5

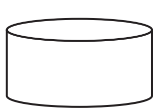
Half Turns and Quarter Turns

Rotations of Figures on the Coordinate Plane

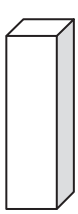
WARM UP

1. Redraw each given figure as described.
   a. so that it is turned 180°
      Before: After:

   b. so that it is turned 90° counterclockwise
      Before: After:



   c. so that it is turned 90° clockwise
      Before: After:



LEARNING GOALS

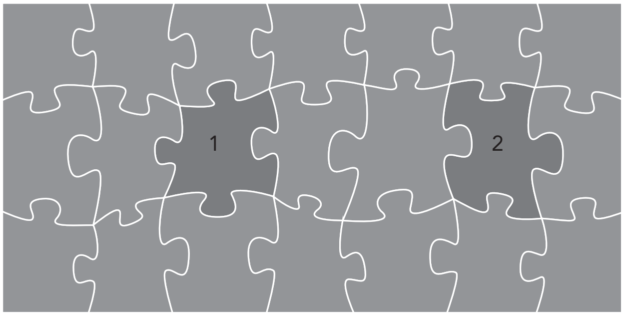
- Rotate geometric figures on the coordinate plane 90°, 180°, 270°, and 360°.
- Identify and describe the effect of geometric rotations of 90°, 180°, 270°, and 360° on two-dimensional figures using coordinates.
- Identify congruent figures by obtaining one figure from another using a sequence of translations, reflections, and rotations.

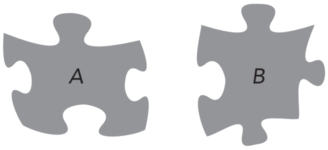
You have learned to model rigid motions, such as translations, rotations, and reflections. How can you model and describe these transformations on the coordinate plane?

Getting Started

Jigsaw Transformations

There are just two pieces left to complete this jigsaw puzzle.





1. **Which puzzle piece fills the missing spot at 1? Describe the translations, reflections, and rotations needed to move the piece into the spot.**

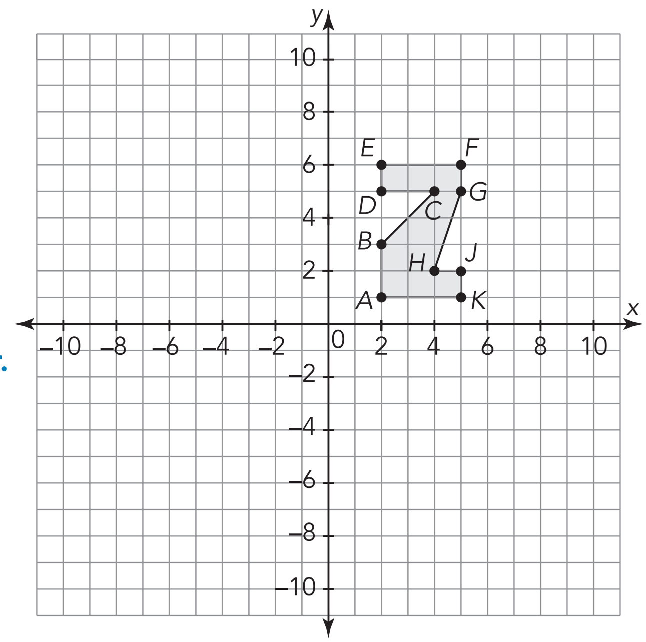
2. **Which puzzle piece fills the missing spot at 2? Describe the translations, reflections, and rotations needed to move the piece into the spot.**

ACTIVITY

5.1

Modeling Rotations on the Coordinate Plane

In this activity, you will investigate rotating pre-images to understand how the rotation affects the coordinates of the image.



1. Rotate the figure 180° about the origin.

a. Place patty paper on the coordinate plane, trace the figure, and copy the labels for the vertices on the patty paper.

b. Mark the origin, (0, 0), as the center of rotation. Trace a ray from the origin on the *x*-axis. This ray will track the angle of rotation.

c. Rotate the figure 180° about the center of rotation. Then, identify the coordinates of the rotated figure and draw the rotated figure on the coordinate plane. Finally, complete the table with the coordinates of the rotated figure.

