Patty Paper, Patty Paper

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Introduction to Congruent Figures

Lesson Overview

Students use patty paper to indirectly measure segments and angles and use folds to make observations about a figure. They determine if figures are the same size and shape. The term *congruent figures* is defined. Students use patty paper to determine if figures are congruent. They then make conjectures about congruence, investigate their conjectures, and justify their conjectures using informal transformation language.

Grade 8
Two-Dimensional Shapes
(10) The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

> (A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane.

ELPS

1.A, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E

Essential Ideas

- Patty paper is a hands-on tool for investigating geometric ideas.
- If two figures are congruent figures, all corresponding sides and all corresponding angles have the same measure.
- Corresponding sides are sides that have the same relative position in geometric figures.
- Corresponding angles are angles that have the same relative position in geometric figures.
- A rigorous study of geometry requires making conjectures, investigating conjectures, and justifying true results.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: It's Transparent!
Students recall geometry vocabulary and notation as they describe a provided shape. They use patty paper to investigate and verify characteristics of the shape. One goal of this activity is to familiarize students with patty paper.

Develop

Activity 1.1: Analyzing Size and Shape
Students sort figures into categories based on their shapes and sizes. They distinguish between figures that have the same shape but not the same size and those that have both the same shape and size. The term *congruent figures* is defined.

Day 2

Activity 1.2: Congruent or Not?
Students learn about the geometric process of conjecture, investigate, and justify. They engage in this process as they decide which figures are congruent to other provided figures. Students make conjectures, investigate with patty paper, and then explain how they could slide, flip, or spin the original figure to obtain each congruent figure.

Demonstrate

Talk the Talk: The Core of Congruent Figures
Students revisit the definition of congruent figures. They use measuring tools to identify corresponding sides and corresponding angles of two congruent figures.

Facilitation Notes

In this activity, students recall geometry vocabulary and notation as they describe a provided shape. They use patty paper to investigate and verify characteristics of the shape. One goal of this activity is to familiarize students with patty paper if they have not used it in the past.

Provide students with patty paper to complete this activity.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Differentiation strategy

Suggest the use of more than one piece of patty paper when comparing measures.

Questions to ask

- What is a polygon?
- What is a regular polygon?
- Is this an example of a polygon?
- What is the name of this polygon based upon its number of sides?
- If the sides of a polygon are equal in length, are the interior angles of the polygon always equal in measure?
- What would a figure look like if it had congruent sides, but not congruent angles?
- If the interior angles of a polygon are equal in measure, are the length of the sides of the polygon always equal in length?
- Do the folds determine the center point of the polygon?
- Is the center point of the polygon equidistant from the sides of the polygon?

Summary

Patty paper can be used to determine characteristics of geometric shapes.

Activity 1.1
Analyzing Size and Shape

Facilitation Notes

In this activity, students sort figures into categories based on their shapes and sizes. They distinguish between figures that have the same shape but not the same size and those that have both the same shape and size. The term *congruent figures* is defined.

Have students work with partner or in a group to complete Questions 1 through 4. Share responses as a class.

Questions to ask
- What do you know about geometric figures that are congruent?
- What do you know about geometric figures that are similar?
- What is the difference between congruent geometric figures and similar geometric figures?
- What do congruent geometric figures have in common with similar geometric figures?
- Are congruent figures also considered to be similar figures?
- How can patty paper be used to determine congruent geometric figures?
- How can patty paper be used to determine similar geometric figures?
- What is another method that can be used to determine congruent figures?

Differentiation strategy
Have students trace figure(s) onto patty paper, rather than cutting them out.

Misconceptions
- Students may think that if figures are the same shape but not the same size that they are not truly the same shape. You can use a triangle or square drawn with technology (word document, interactive white board, etc.) and show that these figures can be made larger or smaller and still have the original shape.
- Students may overgeneralize and think that two figures have the same shape as long as they are named the same. Disprove their thinking by showing examples of rectangles that have different shapes.

Summary

The corresponding angles and corresponding sides of congruent figures are congruent.

Activity 1.2
Congruent or Not?

Facilitation Notes

In this activity, students learn about the geometric process of conjecture, investigate, and justify. They engage in this process as they decide which figures are congruent to other provided figures. Students make conjectures, investigate with patty paper, and then explain how they could slide, flip, or spin the original figure to obtain each congruent figure.

Provide students with patty paper to complete this activity.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask
- Which figures are congruent to the original flower?
- Which figures are similar to the original flower?
- Which figures are smaller than the original flower?
- Which figures are larger than the original flower?
- Which figures required a slide to determine congruency?
- Which figures required a flip to determine congruency?
- Which figures required a spin to determine congruency?
- Could more than one process (slide, flip or spin) be used to determine congruency?

