# Half Turns and Quarter Turns

5

Rotations of Figures on the Coordinate Plane

## MATERIALS

Patty paper

## Lesson Overview

Students use patty paper to explore rotations of various figures on a coordinate plane. They then generalize about the effects of rotating a figure on its coordinates. Students verify that two figures are congruent by describing a sequence of rigid motions that map one figure onto another.

## Grade 8

## Two-Dimensional Shapes

**(10) The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:**

(A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane.

(C) explain the effect of translations, reflections over the *x*- or *y*-axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation.

## ELPS

1.A, 1.C, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.D, 4.G, 4.K, 5.E

## Essential Ideas

- A rotation is a transformation that turns a figure clockwise or counterclockwise about a fixed point for a given angle and a given direction.
- An angle of rotation is the amount of clockwise or counterclockwise rotation about a fixed point.
- The point of rotation can be a point on the figure, in the figure, or outside the figure.

# Lesson Structure and Pacing: 2 Days

## Day 1

### Engage

**Getting Started: Jigsaw Transformations**

Students are presented with a jigsaw puzzle that has two pieces missing. Students match each missing piece to the open spot of the puzzle and describe the sequence of translations, reflections, and rotations that would move the puzzle piece presented to the matching open spot of the puzzle.

### Develop

**Activity 5.1: Modeling Rotations on the Coordinate Plane**

Students copy figures and the coordinates of their vertices onto patty paper. They perform rotations of 90° counterclockwise, 90° clockwise, 180°, 270° clockwise, and 360° about the origin. Students record the coordinates of the original (pre-image) and rotated (image) figures. They explore how the rotations affect the coordinates of the pre-image to create the image. Students end the activity by making conjectures about the effects of rotations on an arbitrary ordered pair ($x$, $y$).

## Day 2

**Activity 5.2: Rotating Any Points on the Coordinate Plane**

A point with the coordinates ($x$, $y$) is located in the first quadrant. Students perform rotations of 90° counterclockwise, 90° clockwise, and 180° using the origin as the point of rotation and record the coordinates of the images. Next, students begin with a triangle in Quadrant I and perform rotations of 90° clockwise, 90° counterclockwise, 180°, 270° clockwise, and 360°. Lastly, they are given the coordinates of the vertices of a triangle, and without graphing they determine the coordinates of images resulting from different rotations.

**Activity 5.3: Verifying Congruence Using Rigid Motions**

Students examine the change in $x$- and $y$-coordinates to determine the congruence of geometric figures. They decide if a sequence of transformations can be used to prove the congruence of figures shown on a graph and then describe that sequence of rigid motions.

### Demonstrate

**Talk the Talk: Just the Coordinates**

Students verify the congruence of pairs of figures, given only the coordinates of the pre-images and images.

## Facilitation Notes

In this activity, students are presented with a jigsaw puzzle that has two pieces missing. Students match each missing piece to the open spot of the puzzle and describe the sequence of translations, reflections, and rotations that would move the puzzle piece presented to the matching open spot of the puzzle.

Note that the transformations students list in this activity can be described informally—a reference to a line of reflection or center of rotation is not needed. You might want to discuss the difference between a horizontal flip and a vertical flip of a puzzle piece.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

**Questions to ask**

- Which rigid motion transformation will move the puzzle piece from its initial position to an open position on the puzzle board?
- Is there more than one rigid motion transformation or series of transformations that will work?
- Does the order of the transformations make a difference?
- Are the translations used to move the puzzle piece into position horizontal, vertical, or both?
- If a reflection is used to move the puzzle piece into position, where is the location of the line of reflection?
- If a rotation is used to move the puzzle piece into position, where is the location of the center of rotation?

**Differentiation strategy**

To scaffold support, suggest that students copy one or both of the puzzle pieces onto patty paper. They can then use the patty paper to perform the transformations necessary to fit the pieces into the puzzle.

## Summary

Rigid motion transformations such as reflections, rotations, and translations can be used to describe and solve real-world situations.

DEVELOP

# Activity 5.1

## Modeling Rotations on the Coordinate Plane

### Facilitation Notes

In this activity, students copy figures and the coordinates of their vertices onto patty paper. They perform rotations of 90° counterclockwise, 90° clockwise, 180°, 270° clockwise, and 360° about the origin. Students record the coordinates of the original (pre-image) and rotated (image) figures. They explore how the rotations affect the coordinates of the pre-image to create the image. Students end the activity by making conjectures about the effects of rotations on an arbitrary ordered pair ($x$, $y$).

Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

**Questions to ask**

- Why is a rotation of 180° considered a half turn?
- Is the figure rotated 180° clockwise about the origin or 180° counterclockwise about the origin? How do you know?
- What is the difference between rotating the figure 180° clockwise about the origin and rotating the figure 180° counterclockwise about the origin?
- Is there any relationship between the *x*-coordinate of the pre-image and the *x*-coordinate of the image? Is this true for all vertices?
- Is there any relationship between the *y*-coordinate of the pre-image and the *y*-coordinate of the image? Is this true for all vertices?
- Could this image have also been created by a translation? Explain.
- Could this image have also been created by a reflection? Explain.
- Why is a rotation of 90° considered a quarter turn?
- What is the difference between rotating the figure 90° clockwise about the origin and rotating the figure 90° counterclockwise about the origin?
- How are the coordinate changes evident in the position of the diagram?
- What quadrant(s) do you conjecture the image will lie in?
- How will the orientation of the image compare to the pre-image?

- What is the same about rotating a figure 90° counterclockwise about the origin and rotating a figure 270° about the origin?
- What do you notice about rotating a figure 360° about the origin?
- Can you think of any real-world examples of a 360° rotation?

**Differentiation strategy**

To extend the activity, have students (1) rotate the figure 90° clockwise about the origin, (2) reflect the figure across the $x$-axis, (3) reflect the figure across the $y$-axis, and (4) reflect the figure first across the $x$-axis and then across the $y$-axis. Compare these images with the two completed in the activity.

## Summary

A point $(x, y)$ rotated 180° about the origin becomes the point $(-x, -y)$, and when it is rotated 90° counterclockwise about the origin becomes the point $(-y, x)$. A point $(x, y)$ rotated 270° clockwise about the origin becomes the point $(-y, x)$. A point rotated 360° about the origin becomes the point $(x, y)$.

# Activity 5.2

## Rotating Any Points on the Coordinate Plane

## Facilitation Notes

A point with the coordinates $(x, y)$ is located in the first quadrant. Students perform rotations of 90° counterclockwise, 90° clockwise, and 180° using the origin as the point of rotation and record the coordinates of the images in terms of $x$ and $y$. Next, students begin with a triangle in Quadrant I and perform rotations of 90° clockwise, 90° counterclockwise, 180°, 270° clockwise, and 360°. Lastly, they are given the coordinates of the vertices of a triangle, and without graphing they determine the coordinates of images resulting from different rotations.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

**Questions to ask**

- When the figure was rotated 90° counterclockwise about the origin, did the rotation change the $x$-coordinate of each vertex?
- When the figure was rotated 90° counterclockwise about the origin, did the rotation change the $y$-coordinate of each vertex?

- Which coordinate in every point of the pre-image changed as a result of the 90° counterclockwise rotation about the origin?
- When the figure was rotated 90° clockwise about the origin, did the rotation change the $x$-coordinate of each vertex?
- When the figure was rotated 90° clockwise about the origin, did the rotation change the $y$-coordinate of each vertex?
- Does a 180° clockwise rotation about the origin bring about the same results as a 180° counterclockwise rotation about the origin?
- What rotations cause a switch between the $x$-coordinates and $y$-coordinates? Why do you think that is the case?
- If a rotation causes a switch in the $x$-coordinates and $y$-coordinates as well as a sign change, does it matter what order those changes are made? Explain.

Have students work with a partner or in a group to complete Question 2 and 3. Share responses as a class.

**Questions to ask**

- If the point $(x, y)$ is rotated 90° counterclockwise about the origin, how does the $x$-coordinate change?
- If the point $(x, y)$ is rotated 90° counterclockwise about the origin, how does the $y$-coordinate change?
- If the point $(x, y)$ is rotated 90° clockwise about the origin, how does the $x$-coordinate change?
- If the point $(x, y)$ is rotated 90° clockwise about the origin, how does the $y$-coordinate change?
- If the point $(x, y)$ is rotated 180° about the origin, how does the $x$-coordinate change?
- If the point $(x, y)$ is rotated 180° about the origin, how does the $y$-coordinate change?
- If the rotation of a point $(x, y)$ about the origin results in the point
  $(-x, y)$, what do you know about the rotation?
- If the rotation of a point $(x, y)$ about the origin results in the point $(-y, x)$, what do you know about the rotation?
- If the rotation of a point $(x, y)$ about the origin results in the point $(-x, -y)$, what do you know about the rotation?
- If the rotation of a point $(x, y)$ about the origin results in the point $(y, -x)$, what do you know about the rotation?
- If the point $(x, y)$ is rotated 270° about the origin, how does the $x$-coordinate change?
- If the point $(x, y)$ is rotated 270° about the origin, how does the $y$-coordinate change?

- If the point $(x, y)$ is rotated 360° about the origin, how does the $x$-coordinate change?
- If the point $(x, y)$ is rotated 360° about the origin, how does the $y$-coordinate change?