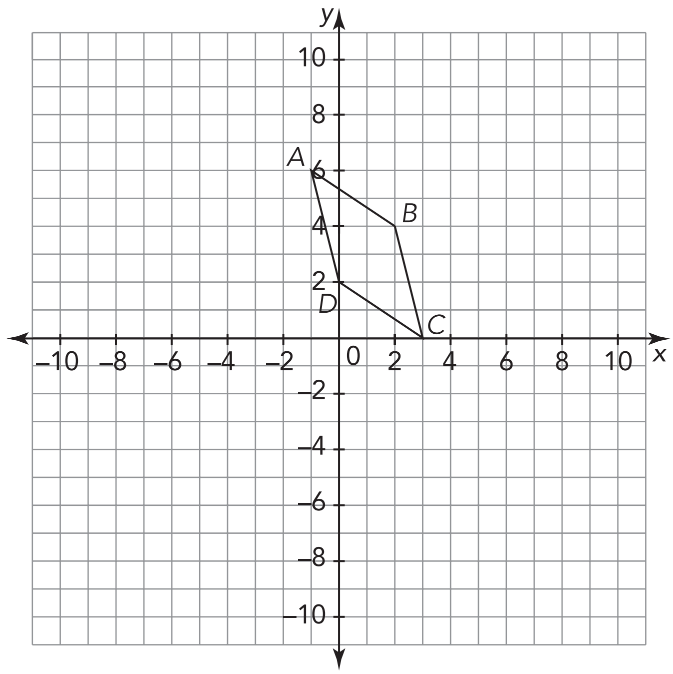
Coordinates of Pre-Image	Coordinates of Image
A (2, 1)	
B (2, 3)	
C (4, 5)	
D (2, 5)	
E (2, 6)	
F (5, 6)	
G (5, 5)	
H (4, 2)	
J (5, 2)	
K (5, 1)	

d. Compare the coordinates of the rotated figure with the coordinates of the original figure. How are the values of the coordinates the same? How are they different? Explain your reasoning.

NOTES

Now, let's investigate rotating a figure 90° about the origin.

2. Consider the parallelogram shown on the coordinate plane.



a. Place patty paper on the coordinate plane, trace the parallelogram, and then copy the labels for the vertices.

b. Rotate the figure 90° counterclockwise about the origin. Then, identify the coordinates of the rotated figure and draw the rotated figure on the coordinate plane.

c. Complete the table with the coordinates of the pre-image and the image.

Coordinates of Pre-Image	Coordinates of Image

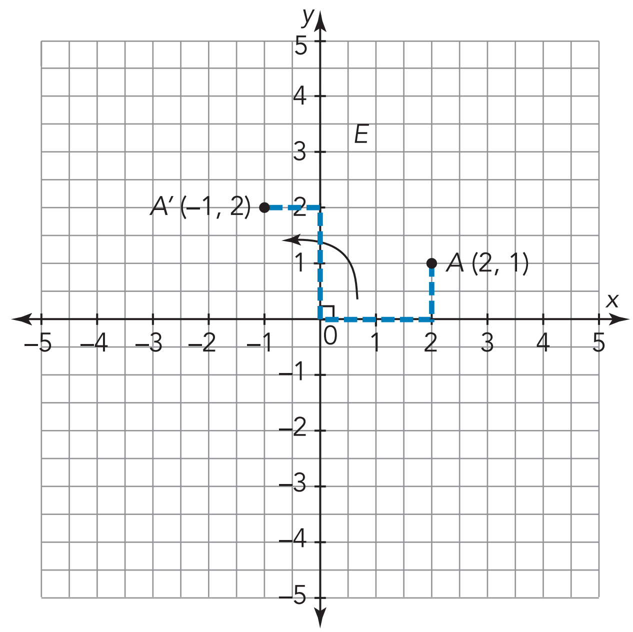
d. Compare the coordinates of the image with the coordinates of the pre-image. How are the values of the coordinates the same? How are they different? Explain your reasoning.

