Lateral Moves 3

Translations of Figures on the Coordinate Plane

MATERIALS

Patty paper
Colored pencils

Lesson Overview

Students use patty paper to explore translations of various figures on a coordinate plane. They then generalize about the effects of translating a figure on its coordinates. Students verify that two figures are congruent by describing a sequence of translations that map one figure onto another.

Grade 8
Two-Dimensional Shapes

(10) The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

(A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane.
(C) explain the effect of translations, reflections over the *x*- or *y*-axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation.

ELPS

1.A, 1.C, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.D, 4.G, 4.K, 5.E

Essential Ideas

- A translation is a transformation that moves each point of a figure the same distance and direction.
- A point with the coordinates (x, y), when translated horizontally by c units, has new coordinates $(x + c, y)$. A point with the coordinates (x, y), when translated vertically by c units, has new coordinates $(x, y + c)$.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Stopping for Directions

Students solve a corridor maze. They then write the steps—in the form of left, right, up, and down moves—they used to solve the maze. This activity is designed to engage students in thinking about the necessity for more precise descriptions of translations as they apply what they know about translations to movement on the coordinate plane.

Develop

Activity 3.1: Modeling Translations on the Coordinate Plane

Students copy figures and the coordinates of their vertices onto patty paper and perform horizontal and vertical translations of the figures. They record the coordinates of the original (pre-image) and translated (image) figures and explore how the translation affects the coordinates of the pre-image. Students end the activity by making a general conjecture about the effect of a horizontal or vertical translation on an arbitrary ordered pair (x, y).

Day 2

Activity 3.2: Translating Any Points on the Coordinate Plane

Students perform translations and record the coordinates of the images in terms of x and y. They generalize by describing the translation in terms of x and y that would move the point (x, y) into each of the quadrants. Students are then given the coordinates of three vertices of a triangle and its graph. Using translations, they form two different triangles and record the coordinates of the vertices of the images. Finally, students are given the coordinates of the vertices of a triangle. Without graphing, they determine the coordinates of images resulting from different translations.

Activity 3.3: Verifying Congruence Using Translations

Students solve problems in order to demonstrate, using translations, that two figures are congruent if the same sequence of translations moves all the points of one figure onto all the points of the other figure.

Demonstrate

Talk the Talk: Left and Right, Up and Down

Students summarize what they learned in this lesson by describing how a general horizontal or vertical translation of c affects an arbitrary ordered pair (x, y). They write the ordered pair for such general translations. Students then describe why a sequence of translations cannot be used to verify the congruence of two right triangles, since the triangles are congruent due to a reflection across the y-axis. Reflections are investigated in the next lesson.

Getting Started: Stopping for Directions

ENGAGE

Facilitation Notes

In this activity, students solve a corridor maze. They then write the steps—in the form of left, right, up, and down moves—they used to solve the maze. This activity is designed to engage students in thinking about the necessity for more precise descriptions of translations as they apply what they know about translations to movement on the coordinate plane.

Note that writing clear and precise directions as students do in this activity is often an entry point for learning to code. Many software animations are written using step-by-step transformations like this. The precision is necessary for the computer to correctly interpret the intent of the programmer.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What is the first direction the turtle moved?
- What is the direction taken to move from the end toward the turtle?
- How many total moves were made to complete this maze?
- Is there more than one way to solve this maze?

Summary

Translations on a plane are used to explain real-world problem situations.

Activity 3.1
Modeling Translations on the Coordinate Plane

DEVELOP

Facilitation Notes

In this activity, students copy figures and the coordinates of their vertices onto patty paper and perform horizontal and vertical translations of the figures. They record the coordinates of the original (pre-image) and translated (image) figures and explore how the translation affected the coordinates of the pre-image. Students end the activity by making a general conjecture about the effect of a horizontal or vertical translation on an arbitrary ordered pair (x, y).

Emphasize that translations move figures along a line. Repeated translations (performing the same translation sequence over and over on a figure) move figures along the same line. This idea is important as it connects to other concepts in the course.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- How does the length of side *AB* compare to the length of side *A′B′*?
- How does angle *A* compare to angle *A′*?
- Are all of the corresponding sides congruent?
- Are all of the corresponding angles congruent?
- How does the *x*-coordinate for point *A* compare to the *x*-coordinate for point *A′*? Is this true for all of the *x*-coordinates of the corresponding vertices?
- How does the *y*-coordinate for point *A* compare to the *y*-coordinate for point *A′*? Is this true for all of the *y*-coordinates of the corresponding vertices?