**Note** that when students rotate the original triangle 270° about the origin, the coordinates are the same as the 90° counterclockwise rotation. When they rotate the original triangle 360° about the origin, the coordinates are the same as the original triangle.

**Differentiation strategies**

To scaffold support when students must consider rotations without graphing:

- Allow them to graph the vertices.
- Refer them to the general form at the start of the activity to make a connection to their work and the general form.

## Summary

A point $(x, y)$ when rotated 90° counterclockwise becomes the point $(-y, x)$, when rotated 90° clockwise becomes the point $(y, -x)$, and when rotated 180° becomes the point $(-x, -y)$.

# Activity 5.3

### Verifying Congruence Using Rigid Motions

## Facilitation Notes

In this activity, students examine the change in $x$- and $y$-coordinates to determine the congruence of geometric figures. They decide if a sequence of transformations can be used to prove the congruence of figures shown on a graph and then describe that sequence of rigid motions.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

**Differentiation strategies**

To scaffold support,

- Suggest that students use patty paper to determine the required transformations.
- Ask students if the orientation of the figure remained the same, and if so, suggest they use translations (even if other rigid motions may work as well).

**Questions to ask**

- Is a translation involved in this situation? How do you know?
- Is a rotation involved in this situation? How do you know?
- Is a reflection involved in this situation? How do you know?
- What is another set of rigid motions that would create the same image?
- What is the $x$-coordinate of each point in the pre-image? What is the $x$-coordinate of each point in the image?
- Is the relationship between the $x$-coordinate of each point in the pre-image and its corresponding $x$-coordinate in the image the same for all pairs of corresponding points?
- What is the $y$-coordinate of each point in the pre-image? What is the $y$-coordinate of each point in the image?
- Is the relationship between the $y$-coordinate of each point in the pre-image and its corresponding $y$-coordinate in the image the same for all pairs of corresponding points?
- How can you check if you are correct?

### Summary

Two figures are congruent if the same sequence of reflections, rotations, and translations moves all the points of one figure to all the points of the other figure.

DEMONSTRATE

## Talk the Talk: Just the Coordinates

### Facilitation Notes

In this activity, students use the coordinates of the pre-image and image to describe the rigid motion transformations associated with two congruent geometric figures.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

**Differentiation strategy**

To assist all students, suggest they use graphs and patty paper and/or tables to visualize these problems.

**Questions to ask**

- What do you know to be true about the coordinates of points that have undergone a vertical translation?
- What do you know to be true about the coordinates of points that have undergone a horizontal translation?

- What do you know to be true about the coordinates of points that have undergone a reflection across the $x$-axis?
- What do you know to be true about the coordinates of points that have undergone a reflection across the $y$-axis?
- What do you know to be true about the coordinates of points that have undergone a 90° counterclockwise rotation about the origin?
- What do you know to be true about the coordinates of points that have undergone a 90° clockwise rotation about the origin?
- What do you know to be true about the coordinates of points that have undergone a 180° rotation about the origin?
- What is another set of rigid motion transformations to create this same image?

## Summary

Rigid motion transformations can be used to verify the congruence of geometric figures.

NOTES

# Half Turns and Quarter Turns

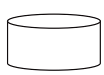
# 5

Rotations of Figures on the Coordinate Plane

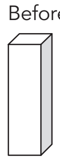
## WARM UP

1. Redraw each given figure as described.
   a. so that it is turned 180°
      Before: After:

   b. so that it is turned 90° counterclockwise
      Before: After:



   c. so that it is turned 90° clockwise
      Before: After:



## LEARNING GOALS

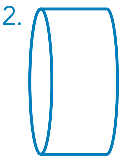
- Rotate geometric figures on the coordinate plane 90°, 180°, 270°, and 360°.
- Identify and describe the effect of geometric rotations of 90°, 180°, 270°, and 360° on two-dimensional figures using coordinates.
- Identify congruent figures by obtaining one figure from another using a sequence of translations, reflections, and rotations.

You have learned to model rigid motions, such as translations, rotations, and reflections. How can you model and describe these transformations on the coordinate plane?



## Warm Up Answers

1. 

2. 

3. 

### ELL Tip

Before beginning the lesson, remind students of the difference between *clockwise* and *counterclockwise*. Ask students to discuss the two words with one another and have them write down a few pieces of information about each word. Write the two words on the board, and have students come to the board and write down their pieces of information, not duplicating the information written previously.

