps given as you investigate each shape.
• Make a conjecture about which figures are congruent to Figure A.
• Use patty paper to investigate your conjecture.
• Justify your conjecture by stating how you can move from Figure A to each congruent figure by sliding, flipping, or spinning Figure A.

Figure A Figure B Figure C

Figure D

9 Stretch
The figure on the left was reflected, or flipped, over a line of reflection to create the figure on the right. Determine the location of the line of reflection.

10 Review
1. Determine each sum or difference.
a. $-14 + 25$ b. $-14 - 25$
2. Calculate the area of each figure.
a.  b. 
3. Write the ordered pair for each point plotted on the coordinate plane.



2 • TOPIC 1: Rigid Motion Transformations

6. Write
Reflect on your work and clarify your thinking.

7. Remember
Take note of the key concepts from the lesson.

8. Practice
Use the concepts learned in the lesson to solve problems.

9. Stretch
Ready for a challenge?

10. Review
Remember what you've learned by practicing concepts from previous lessons and topics.

Assignment • 19

Problem Types You Will See

Lessons include a variety of problem types to engage students in reasoning about the math.

Worked Examples

Worked Examples help students develop their skills as they question their understanding, make connections with the steps, and ultimately explain the progression of the steps towards the final outcome. They represent and mimic an internal dialog about the mathematics and the strategies, and the questions that follow them are designed to serve as a model for self-questioning and self-explanations, while making sure that students don't skip over a Worked Example without interacting with it, thinking about it, and responding to its accompanying questions. This approach aids students as they develop their desired habits of mind for being conscientious about the importance of steps and their order.

Problem Types You Will See

Worked Example

When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connections between steps.

Ask Yourself:

- What is the main idea?
- How would this work if I changed the numbers?
- Have I used these strategies before?

WORKED EXAMPLE

The first right triangle has sides of length 3 units, 4 units, and 5 units, where the sides of length 3 units and 4 units are the legs and the side with length 5 units is the hypotenuse.

The sum of the squares of the lengths of the legs: $3^2 + 4^2 = 9 + 16 = 25$

The square of the hypotenuse: $5^2 = 25$

Therefore $3^2 + 4^2 = 5^2$, which verifies the Pythagorean Theorem, holds true.

The Pythagorean Theorem can be used to determine unknown side lengths in a right triangle. Evan and Sophi are using the theorem to determine the length of the hypotenuse, c , with leg lengths of 2 and 4. Examine their work.

Sophi

$$c^2 = 2^2 + 4^2$$
$$c^2 = 4 + 16 = 20$$
$$c = \sqrt{20} \approx 4.5$$

The length of the hypotenuse is approximately 4.5 units.

Evan

$$c^2 = 2^2 + 4^2$$
$$c^2 = 6^2$$
$$c = 6$$

The length of the hypotenuse is 6 units.

Thumbs Up

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connections between steps.

Ask Yourself:

- Why is this method correct?
- Have I used this method before?

Thumbs Down

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

Ask Yourself:

- Where is the error?
- Why is it an error?
- How can I correct it?

20 • Problem Types

Thumbs Up / Thumbs Down

Thumbs Up problems give students the opportunity to analyze viable methods and problem-solving strategies. Questions are presented to help students consider the various strategies in-depth, and to focus on an analysis of correct responses. Because research shows that providing only positive examples is less effective for eliminating common student misconceptions than also showing negative examples, incorrect responses are provided alongside the correct responses. From the incorrect responses, students learn to determine where the error in calculation is, why the method is wrong or is being used wrong, and also how to correct the method to calculate the solution properly.

Who's Correct?

"Who's Correct?"

problems are an advanced form of correct vs. incorrect responses. In this problem type, students are not told who is correct. Students have to think more deeply about what the strategies really mean, and whether each of the solutions made sense. Students will determine what is correct and what is incorrect, and then explain their reasoning. These types of problems will help students analyze their own work for errors and correctness.

Isabel says that $2^2 + 2^3 = 2^5$, and Elizabeth says that $2^2 + 2^3 \neq 2^5$. Who is correct? Explain your reasoning.



Who's Correct

When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine correct or not correct.

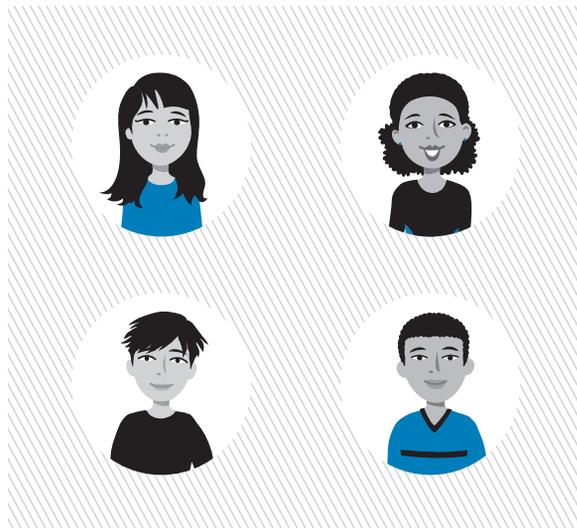
Ask Yourself:

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

Promoting Self-Reflection

The Crew

The Crew is here to help you on your journey. Sometimes they will remind you about things you already learned. Sometimes they will ask you questions to help you think about different strategies. Sometimes they will share fun facts. They are members of your group—someone you can rely on!



Teacher aides will guide you along your journey. They will help you make connections and remind you to think about the details.



The Crew

Characters are embedded throughout the Texas Math Solution to remind students to stop and think in order to promote productive reflection. The characters are used in a variety of ways: they may remind students to recall a previous mathematical concept, help students develop expertise to think through problems, and occasionally, present a fun fact.

Mathematical Process Standards

Note

Each lesson provides opportunities for students to think, reason, and communicate their mathematical understanding. However, it is your responsibility as a teacher to recognize these opportunities and incorporate these practices into your daily rituals. Expertise is a long-term goal, and students must be encouraged to apply these practices to new content throughout their school career.

Mathematical Process Standards

Texas Mathematical Process Standards

Effective communication and collaboration are essential skills of a successful learner. With practice, you can develop the habits of mind of a productive mathematical thinker. The “I can” expectations listed below align with the TEKS Mathematical Process Standards and encourage students to develop their mathematical learning and understanding.

► Apply mathematics to problems arising in everyday life, society, and the workplace.

I can:

- use the mathematics that I learn to solve real world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

► Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem solving process and reasonableness of the solution.

I can:

- explain what a problem “means” in my own words.
- create a plan and change it if necessary.
- ask useful questions in an attempt to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Mathematical Process Standards • 23



Supporting Students to Use Mathematical Tools

Visit the Texas Support Center for strategies to support students as they use mathematical tools, including formula charts and reference sheets.

► **Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate; and techniques including mental math, estimation, and number sense as appropriate, to solve problems.**

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

► **Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.**

I can:

- communicate and defend my own mathematical understanding using examples, models, or diagrams.
- use appropriate mathematical vocabulary in communicating mathematical ideas.
- make generalizations based on results.
- apply mathematical ideas to solve problems.
- interpret my results in terms of various problem situations.

Note

When you are facilitating each lesson, listen carefully and value diversity of thought, redirect students' questions with guiding questions, provide additional support with those struggling with a task, and hold students accountable for an end product. When students share their work, make your expectations clear, require that students defend and talk about their solutions, and monitor student progress by checking for understanding.

Supporting ALL Learners

Visit the Texas Support Center for facilitation strategies to support ALL students as they engage in the Mathematical Process Standards.



Academic Glossary

It is critical for students to possess an understanding of the language of their text. Students must learn to read for different purposes and write about what they are learning. Encourage students to become familiar with the key words and the questions they can ask themselves when they encounter these words.

It is our recommendation to be explicit about your expectations of language use and the way students write responses throughout the text. Encourage students to answer questions with complete sentences. Complete sentences help students reflect on how they arrived at a solution, make connections between topics, and consider what a solution means both mathematically as well as in context.



Academic Glossary

Visit the Students & Caregivers Portal on the Texas Support Center at www.CarnegieLearning.com/texas-help to access the Mathematics Glossary for this course anytime, anywhere.



Related Phrases

- Examine
- Evaluate
- Determine
- Observe
- Consider
- Investigate
- What do you notice?
- What do you think?
- Sort and match

ANALYZE

Definition
To study or look closely for patterns. Analyzing can involve examining or breaking a concept down into smaller parts to gain a better understanding of it.

Ask Yourself

- Do I see any patterns?
- Have I seen something like this before?
- What happens if the shape, representation, or numbers change?

Related Phrases

- Show your work
- Explain your calculation
- Justify
- Why or why not?

EXPLAIN YOUR REASONING

Definition
To give details or describe how to determine an answer or solution. Explaining your reasoning helps justify conclusions.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Does my reasoning make sense?
- How can I justify my answer to others?

26 • Academic Glossary

Supporting Students at Varying Levels of Language Proficiency

Visit the Texas Support Center for guidance on how to leverage the Academic Glossary to support students at varying levels of language proficiency.

REPRESENT

Definition

To display information in various ways. Representing mathematics can be done using words, tables, graphs, or symbols.

Ask Yourself

- How should I organize my thoughts?
- How do I use this model to show a concept or idea?
- What does this representation tell me?
- Is my representation accurate?

Related Phrases

- Show
- Sketch
- Draw
- Create
- Plot
- Graph
- Write an equation
- Complete the table

ESTIMATE

Definition

To make an educated guess based on the analysis of given data. Estimating first helps inform reasoning.

Ask Yourself

- Does my reasoning make sense?
- Is my solution close to my estimation?

Related Phrases

- Predict
- Approximate
- Expect
- About how much?

DESCRIBE

Definition

To represent or give an account of in words. Describing communicates mathematical ideas to others.

Ask Yourself

- How should I organize my thoughts?
- Is my explanation logical?
- Did I consider the context of the situation?
- Does my reasoning make sense?

Related Phrases

- Demonstrate
- Label
- Display
- Compare
- Determine
- Define
- What are the advantages?
- What are the disadvantages?
- What is similar?
- What is different?

Academic Glossary • 27

Ask Yourself

The Ask Yourself questions help students develop the proficiency to explain to themselves the meaning of problems.