Have students work with a partner or in a group to complete Question 2. Share responses as a class.

Questions to ask

- How does the length of side *AB* compare to the length of side *A′B′*?
- How does angle *A* compare to angle *A′*?
- Are all of the corresponding sides congruent?
- Are all of the corresponding angles congruent?
- How does the *x*-coordinate for point *A* compare to the *x*-coordinate for point *A′*? Is this true for all of the *x*-coordinates of the corresponding vertices?
- How does the *y*-coordinate for point *A* compare to the *y*-coordinate for point *A′*? Is this true for all of the *y*-coordinates of the corresponding vertices?

Summary

A horizontal translation left or right affects the *x*-coordinate of the points translated. A vertical translation up or down affects the *y*-coordinate of the points translated. The magnitude of the translation is added to or subtracted from the *x*- or *y*-coordinate.

Activity 3.2
Translating Any Points on the Coordinate Plane

Facilitation Notes

In this activity, students perform translations, record the coordinates of the images, and then generalize by describing the translation in terms of x and y that would move the point (x, y) into each of the quadrants. Finally, without graphing, students use their generalizations to determine the coordinates of images that result from various translations.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Misconception

Students sometimes believe that any point (x, y) means that both the values of x and y are positive, and the coordinate pair always lies in Quadrant I. The introductory sentence and questions that follow may somehow validate that misconception. If that is the case, complete a second question as practice, placing (x, y) in a different quadrant and answering Question 2 again, referring to the other quadrants.

Questions to ask

- If (x, y) is in the first quadrant, which quadrant of the coordinate plane is the location of the point $(-x, y)$?
- If (x, y) is in the first quadrant, which quadrant of the coordinate plane is the location of the point $(x, -y)$?
- If (x, y) is in the first quadrant, which quadrant of the coordinate plane is the location of the point $(-x, -y)$?
- If a point with the coordinates (x, y) is translated $-x$ units, what are the coordinates of its image?
- If a point with the coordinates (x, y) is translated $-y$ units, what are the coordinates of its image?
- If a point with the coordinates (x, y) is translated less than $-x$ units, how would you describe the coordinates of its image?
- If a point with the coordinates (x, y) is translated less than $-y$ units, how would you describe the coordinates of its image?

Have students work with a partner or in a group to complete Question 3. Share responses as a class.

Questions to ask

- How did you determine the coordinates of the vertices of $\triangle ABC$ when it was translated right 5 units?

- How did you determine the coordinates of the vertices of $\triangle ABC$ when it was translated down 8 units?
- Could you have determined the coordinates of the vertices of the image without graphing?

Have students work with a partner or in a group to complete Question 4. Share responses as a class.

Questions to ask

- How did you determine the coordinates of point D'?
- How did you determine the coordinates of point E'?
- How did you determine the coordinates of point F'?
- How did you determine the coordinates of point D''?
- How did you determine the coordinates of point E''?
- How did you determine the coordinates of point F''?

Differentiation strategy

To extend the activity, ask students to translate a triangle diagonally. For example, graph the triangle in Question 4 so that it is translated diagonally to the left 3 units and up 5 units.

Summary

When a translation is applied to a figure on the plane, addition or subtraction can be performed on the coordinates of a pre-image to determine the coordinates of the resulting image.

Activity 3.3

Verifying Congruence Using Translations

Facilitation Notes

In this activity, students determine if two figures are congruent by comparing the sequences of translations applied when moving all the points of one figure onto all the points of the other figure.

Ask a student to read the introduction aloud. Discuss as a class. Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

As students work, look for

- Errors in counting the translation due to counting lines on the coordinate plane rather than the distance between the lines.
- Errors from starting with the wrong figure.

Questions to ask

- Why is it only necessary to check the vertices of figures to verify if two figures are congruent by translation moves?
- How would you describe the translation performed on each of the vertices?
- Was the same translation performed on each vertex?
- What translation was performed on each pair of corresponding vertices?
- How do you know the corresponding sides are congruent?
- How do you know the corresponding angles are congruent?

Summary

Translations can be used to determine if two figures are congruent. If the figures are congruent, then the same sequence of translations moves all of the points of one figure onto all the points of the other figure.