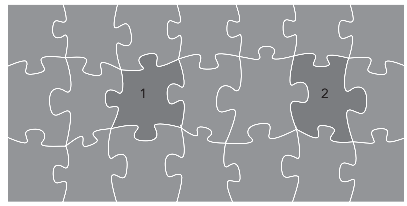
## Answers

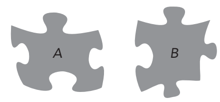
1. Piece B.
   Sample explanations include:
   reflect horizontally then reflect vertically, or rotate 180°, translate
2. Piece A.
   Sample explanations include:
   rotate 90° clockwise, reflect vertically, translate

# Getting Started

## Jigsaw Transformations

There are just two pieces left to complete this jigsaw puzzle.




**1. Which puzzle piece fills the missing spot at 1? Describe the translations, reflections, and rotations needed to move the piece into the spot.**

**2. Which puzzle piece fills the missing spot at 2? Describe the translations, reflections, and rotations needed to move the piece into the spot.**

ACTIVITY 5.1

## Modeling Rotations on the Coordinate Plane

In this activity, you will investigate rotating pre-images to understand how the rotation affects the coordinates of the image.

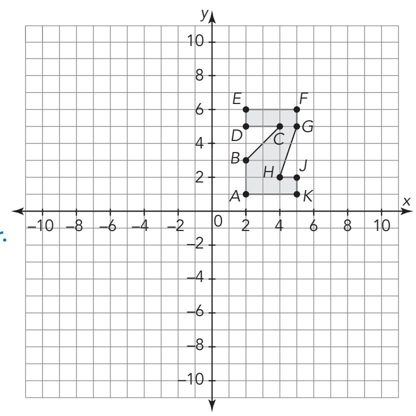
**1. Rotate the figure 180° about the origin.**

**a. Place patty paper on the coordinate plane, trace the figure, and copy the labels for the vertices on the patty paper.**

**b. Mark the origin, (0, 0), as the center of rotation. Trace a ray from the origin on the *x*-axis. This ray will track the angle of rotation.**

**c. Rotate the figure 180° about the center of rotation. Then, identify the coordinates of the rotated figure and draw the rotated figure on the coordinate plane. Finally, complete the table with the coordinates of the rotated figure.**

**d. Compare the coordinates of the rotated figure with the coordinates of the original figure. How are the values of the coordinates the same? How are they different? Explain your reasoning.**



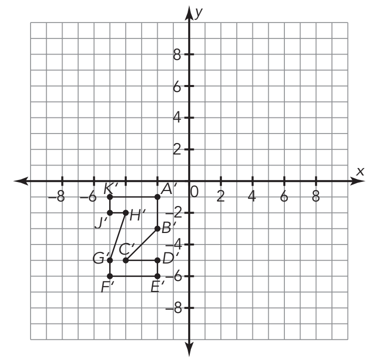

| Coordinates of Pre-Image | Coordinates of Image |
|---|---|
| *A* (2, 1) | |
| *B* (2, 3) | |
| *C* (4, 5) | |
| *D* (2, 5) | |
| *E* (2, 6) | |
| *F* (5, 6) | |
| *G* (5, 5) | |
| *H* (4, 2) | |
| *J* (5, 2) | |
| *K* (5, 1) | |



## Answers

1a, b. Check students' work.

1c.



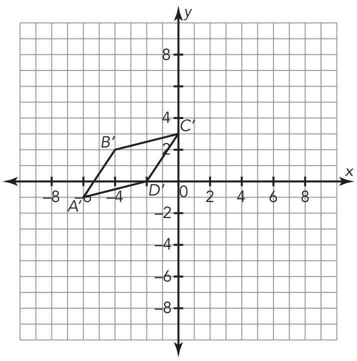

| Coordinates of Pre-Image | Coordinates of Image |
|---|---|
| *A* (2, 1) | *A′* (−2, −1) |
| *B* (2, 3) | *B′* (−2, −3) |
| *C* (4, 5) | *C′* (−4, −5) |
| *D* (2, 5) | *D′* (−2, −5) |
| *E* (2, 6) | *E′* (−2, −6) |
| *F* (5, 6) | *F′* (−5, −6) |
| *G* (5, 5) | *G′* (−5, −5) |
| *H* (4, 2) | *H′* (−4, −2) |
| *J* (5, 2) | *J′* (−5, −2) |
| *K* (5, 1) | *K′* (−5, −1) |

1d. The coordinates are the same values with opposite signs. This is because the image is oriented the same way, but moved from Quadrant I to Quadrant III.

## Answers

2a. Check students' work.

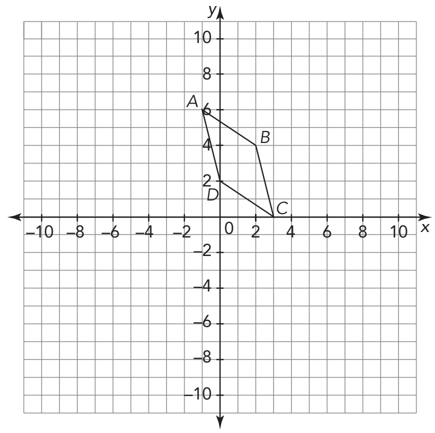
2b.