Real-World Context

Real-world contexts confirm concrete examples of mathematics. The scenarios in the lessons help students recognize and understand that quantitative relationships seen in the real world are no different than quantitative relationships in mathematics. Some problems begin with a real-world context to remind students that the quantitative relationships they already use can be formalized mathematically. Other problems will use real-world situations as an application of mathematical concepts.



Home Connection

Encourage your students to explore the Students & Caregivers portal on the Texas Support Center to access a variety of resources to support their learning at home and elsewhere outside of the classroom.



MATHia Structure

Each unit in MATHia maximizes student learning while collecting critical data about what they do or do not know at every step. Students can access MATHia anywhere, anytime.

ENGAGE

Unit Overview

The Unit Overview page engages students in the learning experience, providing them with a clear set of learning goals, a link to the real world, and a connection back to the math they already know so they can build from it throughout the unit.

The screenshot shows the MATHia interface for a unit overview. The main content area is titled "Problem Solving using Ratio and Rate Reasoning" and includes a "What you'll learn:" section with three bullet points: "Use a table to solve problems involving equivalent ratios and rates," "Use a double number line to solve problems involving equivalent ratios and rates," and "Use a graph to solve problems involving equivalent ratios and rates." Below this is a video player showing a woman writing math equations on a whiteboard: $2+3=3+2$, $5+1=1+5$, and $(2+3)+4=2+(3+4)$. A "Why this matters:" section follows, and then a "What this connects to:" section explaining that students will use their knowledge of ratios to solve problems using ratio tables, double number lines, and graphs. On the right, a "Workspaces" sidebar lists three activities: "Problem Solving with Equivalent Ratios and Rates using Tables" (with a "Review" button), "Problem Solving with Equivalent Ratios and Rates using Double Number Lines" (with a "Let's Go" button), and "Problem Solving with Equivalent Ratios and Rates using Graphs" (with a "25% Complete" indicator and a cartoon character). The top navigation bar includes "Home", "System Help", "Glossary", and "Polly Nomial".

Step by Step

Step by Step demonstrates how to use the tools in a lesson by guiding students step by step through a sample math problem.

The screenshot shows the "Step-by-Step Example" page for a math problem. The problem text reads: "You are writing a book of jokes. Whenever you come up with an idea for a joke, you like to write it down. The trouble is, you might get the idea in your sleep, or while playing ping pong, or during your favorite TV shows, and then when you go to write it down, you totally forget what it is. The table shows the ideas you got and the ideas you forgot." Below the text is a table with two columns: "Ideas You Got" and "Ideas You Forgot". The table contains the following data:

Ideas You Got	Ideas You Forgot
5	1
15	3
25	5
35	7

To the right of the table are two numbered questions: "1. Oh fiddlesticks. You forgot 8 ideas. How many total ideas did you get?" with an input field and a checkbox for "I want to do the optional double number line tasks."; and "2. Phooey. You had 25 cool ideas, but of course you forgot some. How many did you forget?" with an input field and a checkbox for "I want to do the optional double number line tasks." Below the questions is a "Step-by-Step Example" callout box that says: "The ratio Ideas You Got : Ideas You Forgot is constant. Each row of the table represents an equivalent ratio. Is 8 ideas you forgot in the table?" The top navigation bar includes "Home", "System Help", "Glossary", and "Ada Jacent".

Montell is a freelance writer. He takes on the same number of jobs each month. Every few months he looks at his records and determines the number of jobs he has had in that time. The double number line below shows the number of jobs Montell has had and the number of months since he looked at his books.

Use the double number line to calculate the unknown values.

1. How many months did Montell work if he had 336 jobs since the last time he looked at his books?

months

I want to do the optional double number line tasks.

2. If it
hov

Hint
You know the given value is 336 jobs.
Is **Number of Jobs** represented by the top or bottom number line?

Previous Hint 2 of 3 Next

Set Minor Tick Marks

Number of Jobs: 0, 48, 96, 144, 192, 240, 288, 336, 384

Time (months): 0, 6, 12, 18, 24, 30, 36, 42, 48

Problem: emsch7025 Client Version: 3.0.162 Server Version: 3.0.162 © 2017 Carnegie Learning

Hints

Multi-level hints are available throughout the software to help students solve the problems they are working on.

Montell is a freelance writer. He takes on the same number of jobs each month. Every few months he looks at his records and determines the number of jobs he has had in that time. The double number line below shows the number of jobs Montell has had and the number of months since he looked at his books.

Use the double number line to calculate the unknown values.

1. How many months did Montell work if he had 336 jobs since the last time he looked at his books?

Glossary

double bar graph
double number line

double number line

Definition:

A double number line is a model that is made up of two number lines used to represent the equivalence of two related numbers. Each interval on the number line has two sets of numbers and maintains the same ratio.

Example:

The following double number line represents the ratio of mosquitoes to grasshoppers. The ratios

$\frac{3 \text{ mosquitos}}{1 \text{ grasshopper}}$, and $\frac{12 \text{ mosquitos}}{4 \text{ grasshoppers}}$

double number line

Number of Jobs: 0, 48, 96, 144, 192, 240, 288, 336, 384

Time (months): 0, 6, 12, 18, 24, 30, 36, 42, 48

Problem: emsch7025 Client Version: 3.0.162 Server Version: 3.0.162 © 2017 Carnegie Learning

Glossary

The Glossary is available throughout the software. It contains a list of definitions and examples for key mathematical terms used throughout the curriculum.

DEVELOP AND DEMONSTRATE

Formative Assessment

The Develop and Demonstrate phases of our instructional design happen simultaneously. The reports provide the detail to interpret student performance. Facilitation and suggestions for follow-up are available via our online Resource Center.

Progress Bar

The Progress Bar shows a summary of the major skills that are being covered in a given problem-solving workspace as well as students' progress on those skills.

The screenshot shows the MATHia interface for a problem-solving workspace. The browser address bar displays [https://www.carnegielearning.com/...](https://www.carnegielearning.com/). The page title is "MATHia® Problem Solving with Equivalent Ratios and Rates using Double Number Lines". The navigation bar includes "Home", "System Help", "Glossary", and "Ada Jacent". The main workspace contains a problem statement: "Ms. Goodfellow is the director of the Music Program at Union Middle School. She is completing the scheduling for next year's students. She needs to make sure that the same number of students are in each music class. The number of students and the number of music classes are represented on the double number line." Below the problem is a "Progress Bar" with two rows: "Number of Students" and "Music Classes". The "Number of Students" row has major tick marks at 0, 120, 240, 360, 480, 600, 720, 840, and 960. The "Music Classes" row has major tick marks at 0, 6, 12, 18, 24, 30, 36, 42, and 48. A "Set Minor Tick Marks" button is located above the number lines. A "Skills Progress to Mastery" table is visible in the top right corner of the workspace.

Skills	Progress to Mastery
Calculate value on double number line other than halfway between major tick marks.	<input type="checkbox"/>
Calculate value on double number line halfway between major tick marks.	<input type="checkbox"/>
Enter calculated denominator value.	<input type="checkbox"/>
Enter calculated numerator value.	<input type="checkbox"/>
Enter value visible on double number line.	<input type="checkbox"/>
Plot equivalent ratio on double number line.	<input type="checkbox"/>
Identify minor tick mark interval.	<input type="checkbox"/>

Problem Types in MATHia

MATHia features different instructional strategies to engage students as they develop their math skills.

CL Carnegie Learning

https://www.carnegielearning.com/...

MATHia[®] Exploring the Distributive Property with Numeric Expressions

Home System Help Glossary Ada Jacent

< Unit Overview Step-by-Step Hints Progress I'm Done

Use this Explore Tool to investigate number sentence composition and decomposition. You will use this tool in a variety of problems, so take some time to become familiar with how to use it. The rows of the diagram are horizontal, and the columns are vertical.

Drag the handle at the top to make a numeric expression. Then drag the slider at the bottom to create equivalent expressions. Use the Reset button to return the tool to its original state.

$9 \times 12 = 108$

Use the model to represent $11 \times (4 + 5)$.

Analyze the shaded squares in your model. Enter the values for each representation.

The model has shaded rows and shaded columns.

The model has 2 groups of shaded squares: a group of shaded squares on the left and a group of shaded squares on the right.

There are total shaded squares.

Complete the number sentence to represent the shading in the model.

$11 \times (4 + 5) = 11 \times \text{[]} + 11 \times \text{[]}$
 $= \text{[]} + 55$

Problem: ekt/pwano01 Client Version: 3.0.162 Server Version: 3.0.162 © 2017 Carnegie Learning CARNEGIE LEARNING

Explore Tools

Explore Tools provide students the opportunity to investigate different mathematical concepts, search for patterns, and look for structure in ways that make sense to you. These tools also provide optional supports for students as they answer questions and solve problems.

CL Carnegie Learning

https://www.carnegielearning.com/...

MATHia[®] Developing Area Formulas

Home System Help Glossary Ada Jacent

< Unit Overview Step-by-Step Hints Progress I'm Done

Area of a Parallelogram

This animation develops the formula for the area of a parallelogram. Before attempting to answer any questions, watch the animation.

Watch the animation to derive the formula for the area of a parallelogram.

As you answer each question, you can re-watch the video as many times as you need.

Area of a Parallelogram

Watch the animation and then answer each question.

The cutout triangle has which of these important measures to determine the area of the rectangle?

a base length of b a height of h

The area of a parallelogram is equal to the area of a rectangle with the same base and height.

Quadrilateral $ABCD$ is a parallelogram. The length of segment CD is 9 feet, the length of segment AD is 15 feet, and the length of segment BE is 7 feet. What is the area of the parallelogram?

$h = 7$ feet $a = 9$ feet $b = 15$ feet

Problem: da901 Client Version: 3.0.162 Server Version: 3.0.162 © 2017 Carnegie Learning CARNEGIE LEARNING

Animations

Animations provide students with an opportunity to watch, pause, and re-watch demonstrations of various mathematical concepts. They are a way to connect the visual representations of different mathematical ideas to their abstract underpinnings through visual representations and audio narrative.

Classification Tools

Classification Tools allow students to apply their mathematical understanding by categorizing answers based on similarities. These tools also provide students with the means to demonstrate proficiency in recognizing patterns in problem structure.

The screenshot shows a web browser window with the URL [https://www.carnegielearning.com/...](https://www.carnegielearning.com/). The page title is "Using the Distributive Property with Numeric Expressions". The interface includes navigation buttons: "Unit Overview", "Step-by-Step", "Hints", "Progress", and "I'm Done".

