Slides, Flips, and Spins

2

Introduction to Rigid Motions

WARM UP

Draw all lines of symmetry for each letter.

1. A
2. B
3. H
4. X

LEARNING GOALS

- Model transformations of a plane.
- Translate geometric objects on the plane.
- Reflect geometric objects on the plane.
- Rotate geometric objects on the plane.
- Describe a single rigid motion that maps a figure onto a congruent figure.

KEY TERMS

- plane
- transformation
- rigid motion
- pre-image
- image
- translation
- reflection
- line of reflection
- rotation
- center of rotation
- angle of rotation

When you investigated shapes with patty paper, you used slides, flips, and spins to determine if shapes were congruent. What are the formal names for the actions used to carry a figure onto a congruent figure and what are the properties of those actions?

Design Competition

The Kensington Middle School track club is holding a 5K to raise money for new uniforms. They want to create a logo for the race that includes the running man icon. However, they want the logo to include at least four copies of the running man.

1. **Trace the running man onto a sheet of patty paper. Create a logo for the track team on another sheet of patty paper that includes the original running man and three copies, one example each of sliding, flipping, and spinning the picture of the running man.**

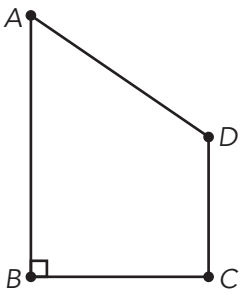
2. **What do you know about the copies of the running man compared with the original picture of the running man?**

Each sheet of patty paper represents a model of a geometric *plane*. A **plane** extends infinitely in all directions in two dimensions and has no thickness.

ACTIVITY
2.1 Translations on the Plane

In this module, you will explore different ways to transform, or change, planes and figures on planes. A **transformation** is the mapping, or movement, of a plane and all the points of a figure on a plane according to a common action or operation. A **rigid motion** is a special type of transformation that preserves the size and shape of the figure. Each of the actions you used to make the running man logo—slide, flip, spin—is a rigid motion transformation.

You are going to start by exploring translations on the plane using the trapezoid shown. Trapezoid *ABCD* has angles *A*, *B*, *C*, and *D*, and sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$.



1. What else do you know about Trapezoid *ABCD*?

2. Use the Translations Mat at the end of the lesson for this exploration.

a. Use a straightedge to trace the trapezoid on the shiny side of a sheet of patty paper.

b. Slide the patty paper containing the trapezoid to align $\overline{AB}$ with one of the segments $\overline{A'B'}$.

c. Record the location of the *image* of Trapezoid *ABCD* on the mat. This image is called Trapezoid *A'B'C'D'*.

Once you have traced the trapezoid on one side, turn the patty paper over and, using a pencil, copy the lines on the back side as well. This will help you to transfer the translated trapezoid back onto the Translations Mat.

$\overline{AB}$ is read, "line segment *AB*."
A' is read, "*A* prime."

The original trapezoid on the mat is called the **pre-image**.

The traced trapezoid is the **image**. It is the new figure that results from the transformation.

3. Examine your pre-image and image.

a. Which angle in Trapezoid *ABCD* maps to each angle of Trapezoid *A'B'C'D'*? Label the vertices on your drawing of the image of Trapezoid *ABCD*.

b. Which side of Trapezoid *ABCD* maps to each side of Trapezoid *A'B'C'D'*?

c. What do you notice about the measures of the corresponding angles in the pre-image and the image?

d. What do you notice about the lengths of the corresponding sides in the pre-image and the image?

e. What do you notice about the relationship of $\overline{A'B'}$ to $\overline{C'D'}$? How does this relate to the corresponding sides of the pre-image?

f. Is the image congruent to the pre-image? Explain your reasoning.

This type of movement of a plane containing a figure is called a *translation*. A **translation** is a rigid motion transformation that "slides" each point of a figure the same distance and direction. Let's verify this definition.

4. On the mat, draw segments to connect corresponding vertices of the pre-image and image.

a. Use a ruler to measure each segment. What do you notice?

b. Compare your translations and measures with your classmates' translations and measures. What do you notice?