NOTES

Now, let's investigate rotating a figure 90° about the origin.

**2. Consider the parallelogram shown on the coordinate plane.**




**a. Place patty paper on the coordinate plane, trace the parallelogram, and then copy the labels for the vertices.**

**b. Rotate the figure 90° counterclockwise about the origin. Then, identify the coordinates of the rotated figure and draw the rotated figure on the coordinate plane.**

**c.** **Complete the table with the coordinates of the pre-image and the image.**

| Coordinates of Pre-Image | Coordinates of Image |
|---|---|
| | |
| | |
| | |
| | |

**d.** **Compare the coordinates of the image with the coordinates of the pre-image. How are the values of the coordinates the same? How are they different? Explain your reasoning.**

**3.** **Make conjectures about how a counterclockwise 90° rotation and a 180° rotation affect the coordinates of any point ($x$, $y$).**



## Answers

2c.

| Coordinates of Pre-Image | Coordinates of Image |
|---|---|
| (3, 0) | (0, 3) |
| (2, 4) | (−4, 2) |
| (−1, 6) | (−6, −1) |
| (0, 2) | (−2, 0) |

2d. The $x$-coordinates of the pre-image became the $y$-coordinates of the image, and the opposites of the $y$-coordinates of the pre-image became the $x$-coordinates of the image.

3. Answers will vary.

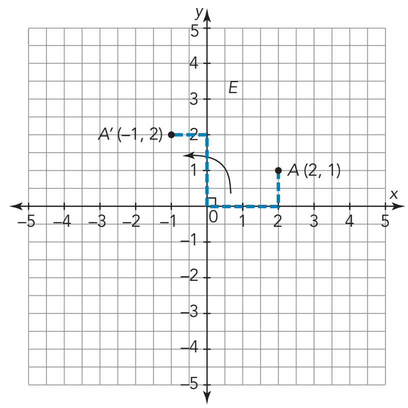
## Answers

4. Answers will vary.

You can use steps to help you rotate geometric objects on the coordinate plane.

Let's rotate a point 90° counterclockwise about the origin.

**Step 1:** Draw a "hook" from the origin to point *A*, using the coordinates and horizontal and vertical line segments as shown.




**Step 2:** Rotate the "hook" 90° counterclockwise as shown.

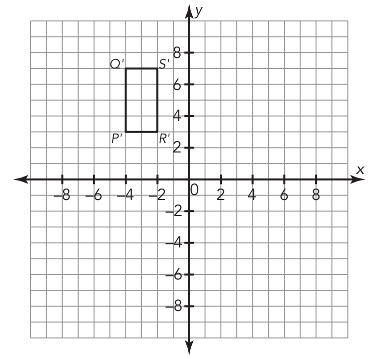
Point *A*′ is located at (–1, 2). Point *A* has been rotated 90° counterclockwise about the origin.

**4. What do you notice about the coordinates of the rotated point? How does this compare with your conjecture?**

## Answers

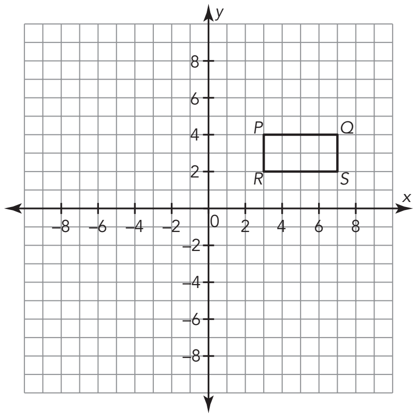
5a. Check students' work.

5b.




Now let's investigate rotating figures more than 180° about the origin.

**5. Consider the parallelogram shown on the coordinate plane.**




**a. Place patty paper on the coordinate plane, trace the parallelogram, and then copy the labels for the vertices.**

**b. Rotate the figure 270° clockwise about the origin. Then, identify the coordinates of the rotated figure and draw the rotated figure on the coordinate plane.**

## Answers

5c.

| Coordinates of Pre-Image | Coordinates of Image |
|---|---|
| *P*(3, 4) | *P*′(−4, 3) |
| *Q*(7, 4) | *Q*′(−4, 7) |
| *R*(3, 2) | *R*′(−2, 3) |
| *S*(7, 2) | *S*′(−2, 7) |

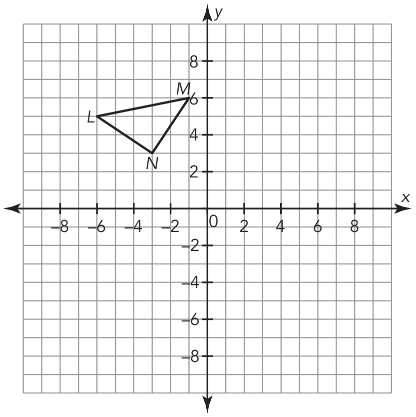
5d. The *x*-coordinates of the pre-image became the *y*-coordinates of the image, and the opposites of the *y*-coordinates of the pre-image became the *x*-coordinates of the image.