The main content area is titled "Distributive Property of Multiplication" and contains the instruction: "Drag each expression into the bin with its equivalent expression. Consider how the expressions in the bins are related to the others in the same bin." Below this instruction are two empty rectangular bins. Above the bins are six draggable expressions: $11(15 + 5)$, $11 \times 15 + 11 \times 5$, $11 \times 4 + 11 \times 5$, $44 + 55$, $165 + 55$, and 11×9 . The first bin contains the expression $11(4 + 5)$ and the second bin contains 11×20 .

Below the bins is a question: "Which statement describes a more efficient way to calculate 11×20 by decomposing 20 and then using the Distributive Property? (A)"

Four radio button options are listed:

- $11(0 + 20)$, because multiplying a number by 0 is 0.
- $11(1 + 19)$, because multiplying a number by 1 is just that number.
- $11(10 + 10)$, because multiplying a number by 10 is easier to calculate.
- $11(11 + 9)$, because multiplying a number by itself is that number squared.

At the bottom of the page, it says "Problem: dm01 Client Version: 3.0.162 Server Version: 3.0.162" and "© 2017 Carnegie Learning CARNEGIE LEARNING".

Problem-Solving Tools

Problem-Solving Tools provide students with highly individualized and self-paced instruction that adapts to their exact needs to deepen their conceptual understanding of the mathematics. Through adaptive learning technologies, they engage in reasoning and sense making.

The screenshot shows a web browser window with the URL [https://www.carnegielearning.com/...](https://www.carnegielearning.com/). The page title is "Problem Solving with Equivalent Ratios and Rates using Double Number Lines". The interface includes navigation buttons: "Unit Overview", "Step-by-Step", "Sample Problem", "Hints", "Progress", and "I'm Done".

The main content area contains a word problem: "The local minor league ball park has hired Kristen to sing the National Anthem before its season opening game. She does such a bad job singing that they ask her to leave before she is done. Phone calls from people complaining pour in, and the operators answering the calls are asked to log the callers as hostile or just complaining. The head operator notices that the ratio of hostile callers to callers that are just complaining is always the same. The table below shows the number of hostile phone callers based upon the number of phone callers who are just calling to complain." Below the text is a table with two columns: "Number of Hostile Phone Callers" and "Number of Complaining Phone Callers".

Two questions are listed:

- If 12 people called just to complain, how many hostile phone calls were there?
30 hostile callers
I want to do the optional double number line tasks.
- How many just complaining callers called if there are 50 hostile phone calls?
20 complaining callers
I want to do the optional double number line tasks.

Below the questions is a double number line diagram. The top number line is labeled "Number of Hostile Phone Callers" and has major tick marks at 0, 15, 30, 45, 60, 75, 90, 105, and 120. The bottom number line is labeled "Number of Complaining Phone Callers" and has major tick marks at 0, 6, 12, 18, 24, 30, 36, 42, and 48. A vertical dashed line connects the tick mark for 30 on the top line to the tick mark for 12 on the bottom line. Another vertical dashed line connects the tick mark for 45 on the top line to the tick mark for 18 on the bottom line. A box highlights the tick mark for 20 on the bottom line.

At the bottom of the page, it says "Problem: emsd025 Client Version: 3.0.162 Server Version: 3.0.162" and "© 2017 Carnegie Learning CARNEGIE LEARNING".

CL Carnegie Learning

https://www.carnegielearning.com/...

MATHia[®] Commutative and Associative Properties

Home System Help Glossary Ada Jacent

Unit Overview Step-by-Step Hints Progress I'm Done

Commutative Property of Addition

Consider the expression $35 + 17 + 105$. You can use the Commutative Property of Addition to simplify this expression.

The Commutative Property of Addition states that changing the order of numbers in an addition expression does not change the sum.

Instead of first adding in order from left to right, use the Commutative Property to rewrite the expression into sums that might be easier to compute mentally.

$35 + 17 + 105 = 35 + 105 + 17$

Now, add the numbers in order from left to right. So, $35 + 105$ is 140 , and then $140 + 17 = 157$.

You can add more efficiently by using the Commutative Property to rearrange the addends in addition expressions.

Examine the worked example and then answer each question.

Let's consider the original expression from the worked example: $35 + 17 + 105$.

Add the numbers in the expression in their original order.

$35 + 17 =$

+ $105 =$

In the worked example, the addends were added in a different order.

The Commutative Property was used so that $35 +$ could be added first.

That result is a number ending with in the ones place.

Problem: casq01 Client Version: 3.0.162 Server Version: 3.0.162 © 2017 Carnegie Learning CARNEGIE LEARNING

Worked Examples

Worked Examples provide students with a tool that allows them to question their understanding, make connections with the steps, and ultimately self-explain. Analyzing Worked Examples also allows students to identify their own misconceptions, make sense of the mathematical concepts, and then ultimately to persevere in problem solving.

Facilitating Student Learning

Visit the Texas Support Center at www.CarnegieLearning.com/texas-help for additional resources to support you anytime, anywhere.



Teacher's Implementation Guide

The Teacher's Implementation Guide (TIG) is designed to fully support a wide range of teachers implementing our materials: from first year teachers to 30-year veterans; from first time Carnegie Learning users to master practitioners.

One goal in developing the Teacher's Implementation Guide was to make our instructional design apparent to the users.

The lessons of each topic were written to be accessible to the full range of learners. With every instructional decision you make, keep in mind your mathematical objectives for the topic and module and the course. Plan each lesson by thinking about how you will create access for your particular group of students, maintain access and pace throughout the lesson, and assess their understanding along the way. We recommend that you do the math in each topic before implementing the activities with your specific group of students.

What makes this Teacher's Implementation Guide useful?

Effective Lesson Design: Each lesson has a consistent structure for teachers and students to follow. The learning experiences are engaging and effective for students.

Pacing: Each course is designed to be taught in a 180-day school year. Pacing suggestions are provided for each lesson. Each day in the pacing guide is an equivalent to about a 45 minute instructional period.

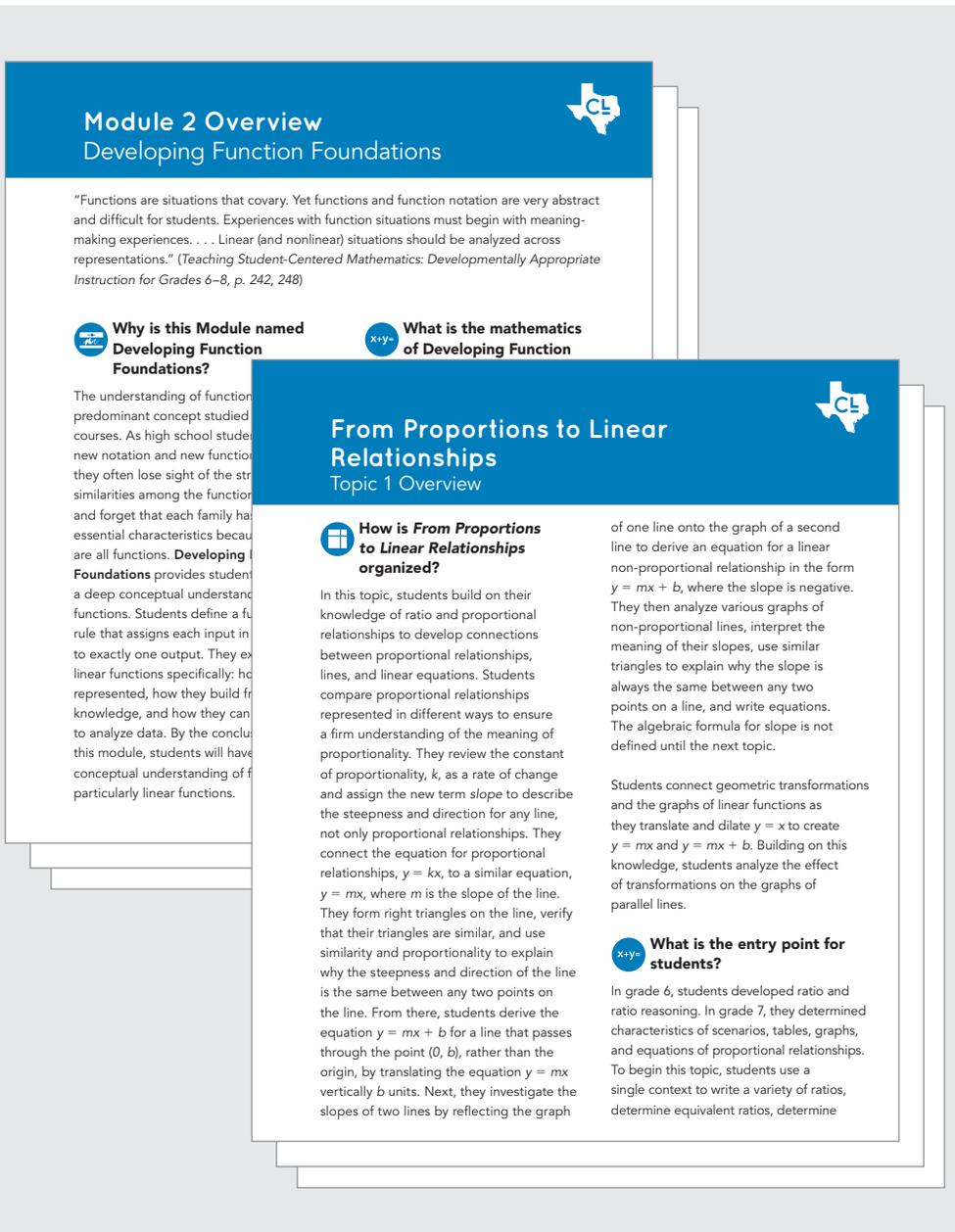
Instructional Supports: Guiding questions are provided for teachers to use as they're circulating the room, as well as differentiation strategies, common student misconceptions, and student look fors.

Clearly Defined Mathematics: The content and instructional goals are clearly described at the module, topic, lesson, and activity levels.

The TIG is critical to understanding how the mathematics that students encounter should be realized in the classroom. The TIG describes the depth of understanding that students need to develop for each standard and a pathway for all learners to be successful. It provides differentiation strategies to support students who struggle, to extend certain activities for students who are advanced in their understanding of the content, and to support English Language Learners.