Talk the Talk: Left and Right, Up and Down

DEMONSTRATE

Facilitation Notes

In this activity, students describe how a general horizontal or vertical translation of *c* units affects an arbitrary ordered pair (x, y) by writing the ordered pair for such general translations. They also decide if a sequence of translations can be used to determine the congruence of two right triangles.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- If the value of *c* is negative, how does this affect the direction of the translation?
- If the value of *c* is positive, how does this affect the direction of the translation?
- If the value of *d* is negative, how does this affect the direction of the translation?
- If the value of *d* is positive, how does this affect the direction of the translation?
- Was does it make sense to always use an addition symbol to demonstrate a translation?

- What translation describes the move from point A onto point A'?
- What translation describes the move from point B onto point B'?
- What translation describes the move from point C onto point C'?
- Is the same translation used on each of the vertices?
- If a translation was not used on the vertices of a figure, does that mean that the two figures are definitely not congruent? Why or why not?

Summary

Given a point (x, y), a horizontal translation c units results in the point $(x + c, y)$, and a vertical translation d units results in the point $(x, y + d)$.

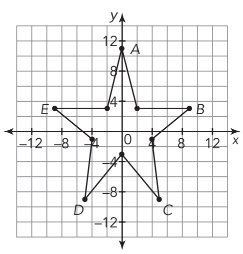
Lateral Moves

3

Translations of Figures on the Coordinate Plane

WARM UP

1. Identify the ordered pairs associated with each of the five labeled points of the star.



LEARNING GOALS

- Translate geometric figures on the coordinate plane.
- Identify and describe the effect of geometric translations on two-dimensional figures using coordinates.
- Identify congruent figures by obtaining one figure from another using a sequence of translations.

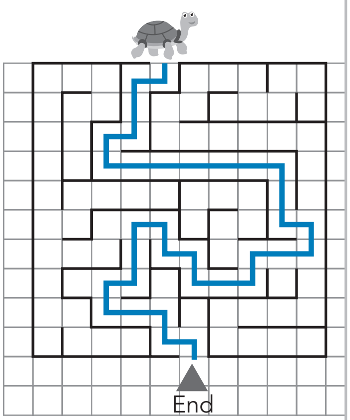
You have learned to model transformations, such as translations, rotations, and reflections. How can you model and describe these transformations on the coordinate plane?

Warm Up Answers

1. point A: (0, 11),
   point B: (9, 3),
   point C: (5, −9),
   point D: (−5, −9),
   point E: (−9, 3)

Answers

1.



Down 1, left 1, down 2, left 1, down 1, right 6, down 2, right 1, down 1, left 2, down 1, left 2, up 1, left 1, up 1, left 1, down 2, left 1, down 1, right 2, down 1, right 1, down 1

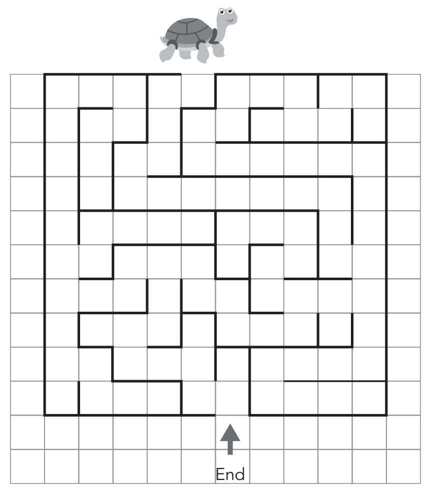
2. Start at the end of the list and reverse all of the steps. The turtle would start with up 1, left 1, up 1, and so on.

Getting Started

Stopping for Directions

Consider the maze shown.

1. **Navigate this maze to help the turtle move to the end. Justify your solution by writing the steps you used to solve the maze.**

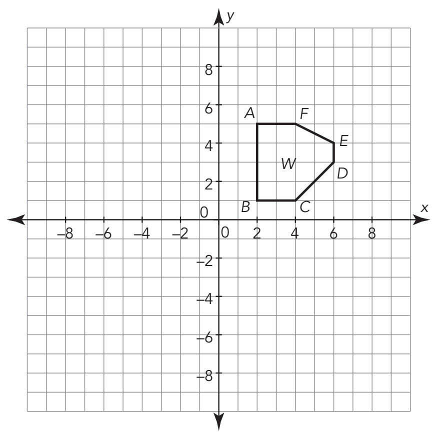


2. **How would your steps change if the turtle started at the end and had to make its way to the start of the maze?**

ACTIVITY 3.1

Modeling Translations on the Coordinate Plane

You know that translations are transformations that "slide" each point of a figure the same distance and the same direction. Each point moves in a line. You can describe translations more precisely by using coordinates.