5. Consider the translation you created, as well as your classmates' translations.

a. What changes about a figure after a translation?

b. What stays the same about a figure after a translation?

c. What information do you need to perform a translation?

A figure can be translated in any direction. Two special translations are vertical and horizontal translations. Sliding a figure only left or right is a horizontal translation, and sliding it only up or down is a vertical translation.

NOTES

ACTIVITY 2.2

Reflections on the Plane

The first transformation you explored was a translation. Now, let's see what happens when you flip, or reflect, the trapezoid. Trace Trapezoid *ABCD* onto a sheet of patty paper. Imagine tracing the trapezoid on one side of the patty paper, folding the patty paper in half, and tracing the trapezoid on the other half of the patty paper.

1. **Make a conjecture about how the image and pre-image will be alike and different.**

To verify or refine your conjecture, let's explore a reflection using patty paper and the Reflections Mat located at the end of the lesson. Trace the trapezoid from the previous activity on the lower left corner of a new piece of patty paper.

2. **Align the trapezoid on the patty paper with the trapezoid on the Reflections Mat. Fold the patty paper along ℓ_1. Trace the trapezoid on the other side of the crease and transfer it onto the Reflections Mat. Label the vertices of the image, Trapezoid *A'B'C'D'*.**

3. **Compare the pre-image and image that you created.**

   a. **What do you notice about the measures of the corresponding angles in the pre-image and the image?**

   b. **What do you notice about the lengths of the corresponding sides in the pre-image and the image?**

c. **What do you notice about the relationship of $\overline{A'B'}$ to $\overline{C'D'}$? How does this relate to the corresponding sides of the pre-image?**

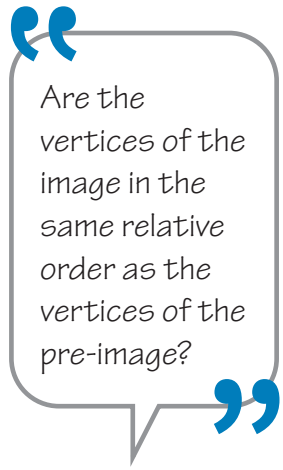
d. **Is the image congruent to the pre-image? Explain your reasoning.**

e. **Draw segments connecting corresponding vertices of the pre-image and image. Measure the lengths of these segments and the distance from each vertex to the fold. What do you notice?**

Notice that the segments you drew are perpendicular to the crease of the patty paper. Why do you think this is true?

4. **Repeat the reflection investigation using Trapezoid *ABCD* and folding along ℓ_2. Record your observations.**

5. **Repeat the reflection investigation using Trapezoid *ABCD* and folding along ℓ_3. Record your observations.**



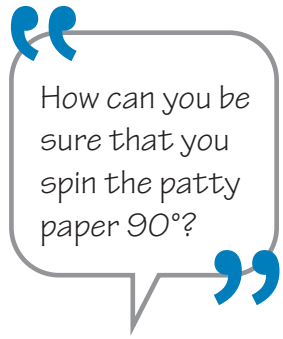
This type of movement of a plane containing a figure is called a *reflection*. A **reflection** is a rigid motion transformation that "flips" a figure across a *line of reflection*. A **line of reflection** is a line that acts as a mirror so that corresponding points are the same distance from the line.

6. Consider the reflections you created.

a. What changes about a figure after a reflection?

b. What stays the same about a figure after a reflection?

c. What information do you need to perform a reflection?

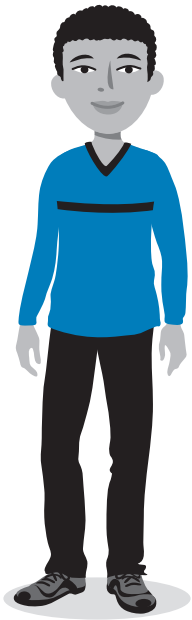


ACTIVITY
2.3 Rotations on the Plane

You have now investigated translating and reflecting a trapezoid on the plane. Let's see what happens when you spin, or rotate, the trapezoid. You are going to use the Rotations Mat found at the end of the lesson for this investigation.