Module and Topic Overviews

You are responsible for teaching the essential concepts associated with a particular course. You need to understand how activities within lessons build to achieve understanding within topics, and how topics build to achieve understanding throughout the course. In the Texas Math Solution, Carnegie Learning seeks to establish a shared curriculum vision with you.



Module Overview

Each module begins with an overview that describes the reasoning behind the name, the mathematics being developed, the connections to prior learning, the connections to future learning, and the pacing information.

Topic Overview

A Topic Overview describes how the topic is organized, the entry point for students, how a student will demonstrate understanding, why the mathematics is important, how the activities promote expertise in the practice standards, descriptions of the learning individually opportunities, and more detailed information to help with pacing.

“Teachers must first develop their ideas about where the curriculum program is going mathematically (curriculum vision) before deciding whether the curriculum materials will help them reach that mathematical goal (curriculum trust)” (Drake & Sherin, 2009, p. 325).

Facilitation Notes

For each lesson, you are provided with detailed facilitation notes to fully support your planning process. This valuable resource provides point-of-use support that serves as your primary resource for planning, guiding, and facilitating student learning.

1. Materials

Materials required for the lesson are identified.

2. Lesson Overview

The Lesson Overview sets the purpose and describes the overarching mathematics of the lesson, explaining how the activities build and how the concepts are developed.

3. TEKS Addressed

The focus standards for each lesson are listed. Carnegie Learning recognizes that modeling is not done in isolation but instead in relationship to other standards. You will see these standards interleaved throughout the course, indicated by an asterisk(*).

Jack and Jill Went Up the Hill

2

MATERIALS ①

Patty paper
Straightedge
Scissors

Using Similar Triangles to Describe the Steepness of a Line

②

Lesson Overview

Students connect the previously learned concepts of unit rate, constant of proportionality, and scale factor with the concept of slope, which is introduced here as the rate of change of the dependent quantity compared to the independent quantity. In this lesson, *slope* is defined as the steepness and direction of a line. The formula to calculate slope is introduced in the next topic. Students derive the equation for a proportional relationship, $y = mx$. By translating the line b units, they derive the equation for a non-proportional linear relationship, $y = mx + b$. They practice writing equations from graphs. Students begin with incomplete tables and graphs to create their own proportional and non-proportional linear relationships. They also investigate the slope of a horizontal line.

Grade 8 Proportionality

③

(4) The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope. The student is expected to:

- (A) use similar right triangles to develop an understanding that slope, m , given as the rate comparing the change in y -values to the change in x -values, $\frac{(y_2 - y_1)}{(x_2 - x_1)}$, is the same for any two points (x_1, y_1) and (x_2, y_2) on the same line.
- (B) graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship.
- (C) use data from a table or graph to determine the rate of change or slope and y -intercept in mathematical and real-world problems.

Proportionality

(5) The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions.

The student is expected to:

- (F) distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form $y = kx$ or $y = mx + b$, where $b \neq 0$.

(H) identify examples of proportional and non-proportional functions that arise from mathematical and real-world problems.

4 ELPS

1.A, 1.C, 1.D, 1.E, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.F, 4.K, 5.E

5 Essential Ideas

- A rate of change is used to describe the rate of increase or decrease of one quantity relative to another quantity.
- A unit rate is a comparison of two measurements in which the denominator has a value of one unit.
- The rate of change is the same for any two points on a line.
- The slope of a line describes its steepness.
- An increasing line has a positive slope.

values increase. They then use the slope to represent the walk home.

Activity 2.5: Describing a Line
Students identify graphs that represent a line. They then write equations for the lines and complete tables and graphs.

Demonstrate

Talk the Talk: A Web of Connections
Students summarize what they have learned by connecting the steepness of a line to the constant of proportionality and the connections among these concepts.

6 Lesson Structure and Pacing: 3 Days 7

Day 1

Engage

Getting Started: Let It Steep

Students examine multiple triangles, each with given base and height measurements. They write ratios to represent the relationship between the height and the base of each triangle. They write these ratios as unit rates and use them to compare the steepness of the triangles.

Develop

Activity 2.1: Constant of Proportionality as Rate of Change

Students write an equation based on a situation of Jack and Jill walking to the bus stop at a constant rate and determine if the situation represents a proportional relationship. They complete a table, identify the unit rate for the relationship, and then graph the relationship. Students connect the rate of change with what they know about unit rate and constant of proportionality.

Day 2

Activity 2.2: Slope of a Line

Students trace right triangles along the line that represents the scenario from the previous activity. They determine that the three triangles are similar. Students identify the unit rate triangle as the triangle with a base length of 1 and a height equal to the unit rate. They slide the triangles along the line to show that the slope remains constant between any two points. The steepness and direction of the line is defined as *slope*, which is connected to unit rate, rate of change, and the constant of proportionality. Students derive the equation $y = mx$ to represent any straight line that passes through the origin.

Activity 2.3: Equation for a Line Not Through the Origin

The scenario continues with Jack and Jill walking to the bus stop at the same rate, but this time they leave from a different location. Students compare the slopes and starting points of the two situations to conclude that the slopes are the same, but the starting points are different. They use patty paper to translate the original line of the form $y = mx$ by b units vertically to represent a new line of the form $y = mx + b$. Students generalize that an equation of the form $y = mx + b$ is a non-proportional linear relationship.

Day 3

Activity 2.4: A Negative Unit Rate

Students compare the graph of Jack and Jill walking to the bus stop at a constant rate with the graph of them walking home from the bus stop at the same constant rate. They use patty paper to investigate the slopes of the two lines by reflecting the graph of the original line onto the graph of the second line. Students conclude that the steepness of the two lines is the same, but the direction of the second graph decreases as the independent

4. ELPS Addressed

The English Language Proficiency Standards for each lesson are listed. As you plan, consider these ELPS and determine the instructional strategies that you will use to meet these ELPS.

5. Essential Ideas

These statements are derived from the standards and state the concepts students will develop.

6. Lesson Structure

This section highlights how the parts of the lesson fit within the instructional design: Engage, Develop, and Demonstrate. A summary of each activity is included.

7. Pacing

Lessons often span more than one 45-minute class period. Suggested pacing is provided for each lesson so that the entire course can be completed in a school year.

8. Facilitation Notes by Activity

A detailed set of guidelines walks the teacher through implementing the Getting Started, Activities, and Talk the Talk portions of the lesson. These guidelines include an activity overview, grouping strategies, guiding questions, possible student misconceptions, differentiation strategies, student look fors, and an activity summary.

9. Activity Overview

Each set of Facilitation Notes begins with an overview that highlights how students will actively engage with the task to achieve the learning goals.

8
9

Getting Started: Let It Steep

ENGAGE

Facilitation Notes

In this activity, students write ratios to represent the relationship between the height and the base of different triangles and use unit rates for comparison purposes.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- Is the height of the triangle written in the numerator or denominator of the ratio?
- Is the base of the triangle written in the numerator or denominator of the ratio?
- Is the ratio $\frac{10}{10}$ equal to the ratio $\frac{15}{15}$?
- How did you determine the unit rate?
- Does the smallest rate identify with the least steep or most steep line on the graph?
- Does the largest rate identify with the least steep or most steep line on the graph?

Misconception

If students calculate $\frac{x}{y}$, take the time to interpret the meaning of their calculations.

Summary

Unit rates can be used to compare different ratios.

Activity 2.1

Constant of Proportionality as Rate of Change



DEVELOP

Facilitation Notes

In this activity, students are given a scenario and asked to write an equation, complete a table, identify the unit rate, and graph the relationship. Connections are made between the rate of change, the unit rate, and the constant of proportionality.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.



Position yourself to take full advantage of the richness of the mathematics addressed in the textbook. The Facilitation Notes provide guidance to reach each student from their current level of understanding to advance to the next stage. Place yourself in the position of the student by experiencing the textbook activities prior to class. Realize your role in the classroom—empower your students! Step back and let them do the math with confidence in their role as learner and your role as facilitator of learning.



Janet Sinopoli, Instructional Designer

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Questions to ask

- Is the distance written in the numerator or denominator of the ratio?
- Is the time written in the numerator or denominator of the ratio?
- Is the ratio $\frac{4}{3}$ equal to the ratio $\frac{3}{4}$?
- Is the equation written in the form $y = kx$?
- How did you determine the time spent walking when the distance from home was given?
- How did you determine the distance from home when the time spent walking was given?
- Does the graph of the line pass through the origin?
- How did you determine the unit rate?
- What point on the graph represents the unit rate?
- What do all the other points on the graph represent?
- Would the graph look the same if they were walking down hill?
- Would the graph look the same if they were walking on level land?
- How can you tell whether the graph is continuous or discrete?
- How long will it take Jack and Jill to walk 5 yards?

11

Differentiation strategies

To extend the activity,

- Calculate Jack and Jill's rate in mph.
- Determine how many minutes it takes Jack and Jill to walk one mile.

Ask a student to read the definition of rate of change aloud. Complete Questions 6 through 8 as a class.

Questions to ask

- What unit of measurement is associated with time in this situation?
- What unit of measurement is associated with distance in this situation?
- Does the amount of time spent walking depend on the distance walked, or does the distance walked depend on the amount of time spent walking?
- Does the distance increase 4 yards for every increase of 3 seconds of time, or does the distance increase 3 yards for every increase of 4 seconds of time?
- Does the distance increase 4 yards for every increase of 3 seconds of time, or does the distance increase $\frac{4}{3}$ yard for every increase of 1 second of time?

Summary

A real-world situation is modeled using proportional relationships. The rate of change, unit rate, and constant of proportionality are equivalent in proportional relationships.

Activity 2.2
Slope of a Line



Facilitation Notes

In this activity, the graph of the scenario from the previous activity is used to draw right triangles and determine triangle

10. Questions to Ask

The overarching questioning strategies throughout each lesson promote analysis and higher-order thinking skills beyond simple yes or no responses. These questions can be used to gather information, probe thinking, make the mathematics explicit, and encourage reflection and justification as students are working together or when they are sharing responses as a class. These questions are an embedded formative assessment strategy to provide feedback as students are actively engaged in learning.

11. Differentiation Strategies

To extend an activity for students who are ready to advance beyond the scope of the activity, additional challenges are provided.

Note

Differentiation strategies are provided that will ensure all students acquire the knowledge of the activity. These strategies provide flexibility within the lesson to allow for varying student acquisition and demonstration of learning. These strategies provide suggestions for all students, including those with learning strengths or learning gaps.