3. Make conjectures about how a counterclockwise 90° rotation and a 180° rotation affect the coordinates of any point (x, y).

You can use steps to help you rotate geometric objects on the coordinate plane.

Let's rotate a point 90° counterclockwise about the origin.

Step 1: Draw a "hook" from the origin to point *A*, using the coordinates and horizontal and vertical line segments as shown.



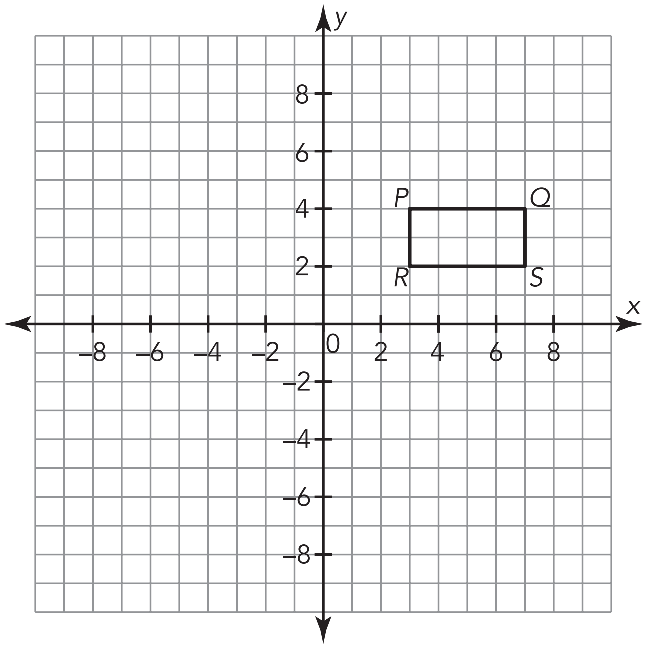
Step 2: Rotate the "hook" 90° counterclockwise as shown.

Point *A*′ is located at (–1, 2). Point *A* has been rotated 90° counterclockwise about the origin.

4. What do you notice about the coordinates of the rotated point? How does this compare with your conjecture?

Now let's investigate rotating figures more than 180° about the origin.

5. Consider the parallelogram shown on the coordinate plane.



a. Place patty paper on the coordinate plane, trace the parallelogram, and then copy the labels for the vertices.

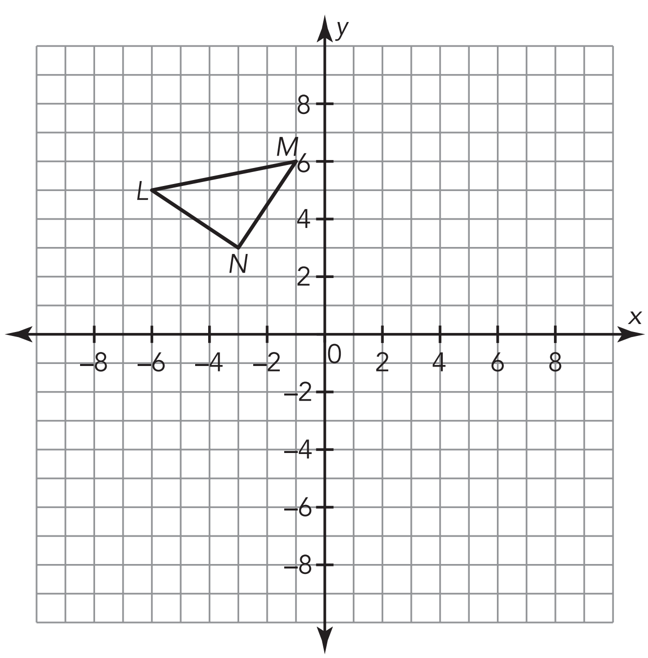
b. Rotate the figure 270° clockwise about the origin. Then, identify the coordinates of the rotated figure and draw the rotated figure on the coordinate plane.

c. **Complete the table with the coordinates of the pre-image and the image.**

Coordinates of Pre-Image	Coordinates of Image

d. **Compare the coordinates of the image with the coordinates of the pre-image. How are the values of the coordinates the same? How are they different? Explain your reasoning.**

6. **Consider the triangle shown on the coordinate plane.**



a. Place patty paper on the coordinate plane, trace the triangle, and then copy the labels for the vertices.

b. Rotate the figure 360° about the origin. Then, identify the coordinates of the rotated figure and draw the rotated figure on the coordinate plane.

c. Complete the table with the coordinates of the pre-image and the image.