1. **Place patty paper on the coordinate plane, trace Figure *W*, and copy the labels for the vertices on the patty paper.**

   a. **Translate the figure down 6 units. Then, identify the coordinates of the translated figure.**

   b. **Draw the translated figure on the coordinate plane with a different color, and label it as Figure *W′*. Then identify the pre-image and the image.**

   c. **Did translating Figure *W* vertically change the size or shape of the figure? Justify your answer.**

   d. **Complete the table with the coordinates of Figure *W′*.**

   e. **Compare the coordinates of Figure *W′* with the coordinates of Figure *W*. How are the values of the coordinates the same? How are they different? Explain your reasoning.**

Coordinates of *W*	Coordinates of *W′*
A (2, 5)	
B (2, 1)	
C (4, 1)	
D (6, 3)	
E (6, 4)	
F (4, 5)	

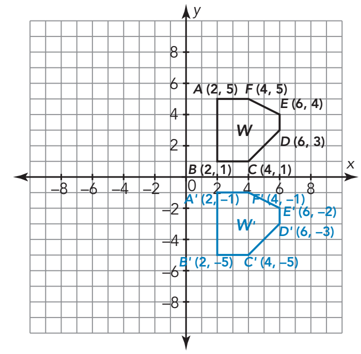
LESSON 3: Lateral Moves • 3

ELL Tip

Before having students continue on to Question 2, have them check their labeled figure with another student. If there is a discrepancy between the two students' labeling, ask students to work to figure out where mistakes were made. Encourage English Language Learners to ask for help if they need it before pairing up with their partner.

Answers

1.



1a. A′ (2, −1), B′ (2, −5), C′ (4, −5), D′ (6, −3), E′ (6, −2), F′ (4, −1)

1b. The pre-image is Figure *W*. The image is Figure *W′*.

1c. The translation did not change the size or shape of Figure *W*. The side lengths are all still the same.

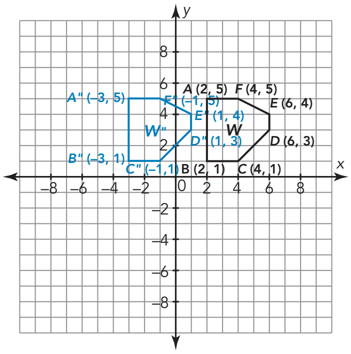
1d.

Coordinates of *W*	Coordinates of *W′*
A (2, 5)	*A′* (2, −1)
B (2, 1)	*B′* (2, −5)
C (4, 1)	*C′* (4, −5)
D (6, 3)	*D′* (6, −3)
E (6, 4)	*E′* (6, −2)
F (4, 5)	*F′* (4, −1)

1e. The *x*-coordinates of the image are all the same as the *x*-coordinates of the pre-image. The *y*-coordinates of the image are different from the *y*-coordinates of the pre-image; the *y*-coordinates of the image are each 6 less than the corresponding *y*-coordinates of the pre-image.

Answers

2a. Check students' work.



2b. The pre-image is Figure *W*. The image is Figure *W*″.

2c. The translation did not change the size or shape of Figure *W*″. The side lengths are all still the same.

2d.

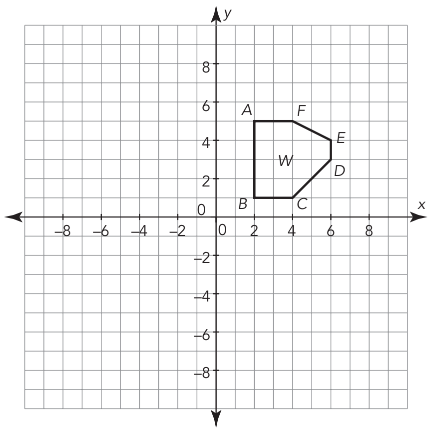
Coordinates of *W*	Coordinates of *W*″
A′ (2, 5)	*A*″ (−3, 5)
B′ (2, 1)	*B*″ (−3, 1)
C′ (4, 1)	*C*″ (−1, 1)
D′ (6, 3)	*D*″ (1, 3)
E′ (6, 4)	*E*″ (1, 4)
F′ (4, 5)	*F*″ (−1, 5)

2e. The *y*-coordinates of the image are all the same as the *y*-coordinates of the pre-image. The *x*-coordinates of the image are each 5 less than the corresponding *x*-coordinates of the pre-image.