12. Differentiation Strategies

To assist all students, instructional strategies are provided that benefit the full range of learners.

13. Grouping Strategies

Suggestions appear to help chunk each activity into manageable pieces and establish the cadence of the lesson.

Learning is social. Whether students work in pairs or in groups, the critical element is that they are engaged in discussion. Carnegie Learning believes, and research supports, that student-to-student discourse is a motivating factor; it increases student learning and supports ongoing formative assessment. Additionally, it provides students with opportunities to have mathematical authority.

Working collaboratively can, when done well, encourage students to articulate their thinking (resulting in self-explanation) and also provides metacognitive feedback (by reviewing other students' approaches and receiving feedback on your own).

The student discussion is then transported to a classroom discussion facilitated by the teacher

similarity relationships. Students identify a unit rate triangle and use it to show that the slope remains constant between any two points on the graph of the line. Connections are made between the steepness of the line (slope), the unit rate, the rate of change, and the constant of proportionality.

Note that the slope of the line is $\frac{3}{4}$. By analyzing the distance on the graph, the unit rate is $\frac{3}{4}$ units per second. The slope of the line is $\frac{3}{4}$.

Ask a student to read the information and definition following Question 7 aloud. Complete Question 8 as a class.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

12

- How is the constant of proportionality helpful when labeling the values on the horizontal and vertical sides of the right triangle?
- What are different interpretations of $\frac{3}{4}$ and $\frac{4}{3}$?
- Are the ratios equivalent? What does this imply?
- Is the unit rate associated with the value when $x = 1$?
- Is the unit rate the same between any two points on the line?
- Are the other rates the same between any two points on the line?

Ask a student to read the information and definition following Question 7 aloud. Complete Question 8 as a class.

Differentiation strategy

- To assist all students,
- Have students circle the terms in the first paragraph that are related: *proportional relationship, rate of change, constant of proportionality, and k.*

13

Activity 2.3 Equation for a Line Not Through the Origin



Facilitation Notes

In this activity, the scenario continues with Jack and Jill walking to the bus stop at the same rate, but this time they leave from a different location. Students compare the slopes and starting points of the two situations to conclude that the slopes are the same, but the starting points are different. They use patty paper to translate the original line of the form $y = mx$ by b units vertically to represent a new line of the form $y = mx + b$. Students generalize that an equation of the form $y = mx + b$ is a non-proportional linear relationship.

Ask a student to read the introduction aloud. Discuss the graph as a class.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Misconceptions

- As students are visualizing the scenario, they may think Aunt Mary's house is to the right of their home, leading them to think they need to start 10 units to the right on the x-axis. Remind them to check the axes labels to determine how to represent Aunt Mary's house on the graph.
- Students may overgeneralize and think that if there is a unit rate, then there is a proportional relationship. Take the point $(1, 11\frac{1}{3})$ from the table to explain the error in their thinking. From this single point, it looks as if Jack and Jill walked $11\frac{1}{3}$ yards in just one second. Even though we know Jack and Jill walk $\frac{4}{3}$ yards per second, it looks as if they also walked that original 10 yards within that one second time period; the original 10 yards affects the computation of the rate. Have students try another point, such as $(6, 18)$, to interpret. Because these rates are not the same, the table does not represent a proportional relationship. The calculation $\frac{18}{6}$ leads to inaccurate calculations of the rate if the relationship is not proportional. Discuss how a non-proportional relationship can be identified from a graph, equation, and situation, in addition to the table.

Differentiation strategy

To scaffold support, the equation $y = 10 + \frac{4}{3}x$ may make more sense because the starting point is written first. Accept this

to guarantee all necessary mathematics is addressed, once again, with the same benefits of discussion.

Alternative Grouping Strategies

Differentiation strategies will also provide other grouping strategies, such as whole class participation and the jigsaw method, are sometimes recommended for specific activities. These are listed as Differentiation Strategies.

More information about grouping strategies is available online in the Texas Support Center at www.CarnegieLearning.com/texas-help.

response; it is mathematically correct and reflects students' understanding of rate and starting position. Discuss what property demonstrates that $y = 10 + \frac{4}{3}x$ and $y = \frac{4}{3}x + 10$ are equivalent. Allow for flexibility in the formats of student responses.

Questions to ask

- What location or point on the graph represents Aunt Mary's house?
- Is the point (0, 10) or the point (0, 30) Aunt Mary's house?
- Is Aunt Mary's house located where on the graph did the line begin?
- Where on the graph does this line begin when they started from Aunt Mary's house?
- If the graph of a line does not pass through the origin, does the line represent a proportional relationship?
- Does the translation add 10 to each coordinate?
- Do translations preserve unit rates?
- Why can't the equation $y = \frac{4}{3}x + 10$ represent this situation?
- Can this equation be written in slope-intercept form? Why not?

Ask a student to read the information aloud. Complete Question 4 as a class.

15

Summary

When a proportional relationship $y = mx$ is translated vertically b units, the equation is $y = mx + b$, where m represents the slope and b represents the y -intercept.

Activity 2.4
A Negative Unit Rate

Facilitation Notes

In this activity, students compare the slopes of two lines representing the distance from the bus stop at a constant rate as they walk home from the bus stop at the same time. They use the same paper to investigate the slopes of the graph of the original line onto the graph of the translated line and conclude that the steepness of the

direction of the second graph decreases as the independent values increase. They then write an equation of the form $y = mx + b$, where m is negative, to represent the walk home from the bus stop.

Note that this activity is designed for students to understand the concept of slope as the direction (either increasing or decreasing from left to right) and steepness (the ratio of the change in vertical distance to the change in horizontal distance between any two points on the line) of a line.

Ask a student to read the introduction aloud. Discuss the graphs as a class.

Have students work with a partner or in a group to complete Questions 1 through 4. Students may try a variety of transformations. Remind them of the units that define each axis. To maintain that relationship, they should reflect the triangle across a horizontal line of reflection. Share responses as a class.

Questions to ask

- What location or point on the graph represents the bus stop?
- Is the point (0, 30) or the point (30, 0) the location of the bus stop?
- Is the bus stop located on the x - or y -axis?
- Where on the graph did the original line begin?
- Where on the graph does this line begin?
- Where on the graph did the line begin when they started from Aunt Mary's house?
- If the graph of a line does not pass through the origin, does the line represent a proportional relationship?
- Is the constant of proportionality the same for every set of points on the line?
- Is the line increasing or decreasing as you read the graph from left to right?
- Does the unit rate remain constant between any two points on the line?
- Does the translation add 30 to the x -value or y -value for each coordinate?
- Do translations preserve unit rates?

14

Misconception

Some students have difficulty understanding how you can tell whether a line is increasing or decreasing because it looks to them

14. Misconceptions

Common student misconceptions are provided in places where students may overgeneralize mathematical relationships or have confusion over the vocabulary used. Suggestions are provided to address the given misconception.

15. Summary

The summary brings the activity to closure. This statement encapsulates the big mathematical ideas of the particular activity.

16. As Students Work, Look For

These notes provide specific language, strategies, and/or errors to look and listen for you as you circulate and monitor students working in pairs or groups. You can incorporate these ideas when students share their responses with the class.

Note

Talk the Talk helps you to assess student learning and to make decisions about helpful connections you need to make in future lessons.

17. White Space

The white space in each margin is intentional. Use this space to make additional planning notes or to reflect on the implementation of the lesson.

16

As students work, look for

- Use of the number of spaces, rather than considering the scale on the axes, to determine the slope.
- Selection of one point (x, y) and the use of $\frac{y}{x}$ to determine the slope in non-proportional relationships.

Questions to ask

- What are the characteristics of a graph representing a proportional relationship?
- What are the characteristics of a graph representing a non-proportional relationship?
- Where is b represented in the graph?
- What is the meaning of b in the context?
- What is the slope of the line?
- How did you determine the slope of the line?
- What does x represent in this situation?
- What does y represent in this situation?

Have student

Questions 2

Differenti

- To sci
- of val
- Ques
- To ex
- "Two
- 3 and
- includ
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Questions

- How 1
- Does
- Why i
- value
- Why 2
- than p
- How i
- would
- How i
- starts

Summary

An equation relationship.

graph of a line with slope m that passes through the point $(0, 0)$. An equation of the form $y = mx + b$, where b is not equal to zero, represents a non-proportional relationship. This equation represents every point (x, y) on the graph of a line with slope m that passes through the point $(0, b)$.

Talk the Talk: A Web of Connections

DEMONSTRATE

Facilitation Notes

In this activity, student use a graphic organizer to make connections between the steepness of the graph of a line, slope, rate of change, unit rate, and constant of proportionality.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- What are the characteristics of a graph representing a proportional relationship?
- What are the characteristics of a graph representing a non-proportional relationship?
- Does the equation $y = mx$ represent a proportional or non-proportional relationship?
- Does the equation $y = mx + b$ (where b does not equal zero) represent a proportional or non-proportional relationship?
- Can the slope of a proportional relationship have a negative value?
- Can the slope of a non-proportional relationship have a negative value?
- How is the unit rate related to the unit rate triangle?
- Is the rate of change always equal to the slope?
- Is the constant of proportionality always equal to the rate of change?
- Is the unit rate always equal to the slope?
- What is the difference between a unit rate and a rate?

Summary

An equation of the form $y = mx$ represents every point (x, y) on the graph of a line with slope m . It describes a proportional relationship where the steepness of the graph, the slope, the rate of change, the unit rate, and the constant of proportionality are the same.

17

Supporting English Learners

English learners often face multiple challenges in the mathematics classroom beyond language development skills, including a lack of confidence, peer-to-peer understanding, and building solid conceptual mastery. The Carnegie Learning Texas Math Solution seeks to support English Learners (ELs) as they develop skills in both mathematics and language.

Answers

- Figure A: $\frac{10}{10}$;
Figure B: $\frac{15}{4}$;
Figure C: $\frac{15}{15}$;
Figure D: $\frac{12}{3}$
- Figure A: 1;
Figure B: 3.75;
Figure C: 1;
Figure D: 4
- Sample answer.
Figure A and Figure C have the same steepness. Figure D is the steepest triangle.

Getting Started

Let It Steep

Examine each triangle shown.