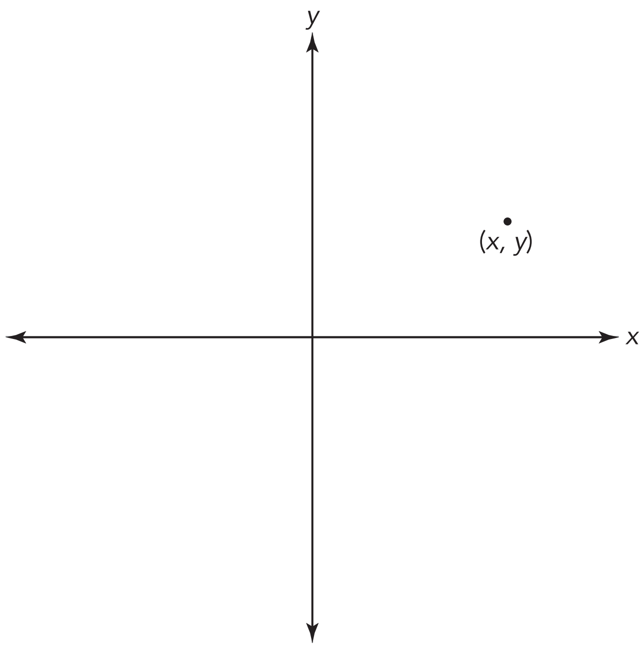
Coordinates of Pre-Image	Coordinates of Image

d. Compare the coordinates of the image with the coordinates of the pre-image. How are the values of the coordinates the same? How are they different? Explain your reasoning.

ACTIVITY
5.2

Rotating Any Points on the Coordinate Plane

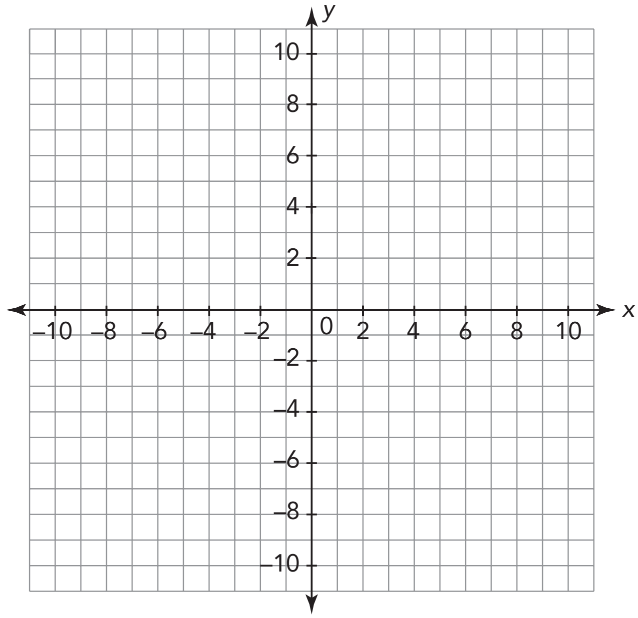
Consider the point (*x*, *y*) located anywhere in the first quadrant.



1. Use the origin, (0, 0), as the point of rotation. Rotate the point (*x*, *y*) as described in the table and plot and label the new point. Then record the coordinates of each rotated point in terms of *x* and *y*.

Original Point	Rotation About the Origin 90° Counterclockwise	Rotation About the Origin 90° Clockwise	Rotation About the Origin 180°
(*x*, *y*)			

2. Graph ΔABC by plotting the points A (3, 4), B (6, 1), and C (4, 9).



Use the origin, (0, 0), as the point of rotation. Rotate ΔABC as described in the table, graph and label the new triangle. Then record the coordinates of the vertices of each triangle in the table.