3. Answers will vary. A horizontal translation left or right affects the *x*-coordinate of the points translated. A vertical translation up or down affects the *y*-coordinate of the points translated. The magnitude of the translation is added to or subtracted from the *x*- or *y*-coordinate.

Now, let's investigate translating Figure *W* horizontally.

2. Place patty paper on the coordinate plane, trace Figure *W*, and write and copy the labels for the vertices.



a. Translate the figure left 5 units.

b. Draw the translated figure on the coordinate plane with a different color, and label it as Figure *W*″. Then identify the pre-image and the image.

c. Did translating Figure *W* horizontally change the size or shape of the figure? Justify your answer.

Coordinates of *W*	Coordinates of *W*″
A (2, 5)	
B (2, 1)	
C (4, 1)	
D (6, 3)	
E (6, 4)	
F (4, 5)	

d. Complete the table with the coordinates of Figure *W*″.

e. Compare the coordinates of Figure *W*″ with the coordinates of Figure *W*. How are the values of the coordinates the same? How are they different? Explain your reasoning.

3. Make a conjecture about how a vertical or horizontal translation affects the coordinates of any point (*x*, *y*).

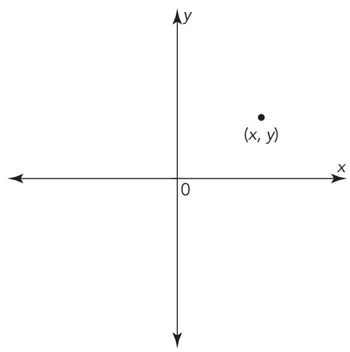
ELL Tip

Students may have a difficult time remembering the difference between *pre-image* and *image*. Have students focus on the word *pre-image*. Ask students if they know anything about the prefix *pre-*. Explain to students that the prefix *pre-* means to come "before", so the *pre-image* is the image before it has been transformed.

ACTIVITY
3.2

Translating Any Points on the Coordinate Plane

Consider the point (x, y) located anywhere in the first quadrant on the coordinate plane.

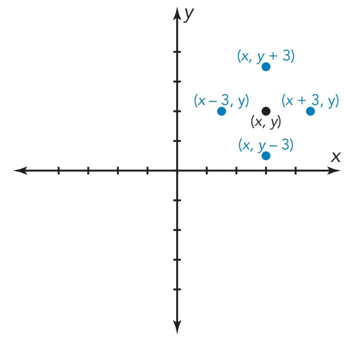


1. Consider each translation of the point (x, y) according to the descriptions in the table shown. Record the coordinates of the translated points in terms of x and y.

Translation	Coordinates of Translated Point
3 units to the left	
3 units down	
3 units to the right	
3 units up	

Answers

1.



Translation	Point (x, y) located in Q_1
3 units to the left	$(x - 3, y)$
3 units down	$(x, y - 3)$
3 units to the right	$(x + 3, y)$
3 units up	$(x, y + 3)$

ELL Tip

Provide students with a graphic organizer about the coordinate plane. Work with students to label the four quadrants, as well as the possible values of each coordinate written in algebraic format. For example if $x > 0$ and $y > 0$: (x, y), $(-x, -y)$, $(-x, y)$ and $(x, -y)$. Support students in plotting coordinates in each quadrant.

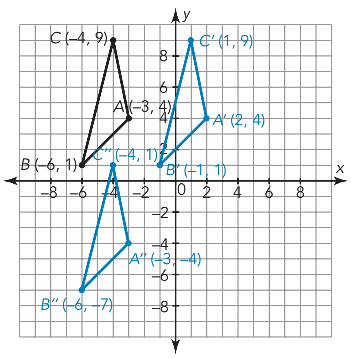
Answers

2a. The point would have to be translated more than x units to the left.

2b. The point would have to be translated more than x units to the left and more than y units down.

2c. The point would have to be translated more than y units down.

3.