Trace Trapezoid *ABCD* onto the center of a sheet of patty paper. Imagine spinning the patty paper so that the trapezoid is no longer aligned with the trapezoid on the mat.

1. Make a conjecture about how the image and pre-image will be alike and different.



Let's investigate with patty paper to verify or refine your conjecture.

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2. Align your trapezoid with the trapezoid on the Rotations Mat.

Put your pencil on point O_1 and spin the patty paper 90° in a clockwise direction.

Then copy the rotated trapezoid onto the Rotations Mat and label the vertices.

3. Compare the pre-image and image created by the rotation.

a. What do you notice about the measures of the corresponding angles in the pre-image and the image?

b. What do you notice about the lengths of the corresponding sides in the pre-image and the image?

c. What do you notice about the relationship of $\overline{A'B'}$ to $\overline{C'D'}$? How does this relate to the corresponding sides of the pre-image?

d. Is the image congruent to the pre-image? Explain your reasoning.

4. Draw two segments: one to connect point O_1 to A and another to connect point O_1 to A'.

a. Measure the lengths of these segments. What do you notice?

b. Measure the angle formed by the segments. What do you notice?

5. **Repeat the process from the previous question with B and B'. What do you notice about the segment lengths and angle measures?**

6. **What do you think is true about the segments connecting C and C' and the segments connecting D and D'?**

7. **Repeat the rotations investigation using Trapezoid $ABCD$ and spinning the patty paper 90° in a counterclockwise direction around O_1. Record your observations.**

8. **Repeat the rotations investigation using Trapezoid $ABCD$ and spinning the patty paper 180° around O_3. Record your observations.**

This type of movement of a plane containing a figure is called a *rotation*. A **rotation** is a rigid motion transformation that turns a figure on a plane about a fixed point, called the **center of rotation**, through a given angle, called the **angle of rotation**. The center of rotation can be a point outside the figure, inside the figure, or on the figure.

Rotations can be clockwise or counterclockwise.

9. Consider the rotations you created.

a. Describe the centers of rotation used for each investigation.

b. How do you identify the angle of rotation, including the direction, in your patty paper rotations?

c. What changes about a figure after a rotation?

d. What stays the same about a figure after a rotation?

e. What information do you need to perform a rotation?

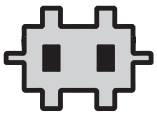
NOTES

ACTIVITY
2.4

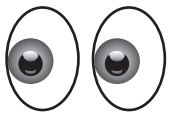
Rigid Motions on the Plane

Use your investigations about the properties of rigid motions to complete each transformation.

1. **Rotate Ali the Alien 180°. Be sure to identify your center of rotation.**



2. **Translate the googly eyes horizontally to the right.**



3. **Rotate the letter E 90° clockwise. Be sure to identify your center of rotation.**

E

4. **Transform the running man so that he is running in the opposite direction.**

NOTES

TALK the TALK

Congruence in Motion

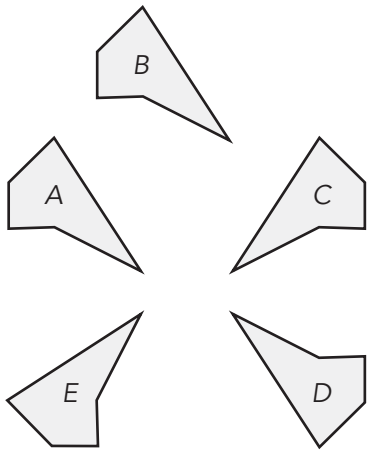
1. Describe a transformation that maps one figure onto the other. Be as specific as possible.

   a. Figure *A* onto Figure *B*

   b. Figure *A* onto Figure *C*

   c. Figure *A* onto Figure *E*

   d. Figure *C* onto Figure *D*



2. Explain what you know about the images that result from translating, reflecting, and rotating the same pre-image. How are the images related to each other and to the pre-image?