Summary

Geometric figures are congruent if you can obtain one figure by a combination of sliding, flipping, and spinning the figure until it lies on the other figure.

Talk the Talk: The Core of Congruent Figures

Facilitation Notes

In this activity, students revisit the definition of congruent figures. They use measuring tools to identify corresponding sides and corresponding angles of two congruent figures.

The focus in this activity is not writing correspondence statements or on notation, but rather for students to better understand the full meaning of congruent figures. Writing congruence statements using proper notation will come later.

Have students work with partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask
- Angle *A* corresponds to which angle in Figure *RSTUV*?
- Side *VU* corresponds to which side in Figure *ABCDE*?
- Is there more than one way to obtain the figure on the right?

Summary

Patty paper is a construction tool that can be used to determine the congruence of geometric figures.

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Introduction to Congruent Figures

WARM UP

Draw an example of each shape.

1. parallelogram

2. trapezoid

3. pentagon

4. regular hexagon

LEARNING GOALS

- Define congruent figures.
- Use patty paper to verify experimentally that two figures are congruent by obtaining the second figure from the first using a sequence of slides, flips, and/or turns.
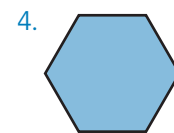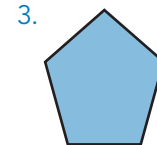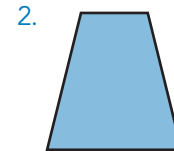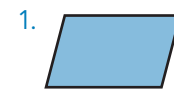- Use patty paper to determine if two figures are congruent.

KEY TERMS

- congruent figures
- corresponding sides
- corresponding angles

You have studied figures that have the same shape or measure. How do you determine if two figures have the same size and the same shape?

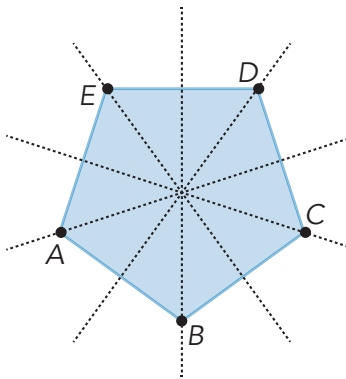## Warm Up Answers

Answers will vary.

1.

2.

3.

4.

ELL Tip

Provide students with a three-column chart at the beginning of the lesson. Have students add one Key Term to each of the columns at the top of the paper. While going through the lesson, stop at different times to allow students to add information they find in the text to their chart. At the end of the lesson, have students share what pieces of information they added to the chart.
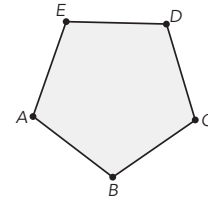
Getting Started

It's Transparent!

Let's use patty paper to investigate the figure shown.

1. **List everything you know about the shape.**

Patty paper is great paper to investigate geometric properties. You can write on it, trace with it, and see creases when you fold it.

> Patty paper was originally created for separating patties of meat! Little did the inventors know that it could also serve as a powerful geometric tool.

2. **Use patty paper to compare the sizes of the sides and angles in the figure.**

    a. **What do you notice about the side lengths?**

    b. **What do you notice about the angle measures?**

    c. **What can you say about the figure based on this investigation?**

Trace the polygon onto a sheet of patty paper.

3. **Use five folds of your patty paper to determine the center of each side of the shape. What do you notice about where the folds intersect?**

ACTIVITY 1.1

Analyzing Size and Shape

Cut out each of the figures provided at the end of the lesson.

1. **Sort the figures into at least two categories. Provide a rationale for your classification. List your categories and the letters of the figures that belong in each category.**

Figures with the same shape but not necessarily the same size are *similar figures*, which you will study in later lessons.

2. **List the figures that are the same shape as Figure A. How do you know they are the same shape?**

3. **List the figures that are both the same shape and the same size as Figure A. How do you know they are the same shape and same size?**

Figures that have the same size and shape are **congruent figures**. If two figures are congruent, all *corresponding sides* and all *corresponding angles* have the same measure.

4. **List the figures that are congruent to Figure C.**

Corresponding sides are sides that have the same relative position in geometric figures.

Corresponding angles are angles that have the same relative position in geometric figures.

Answers

1. Answers will vary. Students may sort the by general shape. Boot-like shape: *A*, *B*, *D*, *E*, *F*, *H*, and *J* and *Z*-like shape: *C*, *G*, *I*, *K*, and *L*. They may also sort by relative size. The figures that are all about the same size are *A*, *C*, *E*, *F*, *G*, *H*, *J*, and *L*, and the smaller shapes are *B*, *D*, *I*, and *K*.