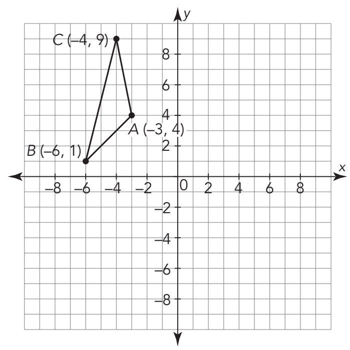
2. **Describe a translation in terms of x and y that would move any point (x, y) in Quadrant I into each quadrant.**

a. **Quadrant II**

b. **Quadrant III**

c. **Quadrant IV**

Let's consider Triangle *ABC* shown on the coordinate plane.



3. **Use the table to record the coordinates of the vertices of each translated triangle.**

a. **Translate Triangle *ABC* 5 units to the right to form Triangle *A′B′C′*. List the coordinates of points *A′*, *B′*, and *C′*. Then graph Triangle *A′B′C′*.**

b. **Translate Triangle *ABC* 8 units down to form Triangle *A″B″C″*. List the coordinates of points *A″*, *B″*, and *C″*. Then graph Triangle *A″B″C″*.**

Original Triangle	Triangle Translated 5 Units to the Right	Triangle Translated 8 Units Down
ΔABC	$\Delta A'B'C'$	$\Delta A''B''C''$
A (−3, 4)		
B (−6, 1)		
C (−4, 9)		

Let's consider translations of a different triangle without graphing.

4. The vertices of Triangle *DEF* are *D* (−7, 10), *E* (−5, 5), and *F* (−8, 1).

a. If Triangle *DEF* is translated to the right 12 units, what are the coordinates of the vertices of the image? Name the triangle.

b. How did you determine the coordinates of the image without graphing the triangle?

c. If Triangle *DEF* is translated up 9 units, what are the coordinates of the vertices of the image? Name the triangle.

d. How did you determine the coordinates of the image without graphing the triangle?

NOTES

Answers

3. See table below.

4a. The coordinates of the vertices of $\triangle D'E'F'$ are $D'(5, 10)$, $E'(7, 5)$, and $F'(4, 1)$.

4b. The coordinates of the vertices of the image were determined by adding 12 to each of the *x*-coordinates. The *y*-coordinates stayed the same.

4c. The coordinates of the vertices of $\triangle D''E''F''$ are $D''(-7, 19)$, $E''(-5, 14)$, $F''(-8, 10)$.

4d. The coordinates of the vertices of the image were determined by adding 9 to each of the *y*-coordinates. The *x*-coordinates stayed the same.

3.

Original Triangle	Triangle Translated 5 Units to the Right	Triangle Translated 8 Units Down
ΔABC	$\Delta A'B'C'$	$\Delta A''B''C''$
A (−3, 4)	A' (2, 4)	A'' (−3, −4)
B (−6, 1)	B' (−1, 1)	B'' (−6, −7)
C (−4, 9)	C' (1, 9)	C'' (−4, 1)

Answers

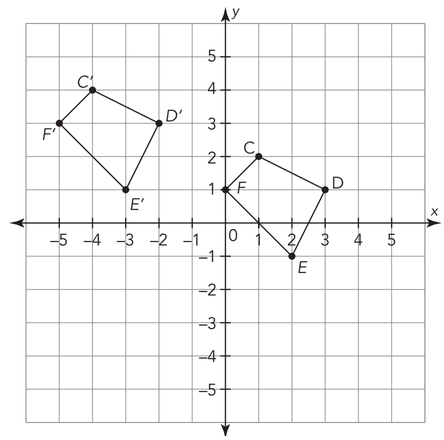
1. See table below.

ACTIVITY 3.3

Verifying Congruence Using Translations

One way to verify that two figures are congruent is to show that the same sequence of translations moves all of the points of one figure to all the points of the other figure.

Consider the two quadrilaterals shown on the coordinate plane.



1. Complete the table with the coordinates of each figure and the translation from each vertex in Quadrilateral *CDEF* to the corresponding vertex in Quadrilateral *C′D′E′F′*.

Coordinates of Quadrilateral *CDEF*	Coordinates of Quadrilateral *C′D′E′F′*	Translations

1.