Original Triangle	Rotation About the Origin 90° Counterclockwise	Rotation About the Origin 90° Clockwise	Rotation About the Origin 180°	Rotation About the Origin 270° Clockwise	Rotation About the Origin 360°
ΔABC	$\Delta A'B'C'$	$\Delta A''B''C''$	$\Delta A'''B'''C'''$	$\Delta A''''B''''C''''$	$\Delta A'''''B'''''C'''''$
A (3, 4)					
B (6, 1)					
C (4, 9)					

2. Consider your table.

   a. What do you notice about the coordinates of the triangle that has been rotated 270° clockwise about the origin? What conjecture can you make about these two triangles?

   b. What do you notice about the coordinates of the triangle that has been rotated 360° about the origin? What conjecture can you make about these two triangles?

NOTES

Let's consider rotations of a different triangle without graphing.

3. The vertices of ΔDEF are D (−7, 10), E (−5, 5), and F (−1, −8).

a. If ΔDEF is rotated 90° counterclockwise about the origin, what are the coordinates of the vertices of the image? Name the rotated triangle.

b. How did you determine the coordinates of the image without graphing the triangle?

c. If ΔDEF is rotated 90° clockwise about the origin, what are the coordinates of the vertices of the image? Name the rotated triangle.

NOTES

d. How did you determine the coordinates of the image without graphing the triangle?

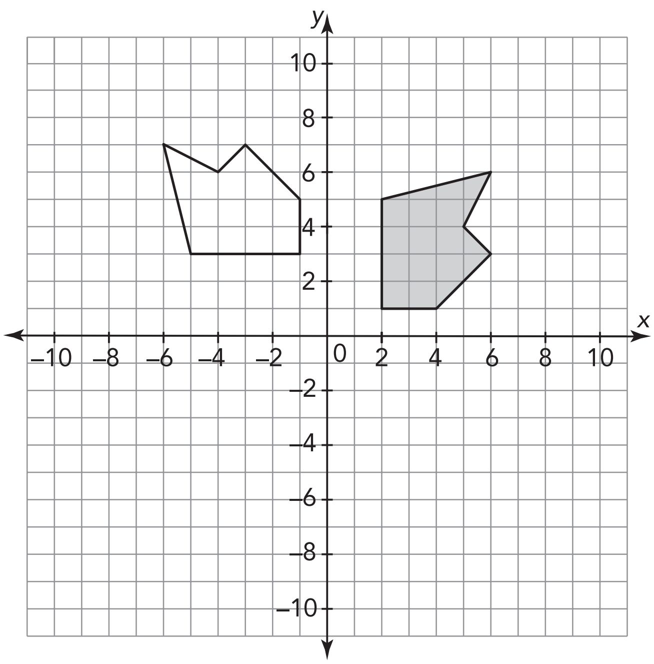
e. If ΔDEF is rotated 180° about the origin, what are the coordinates of the vertices of the image? Name the rotated triangle.

f. How did you determine the coordinates of the image without graphing the triangle?

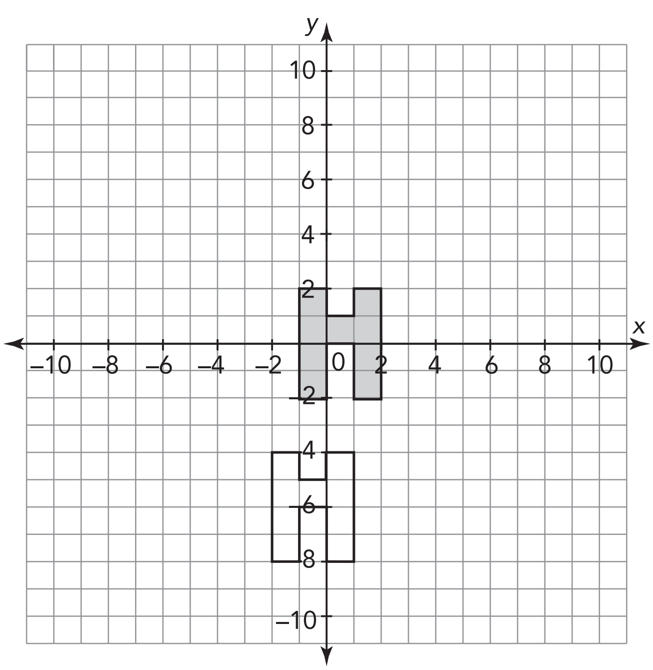
ACTIVITY 5.3 Verifying Congruence Using Rigid Motions

Describe a sequence of rigid motions that can be used to verify that the shaded pre-image is congruent to the image.