Figure A

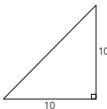


Figure B



Figure C

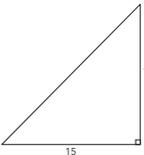


Figure D



- For each triangle, write a ratio that represents the relationship between the height and the base of each triangle.
- Write each ratio as a unit rate.
- How can you use these rates to compare the steepness of the triangles?

2 • TOPIC 1: From Proportions to Linear Relationships

Throughout instruction, EL tips are placed for teachers at point-of-use on the mini-lesson page in the TIG. They provide additional modifications to support this special population.

These tips:

- Inform teachers of potential learning obstacles specific to the lesson.
- Provide engaging activities for learning and assessment.
- Reinforce newly acquired mathematic language to gain an increasing level of comprehension of English.
- Introduce students to language needed to understand a specific context.

Students internalize new content language by using and reusing it in meaningful ways in a variety of different speaking activities that build concept and language attainment.

For More Support

Visit the Texas Support Center for many more resources to support you and your students who are English Learners.



Assessments

Formative assessment tools are provided throughout each lesson and workspace, providing you with ongoing feedback of student performance and encouraging students to monitor their own progress. End of Topic summative assessments are provided to measure student performance on a clearly denoted set of standards. For certain Topics that extend longer than four instructional weeks, a standardized Mid-Topic Assessment is also provided.

End of Topic Assessment

Multiple choice questions help students prepare for standardized tests. All items are multiple choice.

RIGID MOTION TRANSFORMATIONS

End of Topic Assessment

Name _____ Date _____

1. Logan drew $\triangle ABC$ on the coordinate plane, and then reflected the triangle over the y -axis to form $\triangle A'B'C'$. Which statement is **NOT** true about these two triangles?

- a. $\triangle ABC \cong \triangle A'B'C'$
- b. The two triangles have the same angle measures.
- c. The vertices of $\triangle ABC$ and $\triangle A'B'C'$ have the same coordinates.
- d. The triangles have the same side lengths.

2. Blake drew square $ABCD$. Then, he drew the image of it, square $A'B'C'D'$, 2 centimeters to the right of the original figure. Line segment BC is 3 centimeters. How long is line segment $B'C'$?

- a. 1 cm
- b. 3 cm
- c. 5 cm
- d. 6 cm

3. Dianne drew a triangle with coordinates $(1, 3)$, $(3, 2)$, and $(4, 2)$. She drew an image of the triangle with coordinates $(-1, 3)$, $(-3, 2)$, and $(-4, 2)$. How did she make the image?

- a. $(x, y) \rightarrow (x, y - 2)$
- b. $(x, y) \rightarrow (x, y - 6)$
- c. $(x, y) \rightarrow (-x, y)$
- d. $(x, y) \rightarrow (x, -y)$

4. Regina drew a triangle with vertices at $(1, 2)$, $(3, 3)$, and $(4, 1)$. She slides the triangle 2 units down to create an image. What are the vertices of the image?

- a. $(1, 0)$, $(3, 1)$, and $(4, -1)$
- b. $(1, 4)$, $(3, 5)$, and $(4, 3)$
- c. $(-1, 2)$, $(1, 3)$, and $(2, 1)$
- d. $(3, 2)$, $(5, 3)$, and $(6, 1)$

RIGID MOTION TRANSFORMATIONS: Standardized Test • 1

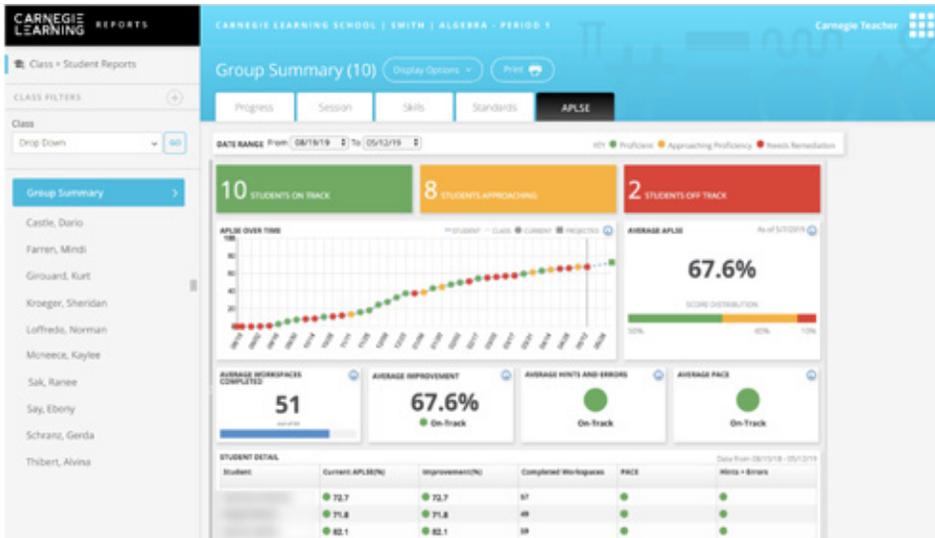


Supporting Students to Use Mathematical Tools

Visit the Texas Support Center for strategies to support students as they use mathematical tools, including formula charts and reference sheets.

Assessing Student Learning in MATHia

MATHia provides easy-to-use reports for you to have insight into your class and individual student’s progress. Data from these reports create action—whether determining how many students are mastering standards, to grouping your students into smaller learning groups, and teacher-student conferencing.

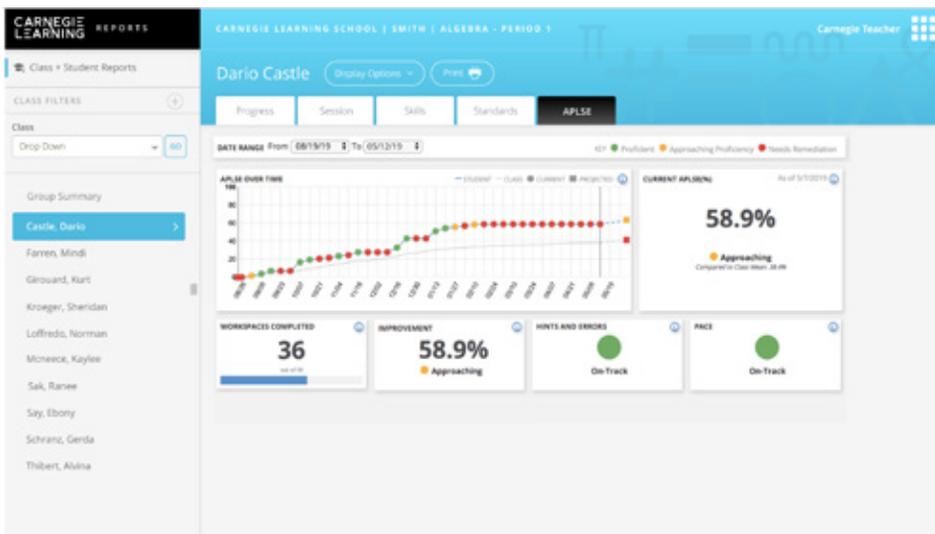


APLSE

The Adaptive Personalized Learning Score (APLSE) Report is a predictive report that displays class and student progress over time. The APLSE Report takes all aspects of a class or student’s work into consideration and provides each class and student with an APLSE Score.

Class View

The class view of the APLSE Report provides insight into the current overall progress of the entire class as well as the current projection to year-end performance



Student View

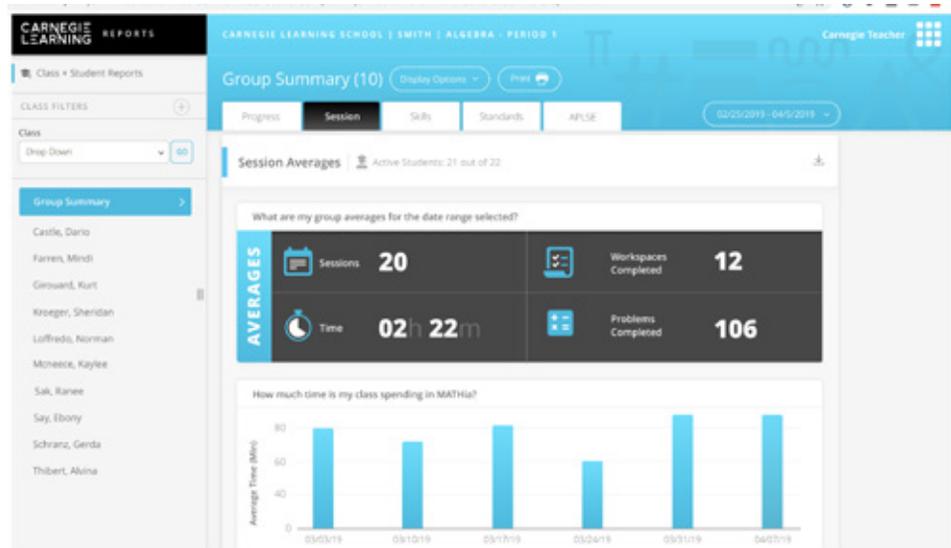
The student view of the APLSE Report displays the student’s current APLSE Score, and whether or not the student is on track to complete the curriculum by the end of the class.

Session Report

The Session Report is designed to give you a day-to-day view of work being completed by students.