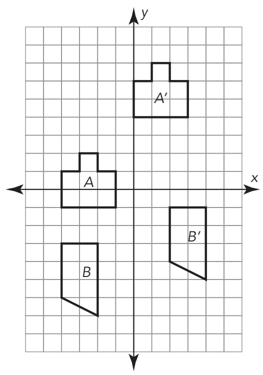
Coordinates of Quadrilateral *CDEF*	Coordinates of Quadrilateral *C′D′E′F′*	Translations
C (1, 2)	*C′* (−4, 4)	Left 5, up 2
D (3, 1)	*D′* (−2, 3)	Left 5, up 2
E (2, −1)	*E′* (−3, 1)	Left 5, up 2
F (0, 1)	*F′* (−5, 3)	Left 5, up 2

2. **Is Quadrilateral *CDEF* congruent to Quadrilateral *C′D′E′F′*? Explain how you know.**

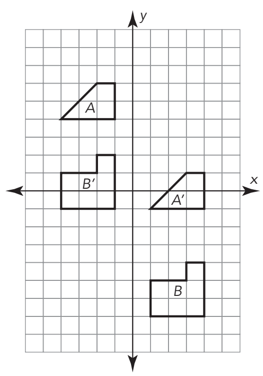
NOTES

3. **Describe a sequence of translations that can be used to show that Figures *A* and *A′* are congruent and that Figures *B* and *B′* are congruent. Show your work and explain your reasoning.**

   a.



   b.



Answers

2. Yes. Quadrilateral *CDEF* is congruent to Quadrilateral *C′D′E′F′*. The same sequence of translations moves the points of *CDEF* to the points of *C′D′E′F′*.

3a. Figure *A* is congruent to Figure *A′*, because all the points of Figure *A* can be translated right 4 and up 5 to create Figure *A′*. Figure *B* is congruent to Figure *B′*, because all the points of Figure *B* can be translated right 6 and up 2 to create Figure *B′*.

3b. Figure *A* is congruent to Figure *A′*, because all the points of Figure *A* can be translated right 5 and down 5 to create Figure *A′*. Figure *B* is congruent to Figure *B′*, because all the points of Figure *B* can be translated left 5 and up 6 to create Figure *B′*.

Answers

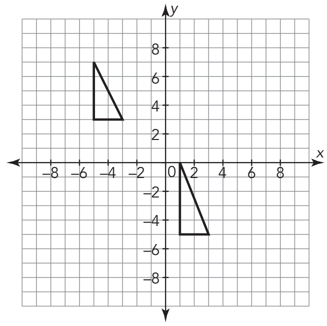
4a. The figures are not congruent.

4b. The figures are congruent. The left figure can be translated 5 units right and 3 units down to create the right figure. The right figure can be translated 5 units left and 3 units up to create the left figure.

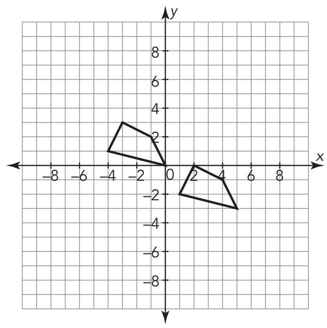
4c. The figures are not congruent.

4. For each example, decide whether the figures given are congruent or not congruent using translations. Show your work and explain your reasoning.

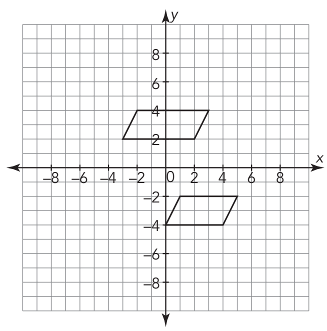
a.



b.



c.



NOTES

TALK the TALK

Left and Right, Up and Down

1. Suppose the point (x, y) is translated horizontally c units.

   a. How do you know if the point is translated left or right?

   b. Write the coordinates of the image of the point.

2. Suppose the point (x, y) is translated vertically d units.

   a. How do you know if the point is translated up or down?

   b. Write the coordinates of the image of the point.

3. Suppose a point is translated repeatedly up 2 units and right 1 unit. Does the point remain on a straight line as it is translated? Draw an example to explain your answer.

Answers

1a. If $c < 0$, the figure will translate to the left. If $c > 0$, the figure will translate to the right.

1b. $(x + c, y)$

2a. If $d < 0$, the figure will translate down. If $d > 0$, the figure will translate up.

2b. $(x, y + d)$

3. Check students' drawings.
Yes, the point remains on a straight line.

Answers

4. No. The triangles are congruent, but you cannot use translations to verify that they are congruent. Each of the vertices is translated differently from Triangle *ABC* to produce Triangle *A′B′C′*.

NOTES

4. Can you verify that these two figures are congruent using only translations? Explain why or why not.