**c. Complete the table with the coordinates of the pre-image and the image.**

| Coordinates of Pre-Image | Coordinates of Image |
|---|---|
| | |
| | |
| | |
| | |

**d. Compare the coordinates of the image with the coordinates of the pre-image. How are the values of the coordinates the same? How are they different? Explain your reasoning.**

**6. Consider the triangle shown on the coordinate plane.**



a. Place patty paper on the coordinate plane, trace the triangle, and then copy the labels for the vertices.

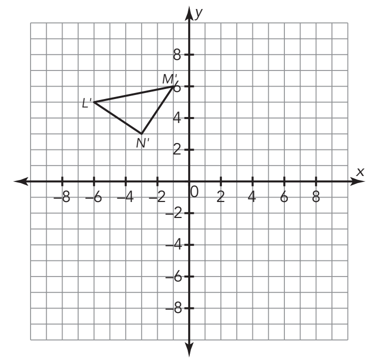
b. Rotate the figure 360° about the origin. Then, identify the coordinates of the rotated figure and draw the rotated figure on the coordinate plane.



## Answers

6a. Check students' work.

6b.



## Answers

6c.

| Coordinates of Pre-Image | Coordinates of Image |
|---|---|
| $L(-6, 4)$ | $L'(-6, 4)$ |
| $M(-1, 6)$ | $M'(-1, 6)$ |
| $N(-3, 3)$ | $N'(-3, 3)$ |

6d. The coordinates are exactly the same because the triangle has made a full rotation or complete turn about the origin.

**c. Complete the table with the coordinates of the pre-image and the image.**

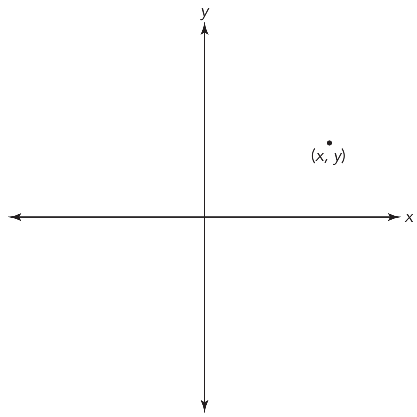
| Coordinates of Pre-Image | Coordinates of Image |
|---|---|
| | |
| | |
| | |

**d. Compare the coordinates of the image with the coordinates of the pre-image. How are the values of the coordinates the same? How are they different? Explain your reasoning.**

ACTIVITY 5.2

## Rotating Any Points on the Coordinate Plane

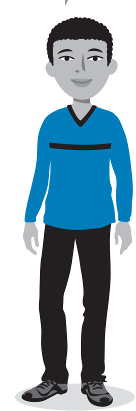
Consider the point $(x, y)$ located anywhere in the first quadrant.




**1. Use the origin, (0, 0), as the point of rotation. Rotate the point $(x, y)$ as described in the table and plot and label the new point. Then record the coordinates of each rotated point in terms of $x$ and $y$.**

| Original Point | Rotation About the Origin 90° Counterclockwise | Rotation About the Origin 90° Clockwise | Rotation About the Origin 180° |
|---|---|---|---|
| $(x, y)$ | | | |

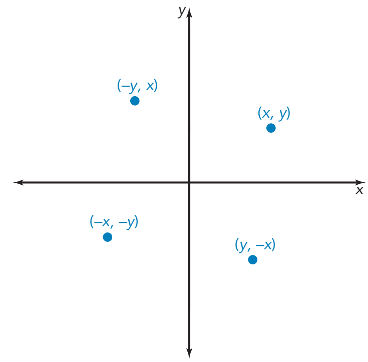
"If your point was at (5, 0), and you rotated it 90°, where would it end up? What about if it was at (5, 1)?"





## Answers

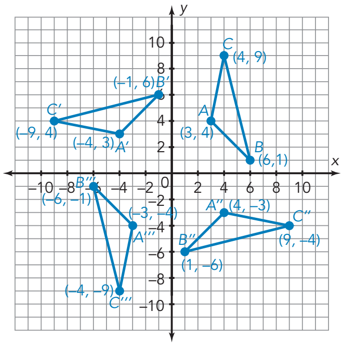
1.




| Rotation About the Origin 90° Counter-clockwise | Rotation About the Origin 90° Clockwise | Rotation About the Origin 180° |
|---|---|---|
| $(-y, x)$ | $(y, -x)$ | $(-x, -y)$ |

## Answers

2.