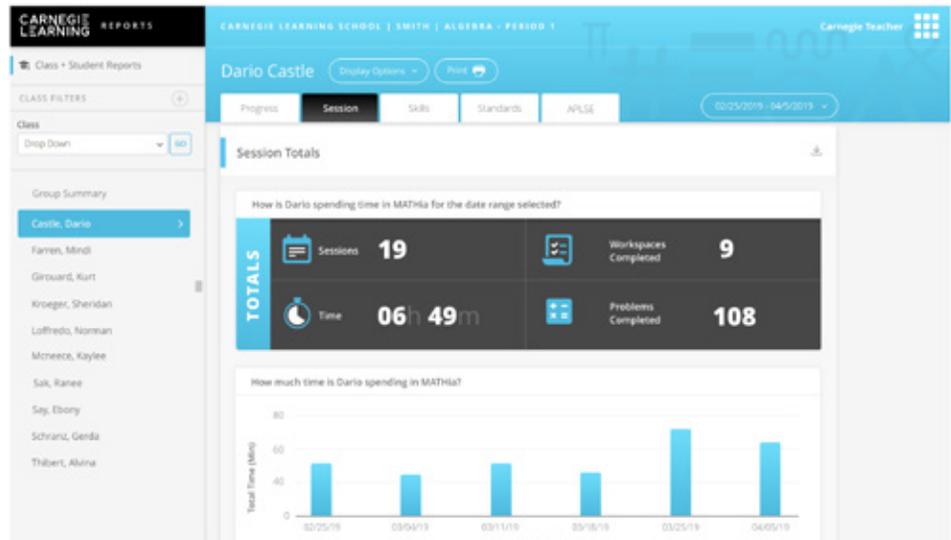
Class View

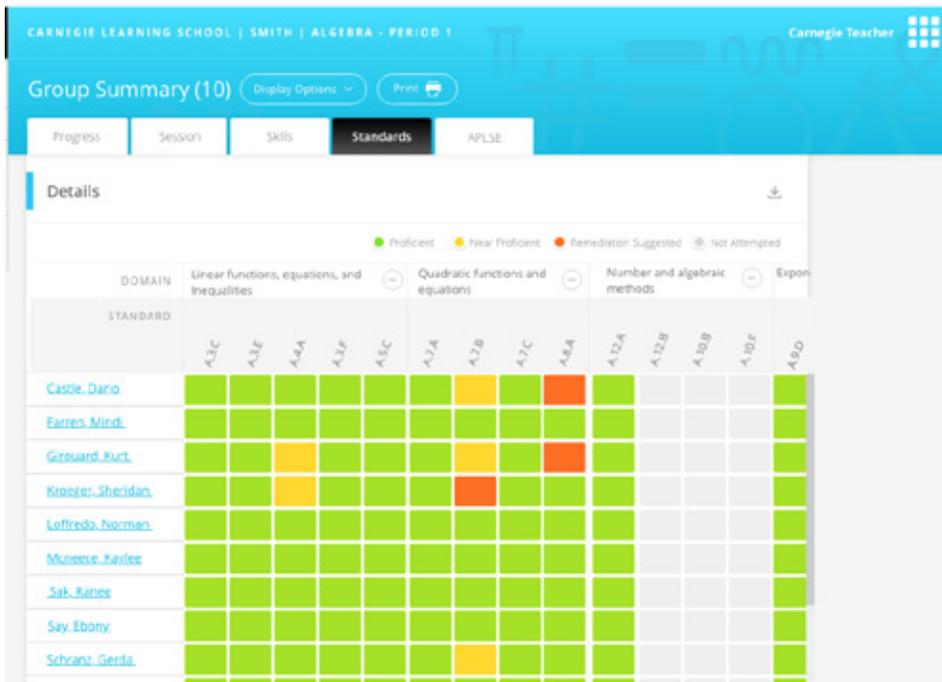
The class view of this report gives you a clear view of student work completed during a single class period, a week in the lab, or up to a five-week stretch.



Student View

All the metrics from the Class Session Report are the same for the Student Session Report, except instead of class averages, you see actual individual student metrics for the selected date range.



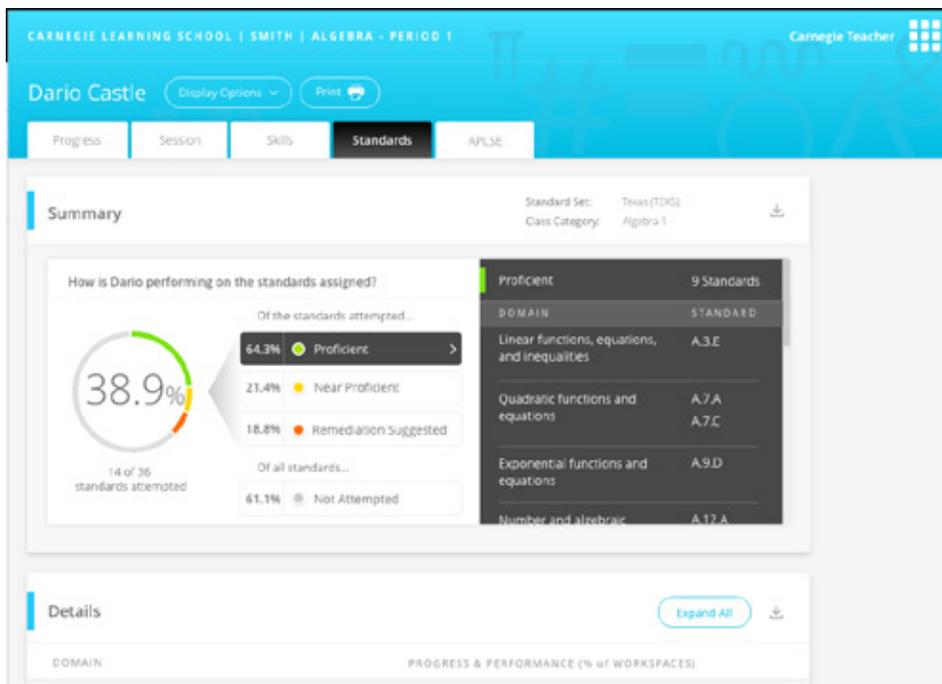


Standards Report

The Standards Report is designed to provide an easy view into how well students are mastering, or have mastered, specific standards.

Class View

The class view of the Standards Report displays summary-level data for progress and performance on the standards assigned in the curriculum.



Student View

The Student Standards Report displays progress and performance data on the standards assigned in the curriculum.

Skills Report

The Skills Report is designed to monitor skill proficiency. It provides detailed information about each student's skill mastery progress organized by module, unit and workspace.

Class View

The class summary view of the Skills Report enables grouping students according to their skill proficiency level and quickly identifying skills that need additional support or remediation.

Student View

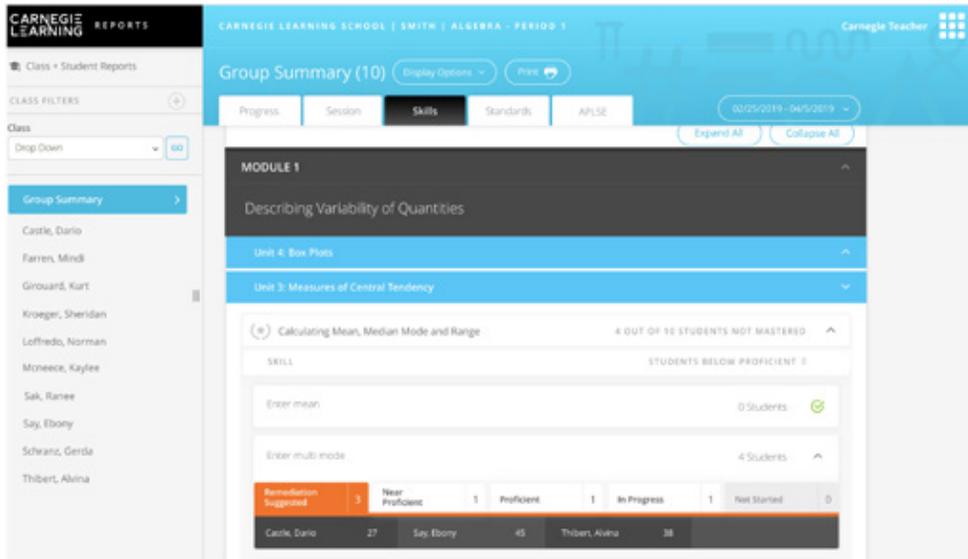
The student view of the Skills Report is designed to monitor skill proficiency for an individual student. All of the metrics from the Group Skills Report are the same for the Student Skills Report, except it shows the individual student skill proficiency for each skill within a workspace.

The screenshot shows the 'Group Summary (10)' view in the Carnegie Learning Reports interface. The interface includes a sidebar with a list of student names: Castle, Dario; Farren, Mindi; Girouard, Kurt; Kroeger, Sheridan; Loffredo, Norman; Monecke, Kaylee; Sak, Rancee; Say, Ebony; Schriani, Gerda; and Thibert, Alvina. The main content area displays 'MODULE 1: Describing Variability of Quantities' with sub-sections for 'Unit 4: Box Plots' and 'Unit 3: Measures of Central Tendency'. A specific skill, 'Calculating Mean, Median Mode and Range', is highlighted, showing that 4 out of 10 students did not master it. Below this, there are input fields for 'Enter mean' (0 Students) and 'Enter multi mode' (4 Students). A proficiency summary table is shown below:

Remediation Suggested	Near Proficient	Proficient	In Progress	Not Started
1	1	1	1	0

At the bottom, a row of student data is visible: Castle, Dario (27); Say, Ebony (45); Thibert, Alvina (38).

The screenshot shows the 'Student View' for Dario Castle in the Carnegie Learning Reports interface. The sidebar lists the same student names as the Group Summary view, with 'Castle, Dario' selected. The main content area displays 'Skill Proficiency Summary' for 'MODULE 1: Describing Variability of Quantities'. It shows 'Unit 4: Box Plots' and 'Unit 3: Measures of Central Tendency'. A specific skill, 'Calculating Mean, Median Mode and Range', is highlighted, showing that 1 of 4 skills was not mastered. Below this, there are input fields for 'Enter mean' (Proficient), 'Enter multi mode' (Remediation Suggested, 55), and 'Enter single mode' (Near Proficient, 76).

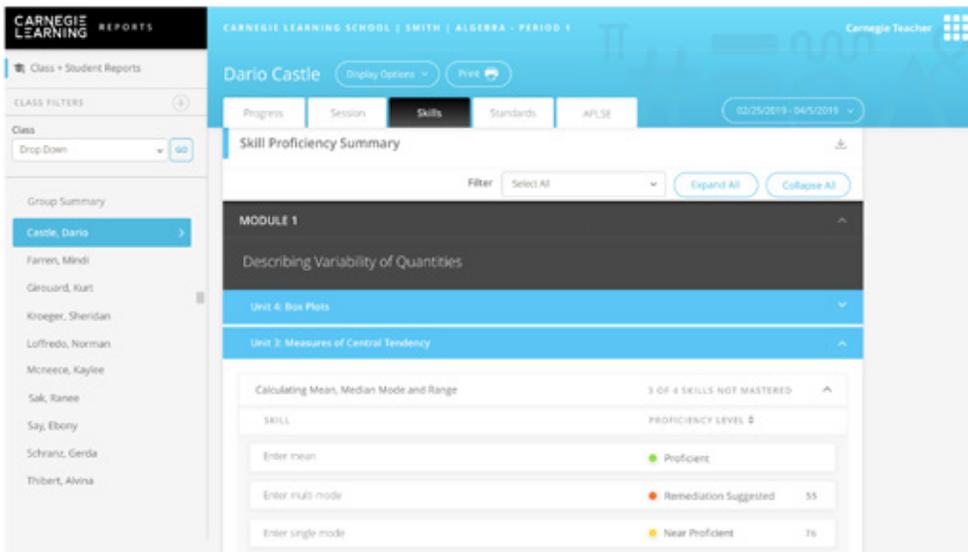


Progress Report

The Student Detail Report provides detailed information about the class and student progress and performance at the module, unit, and workspace levels in MATHia.