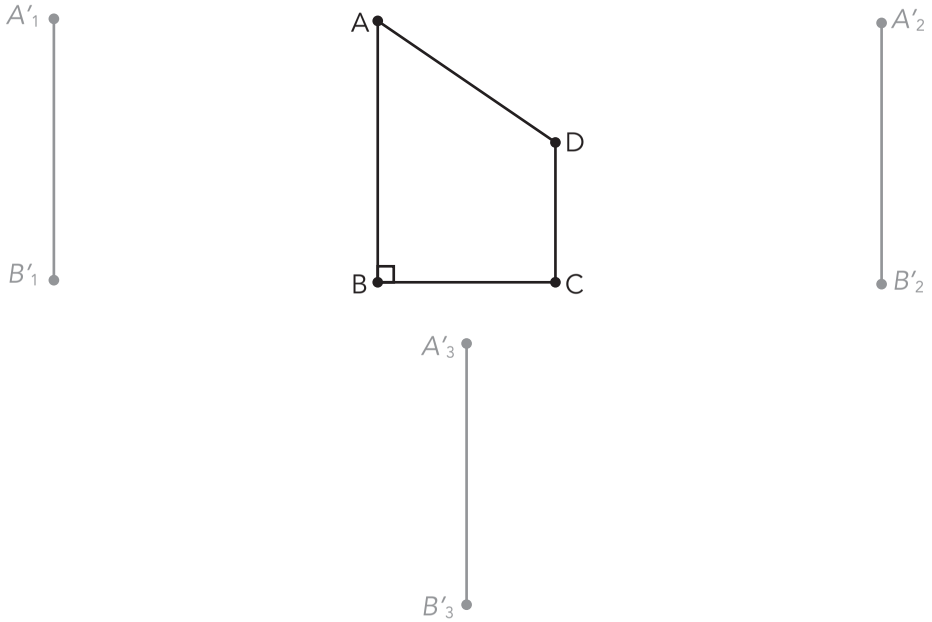
3. If Figure *A* is congruent to Figure *C* and Figure *C* is congruent to Figure *D*, answer each question.

   a. What is true about the relationship between Figures *A* and *D*?

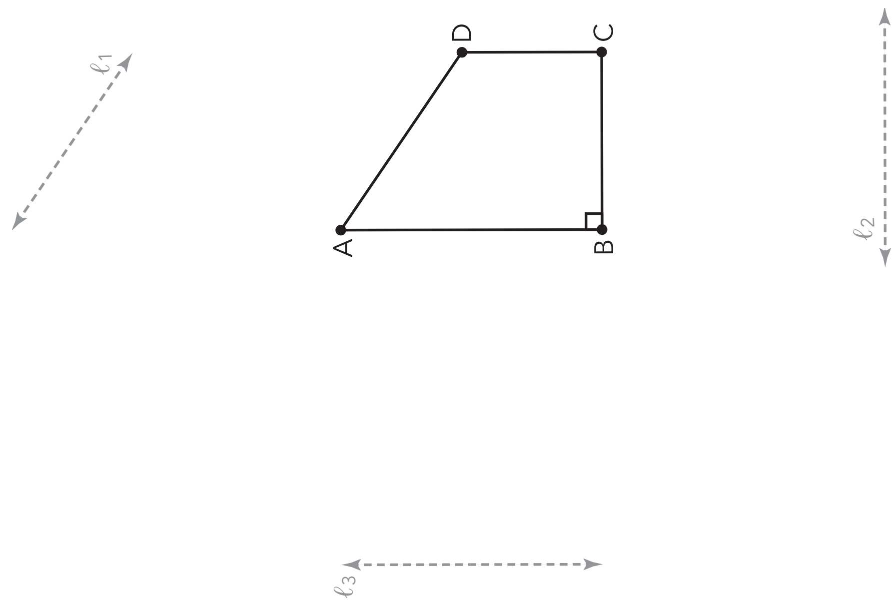
   b. How could you use multiple transformations to map Figure *A* onto Figure *D*?

   c. How could you use a single transformation to map Figure *A* onto Figure *D*?

Translations Mat



Reflections Mat



Rotations Mat