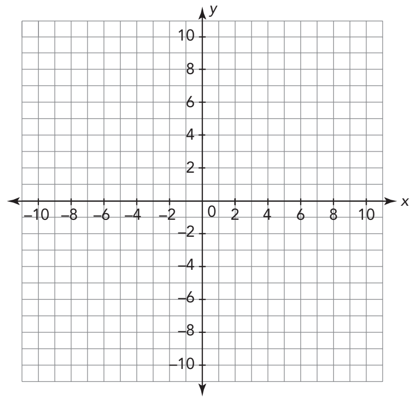



Check students' graphs.The 270° clockwise rotation is congruent to the 90° counterclockwise rotation. The 360° rotation is the same as the original triangle.

2a. The triangle that has been rotated 270° clockwise about the origin has the same coordinates as the triangle that has been rotated 90° counterclockwise about the origin. Triangle $A''''B''''C''''$ is congruent to triangle $A'B'C'$

2b. The triangle that has been rotated 360° about the origin has the same coordinates as the original triangle. The original triangle completed one full rotation. Triangle $A'''''B'''''C'''''$ is congruent to triangle $ABC$.

**2. Graph $\Delta ABC$ by plotting the points $A$ (3, 4), $B$ (6, 1), and $C$ (4, 9).**




**Use the origin, (0, 0), as the point of rotation. Rotate $\Delta ABC$ as described in the table, graph and label the new triangle. Then record the coordinates of the vertices of each triangle in the table.**

| Original Triangle | Rotation About the Origin 90° Counterclockwise | Rotation About the Origin 90° Clockwise | Rotation About the Origin 180° | Rotation About the Origin 270° Clockwise | Rotation About the Origin 360° |
|---|---|---|---|---|---|
| $\Delta ABC$ | $\Delta A'B'C'$ | $\Delta A''B''C''$ | $\Delta A'''B'''C'''$ | $\Delta A''''B''''C''''$ | $\Delta A'''''B'''''C'''''$ |
| $A$ (3, 4) | | | | | |
| $B$ (6, 1) | | | | | |
| $C$ (4, 9) | | | | | |

**2. Consider your table.**

**a. What do you notice about the coordinates of the triangle that has been rotated 270° clockwise about the origin? What conjecture can you make about these two triangles?**

**b. What do you notice about the coordinates of the triangle that has been rotated 360° about the origin? What conjecture can you make about these two triangles?**



| Rotation About the Origin 90° Counterclockwise | Rotation About the Origin 90° Clockwise | Rotation About the Origin 180° | Rotation About the Origin 270° Clockwise | Rotation About the Origin 360° |
|---|---|---|---|---|
| $\Delta A'B'C'$ | $\Delta A''B''C''$ | $\Delta A'''B'''C'''$ | $\Delta A''''B''''C''''$ | $\Delta A'''''B'''''C'''''$ |
| $A'$ (–4, 3) | $A''$ (4, –3) | $A'''$ (–3, –4) | $A''''$ (–4, 3) | $A'''''$ (3, 4) |
| $B'$ (–1, 6) | $B''$ (1, –6) | $B'''$ (–6, –1) | $B''''$ (–1, 6) | $B'''''$ (6, 1) |
| $C'$ (–9, 4) | $C''$ (9, –4) | $C'''$ (–4, –9) | $C''''$ (–9, 4) | $C'''''$ (4, 9) |

Let's consider rotations of a different triangle without graphing.

NOTES

**3. The vertices of $\Delta DEF$ are $D\,(-7, 10)$, $E\,(-5, 5)$, and $F\,(-1, -8)$.**

**a. If $\Delta DEF$ is rotated 90° counterclockwise about the origin, what are the coordinates of the vertices of the image? Name the rotated triangle.**

**b. How did you determine the coordinates of the image without graphing the triangle?**

**c. If $\Delta DEF$ is rotated 90° clockwise about the origin, what are the coordinates of the vertices of the image? Name the rotated triangle.**



## Answers

3a. The coordinates of the vertices of Triangle $D'E'F'$ are $D'\,(-10, -7)$, $E'\,(-5, -5)$, and $F'\,(8, -1)$.

3b. The coordinates of the vertices of the image were determined by writing the opposite of the $y$-coordinates, and then switching the $x$- and $y$-coordinates.

3c. The coordinates of the vertices of Triangle $D''E''F''$ are $D''\,(10, 7)$, $E''\,(5, 5)$, and $F''\,(-8, 1)$.

## Answers

3d. The coordinates of the vertices of the image were determined by writing the opposite of the $x$-coordinates, and then switching the $x$- and $y$-coordinates.