Class View

The class summary view of the Progress Report monitors class-level progress through the software. The data shows the current module placement for all students in the class, displaying totals for percentage of the syllabus completed, time spent on task, and completed modules, units, and workspaces.



Student View

The Progress Report monitors student progress and efforts in very specific content areas. The report identifies student progress across the entire syllabus, including syllabus, module, unit, and workspace completion status, total time spent in each unit, and performance scores for each completed workspace.

Reporting Scenarios

Additional reports are available. The full set of MATHia reports are located in the Teacher's Toolkit in MyCL.

Each time students log in to MATHia, each student's data is constantly recorded and assessed while the software is also adapting programmatically to the mastery level of each individual student. You can use our reporting system to continually assess this progress and use the results to create individualized, data-driven learning plans.

The table shown describes how MATHia reports can be used at the individual student or class level.

If you would like to then, run this report:	Class or Student View
Identify current student placement in a class	Student Detail Report	Class View
Prepare for parent conferences or IEP meetings	APLSE Progress Report or Student Detail Report	Student View
Locate class-level summary data helpful for grading	APLSE Progress Report	Student View
Group students according to standards progress	Standards Report	Class View
Summarize class progress in the curriculum	Student Detail Report	Class View
View a summary of how a student is progressing in the software	Student Detail Report	Student View
Identify a student's most recent session	Session Report	Student View
Summarize student usage data	Session Report or Student Detail Report	Student View

Carnegie Learning recognizes that it is the classroom teachers who make the material come alive for students, transforming the way math is taught. Implementation requires integrating learning together and learning individually.

Prepare for Learning Together

The most important first step you can take in preparing to teach with these instructional materials is to become comfortable with the mathematics.

- Read through the Module 1 Overview and the Topic 1 Overview.
- Do the math of the first Topic, and consider the facilitation notes.
- Prepare team building activities to intentionally create a student-centered environment.

Prepare for Learning Individually

Plan how you will introduce students to MATHia. Explain to them the benefits of working individually and why practice is important.

- Test out the computers or tablets that your students will be using.
- Set up classes in Teacher’s Toolkit.
- Assign yourself to your class so you can work through the math, too.

Prepare for Connecting the Text and MATHia

Think about strategies to help students make connections between the two learning experiences.

- Structure both environments similarly (e.g., warm-up, student work time, and closure). Provide closure around the mathematical concepts encountered each day in either environment to ensure a smooth transition. Additionally, use this time to celebrate student successes.
- As students work in the textbook, specifically ask, “Remember doing this in MATHia?” or “How would you answer this in MATHia?”
- As students work on the software, specifically ask, “How did we solve this in the textbook?” or “Does this look similar to a problem that we’ve done in the textbook?”

PREPARE YOURSELF



Prepare the Environment

The classroom is often considered the third teacher. Consider how to create a learning environment that engages students and fosters a sense of ownership. The use of space in your classroom should be flexible and encourage open sharing of ideas. If you are in person, consider the following:

- Consider how your students are going to use the consumable book. It is the student’s record of their learning. Many teachers have students move an entire topic to a three-ring binder as opposed to carrying the entire book.
- Arrange your desks so students can talk and collaborate with each other.
- Prepare a toolkit for groups to use as they work together and share their reasoning (read the materials list in each Topic Overview).
- Consider where you will display student work, both complete and in-progress.
- Create a word wall of key terms used in the text and MATHia.

Prepare the Learners

If you expect students to work well together, they need to understand what it means to collaborate and how it will benefit them. It is important to establish classroom guidelines and structure groups to create a community of learners.

- Facilitate team building activities and encourage students to learn each others’ names.
- Set clear expectations for how the class will interact:
 - Their text is a record of their learning and is to be used as a reference for any assignments or tests you give.
 - They will be doing the thinking, talking, and writing in your classroom.
 - They will be working and sharing their strategies and reasoning with their peers.
 - Mistakes and struggles are normal and necessary.

Prepare the Support

- Prepare a letter to send home on the first day.
- Encourage guardians to read the introduction of the student book or visit our website at www.CarnegieLearning.com.
- Ensure that guardians receive the Family Guide at the start of the first topic and each subsequent topic.
- Consider a Family Math Night some time within the first few weeks of the school year.
- Encourage guardians to explore the Students & Caregivers Portal on the Texas Support Center at www.CarnegieLearning.com/texas-help.

Home Connection

Research has proven time and again that family engagement greatly improves a student's likelihood of success in school.



The Students & Caregivers Portal on the Texas Support Center provides:

- Getting to Know Carnegie Learning video content to provide an introduction to the instructional materials and research.
- Getting Started Guide with system requirements for MATHia.
- Articles and quick tip videos offering strategies for how guardians can support student learning. **Visit the Texas Support Center regularly to access new content and resources for students and caregivers as they learn mathematics in a variety of environments outside of the classroom.**



Family Guides

Each topic contains a Family Guide that overviews the mathematics of the topic, how that math is connected to what students already know, and how that knowledge will be used in future learning. It also may include an illustration of math from the real-world, a sample standardized test question, information to bust math myths, talking points or questions caregivers can use with their students, and a few of the key terms that students will learn.

We recognize that learning outside of the classroom is crucial to students' success at school. While we don't expect parents to be math teachers, the Family guides are designed to assist caregivers as they talk to their students about what they are learning. Our hope is that both the students and their parents will read and benefit from the guides.

Carnegie Learning Family Guide Grade 8

Module 2: Developing Function Foundations

TOPIC 1: FROM PROPORTIONS TO LINEAR RELATIONSHIPS

In this topic, students build on their knowledge of ratio and proportional relationships to develop connections between proportional relationships, lines, and linear equations. Students compare proportional relationships represented in different ways to ensure a firm understanding of the meaning of proportionality. Students then use similar triangles to explain why the slope of a line is always the same between any two points on the line.

Where have we been?

In grade 6, students developed their understanding of ratios. The next year, they determined characteristics of scenarios, tables, graphs, and equations of proportional relationships. Students review their prior knowledge of ratios and proportional relationships, including unit rate and the constant of proportionality.

Where are we going?

This topic establishes an important link from a major concept of middle school mathematics, ratios and proportional relationships, to a major focus of high school mathematics, functions. In the next topic, students will increase their familiarity and flexibility with determining slope and writing equations of linear relationships from different representations and in different forms.

Using Graphs to Show Proportional and Non-Proportional Relationships

Both of these graphs show linear relationships between time and distance. They both show speeds. The graph on the left shows a proportional linear relationship, because the graph is a straight line through the origin. The graph on the right shows a non-proportional relationship.

TOPIC 1: Family Guide • 1

Myth: There is one right way to do math problems.



Employing multiple strategies to arrive at a single, correct solution is important in life. Suppose you are driving in a crowded downtown area. If one road is backed up, then you can always take a different route. If you know only one route, then you're out of luck.

Learning mathematics is no different. There may only be one right answer, but there are often multiple strategies to arrive at that solution. Everyone should get in the habit of saying: *Well, that's one way to do it. Is there another way? What are the pros and cons? That way, you avoid falling into the trap of thinking there is only one right way because that strategy might not always work, or there might be a more efficient strategy.*

Teaching students multiple strategies is important. This helps students understand the benefits of the more efficient method. In addition, everyone has different experiences and preferences. What works for you might not work for someone else.

#mathmythbusted

Talking Points

You can further support your student's learning by asking them to take a step back and think about a different strategy when they are stuck.

Questions to Ask

- What strategy are you using?
- What is another way to solve the problem?
- Can you draw a model?
- Can you come back to this problem after doing some other problems?

Key Terms

constant of proportionality

In a proportional relationship, the ratio of all y -values to their corresponding x -values is constant. This ratio, $\frac{y}{x}$, is called the constant of proportionality.

slope

In any linear relationship, slope describes the direction and steepness of a line. In a proportional relationship, the constant of proportionality and the slope are the same.

2 • TOPIC 1: From Proportions to Linear Relationships

You Might Be Wondering ...

We're here for you.

The Carnegie Learning Texas Support Team is available to help with any issue at texashelp@carnegielearning.com.

Monday–Friday
8:00 am–8:00 pm CT
via email, phone, or
live chat.

Our expert team provides support for installations, networking, and technical issues, and can also help with general questions related to pedagogy, classroom management, content, and curricula.

Why are the student books consumable?

The Student Edition contains all of the resources students need to complete the course. Students are to actively engage in this textbook, topic by topic, creating a record of their learning as they go. There is room to record answers, take notes, draw diagrams, and fix mistakes.

Why do we believe in our brand of blended: Learning Together and Learning Individually?

There has been a lot of research on the benefits of learning collaboratively. Independent practice is necessary for students to become fluent and automatic in a skill. A balance of these two pieces provides students with the opportunity to develop a deep conceptual understanding through collaboration with their peers, while demonstrating their understanding independently.

Why don't we have a Worked Example at the start of every lesson?

Throughout the Texas Math Solution, we do provide Worked Examples. Sweller and Cooper (1985) argue that Worked Examples are educationally efficient because they reduce working memory load. Ward and Sweller (1990) found that alternating between problem solving and viewing Worked Examples led to the best learning. Students often read Worked Examples with the intent to confirm that they understand the individual steps. However, the educational value of the Worked Example often lies in thinking about how the steps connect to each other and how particular steps might be added, omitted or changed, depending on context.

Where are the colorful graphics to get students' attention?

Color and visuals make for stronger student engagement, right? Not quite. Our instructional materials have little extraneous material. This approach follows from research showing that “seductive details” used to spice up the presentation of material often have a negative effect on student learning (Mayer et al., 2001; Harp & Meyer, 1998). Students may not know which elements of an instructional presentation are essential and which are intended simply to provide visual interest. So, we focus on the essential materials. While we strive to make our educational materials attractive and engaging to students, research shows that only engagement based on the mathematical content leads to learning.