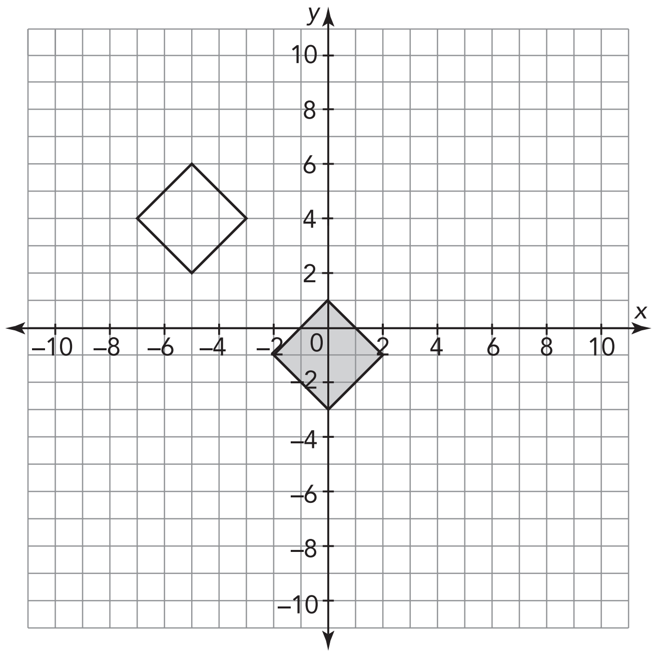
1.



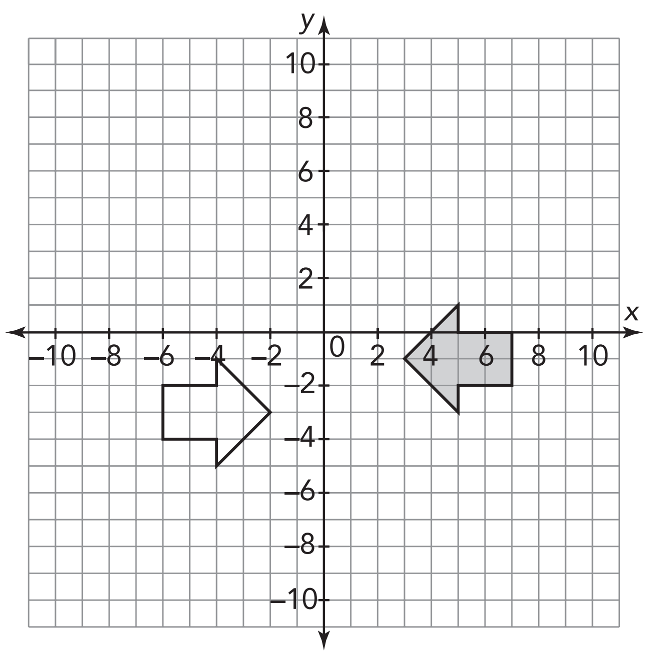
2.



3.



4.



NOTES

TALK the TALK

Just the Coordinates

Using what you know about rigid motions, verify that the figures represented by the coordinates are congruent. Describe the sequence of rigid motions to explain your reasoning.

1. **$\triangle QRS$ has coordinates Q (1, −1), R (3, −2), and S (2, −3). $\triangle Q'R'S'$ has coordinates Q' (5, −4), R' (6, −2), and S' (7, −3).**

2. **Rectangle $MNPQ$ has coordinates M (3, −2), N (5, −2), P (5, −6), and Q (3, −6). Rectangle $M'N'P'Q'$ has coordinates M' (0, 0), N' (–2, 0), P' (–2, 4), and Q' (0, 4).**