3e. The coordinates of the vertices of Triangle $D'''E'''F'''$ are $D'''$ (7, −10), $E'''$ (5, −5), and $F'''$ (1, 8).

3f. The coordinates of the vertices of the image were determined by writing the opposite of each of the $x$-coordinates and the opposite of each of the $y$-coordinates.

NOTES

**d. How did you determine the coordinates of the image without graphing the triangle?**

**e. If $\Delta DEF$ is rotated 180° about the origin, what are the coordinates of the vertices of the image? Name the rotated triangle.**

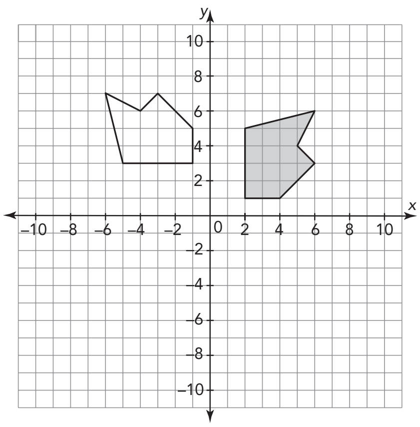
**f. How did you determine the coordinates of the image without graphing the triangle?**

ACTIVITY 5.3

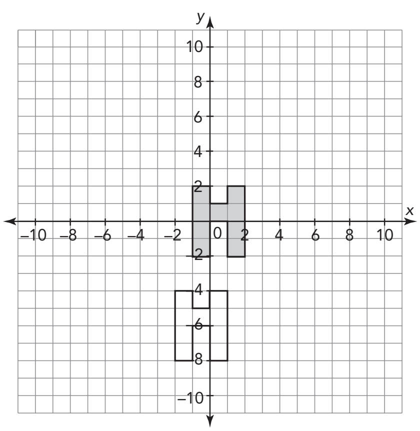
## Verifying Congruence Using Rigid Motions

Describe a sequence of rigid motions that can be used to verify that the shaded pre-image is congruent to the image.

**1.**




**2.**






## Answers

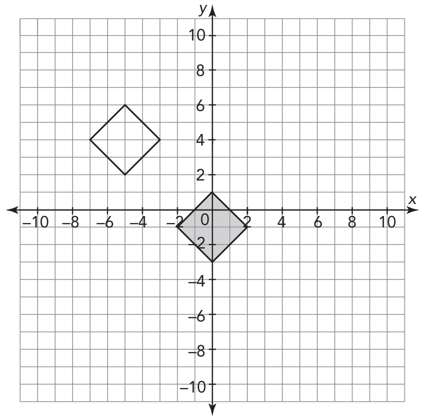
1. Sample answer.
   The figures are congruent by a 90° counterclockwise rotation about the origin, and a translation up 1 unit.
2. Sample answer.
   The figures are congruent by a translation of 1 unit to the left and a translation of 6 units down.

## Answers

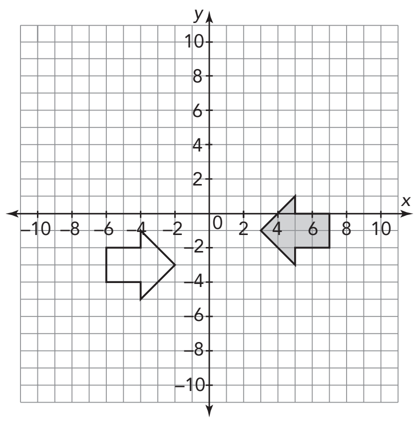
3. Sample answer. The figures are congruent by a translation left 5 units and up 5 units.

4. Sample answer. The figures are congruent by a reflection across the *y*-axis, a translation down two units and a translation to the right one unit.

3.




4.



NOTES

## TALK the TALK

### Just the Coordinates

Using what you know about rigid motions, verify that the figures represented by the coordinates are congruent. Describe the sequence of rigid motions to explain your reasoning.

1. **$\triangle QRS$ has coordinates $Q$ (1, −1), $R$ (3, −2), and $S$ (2, −3). $\triangle Q'R'S'$ has coordinates $Q'$ (5, −4), $R'$ (6, −2), and $S'$ (7, −3).**

2. **Rectangle $MNPQ$ has coordinates $M$ (3, −2), $N$ (5, −2), $P$ (5, −6), and $Q$ (3, −6). Rectangle $M'N'P'Q'$ has coordinates $M'$ (0, 0), $N'$ (−2, 0), $P'$ (−2, 4), and $Q'$ (0, 4).**



## Answers

1. Sample answer.
   Rotate the figure counterclockwise 90° about the origin, translate the figure down 5 units and to the right 4 units.
2. Sample answer.
   Rotate the figure 180° about the origin. Then translate the figure to the right 3 units and down 2 units.