2. The figures with the same shape as Figure *A* are *B*, *D*, *E*, *F*, *H*, and *J*. Explanations may vary.

3. The figures with the same size and shape as Figure *A* are *E*, *F*, *H*, and *J*. Explanations may vary.

4. The figures congruent to Figure *C* are *G* and *L*.

Answers

1. Answers may vary. Sample answers are provided in the table below.

> *A conjecture is a hypothesis or educated guess that is consistent with what you know but hasn't yet been verified.*
>
> *Persevering through multiple conjectures and investigations is an important part of learning in mathematics.*

ACTIVITY 1.2

Congruent or Not?

Throughout the study of geometry, as you reason about relationships, study how figures change under specific conditions, and generalize patterns, you will engage in the geometric process of

- making a conjecture about what you think is true,
- investigating to confirm or refute your conjecture, and
- justifying the geometric idea.

In many cases, you will need to make and investigate conjectures a few times before reaching a true result that can be justified.
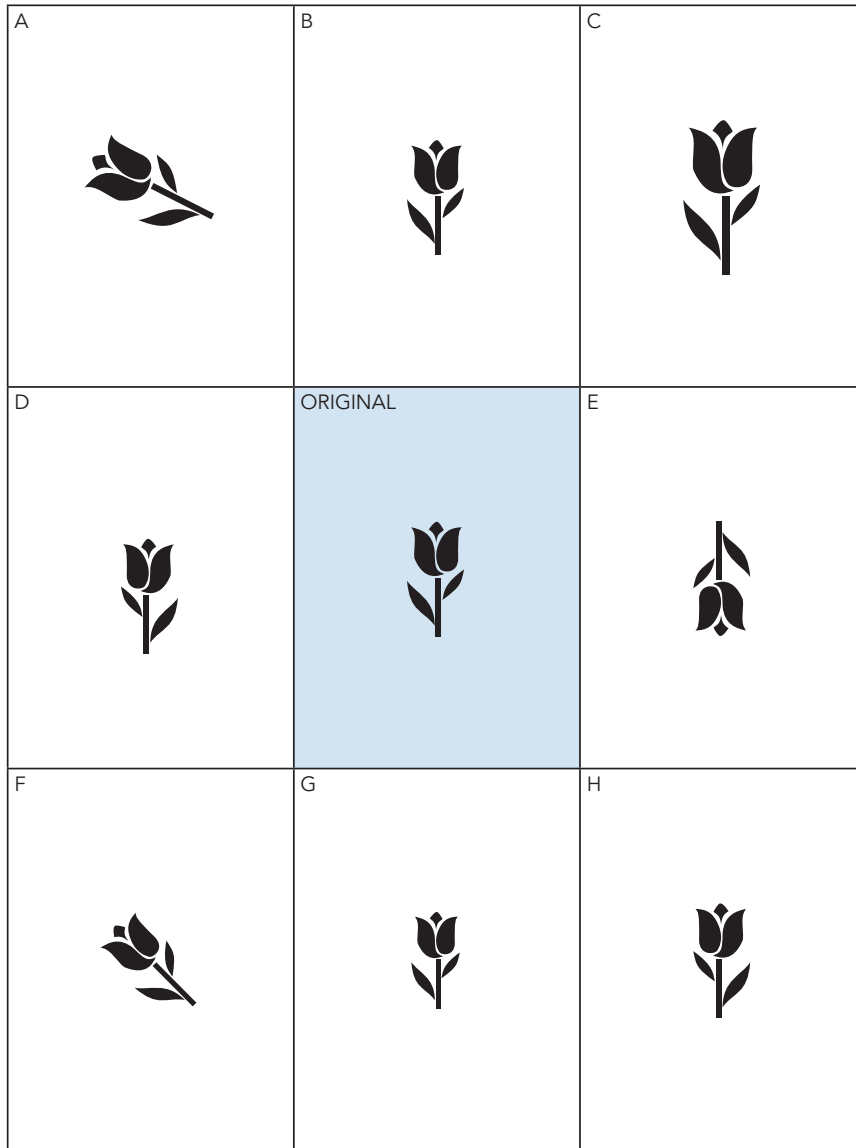
Let's use this process to investigate congruent figures.

If two figures are congruent, you can slide, flip, and spin one figure until it lies on the other figure.

1. Consider the flowers shown following the table. For each flower, make a conjecture about which are congruent to the original flower, which is shaded in the center. Then, use patty paper to investigate your conjecture. Finally, justify your conjecture by stating how you can move from the shaded flower to each congruent flower by sliding, flipping, or spinning the original flower.

Flower	Congruent to Original Flower	How Do You Move the Original Flower onto the Congruent Flower?
A		
B		
C		
D		
E		
F		
G		
H		

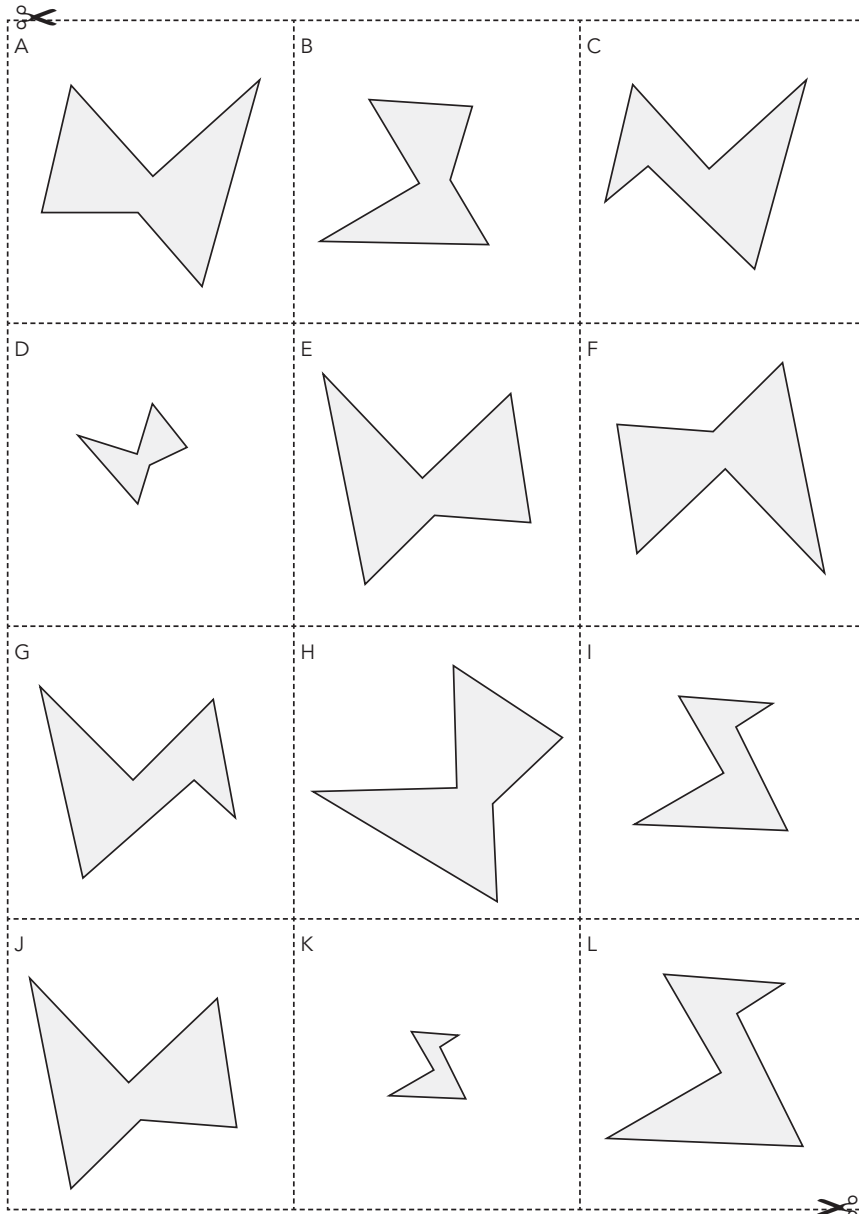| Figure	Congruent to Original Flower	How Do You Move the Original Flower onto the Congruent Figure?
A	No	
B	Yes	Slide original figure up to Figure B.
C	No	
D	Yes	Flip the original figure over a line that is halfway between the original figure and Figure D.
E	Yes	Spin original figure 180° and slide right to Figure E.
F	Yes	Spin the original figure 45 degrees counterclockwise. Slide left to Figure D and down to Figure F.
G	No	
H	Yes	Flip the original figure over a line that is halfway between the original figure and Figure E. Slide down to Figure H.

NOTES

TALK the TALK 💬

The Core of Congruent Figures

Recall that if two figures are congruent, all corresponding sides and all corresponding angles have the same measure.

1. Use patty paper to determine which sides of the congruent figures are corresponding and which angles are corresponding.
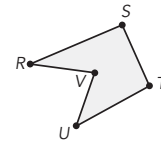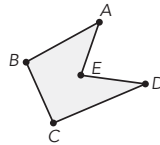
2. How can you slide, flip, or spin the figure on the left to obtain the figure on the right?

A

B

C

D

E

F

G

H

I

J

K

L