Rigid Motion Transformations Summary

KEY TERMS

- congruent figures
- corresponding sides
- corresponding angles
- plane
- transformation
- rigid motion

- pre-image
- image
- translation
- reflection
- line of reflection
- rotation

- center of rotation
- angle of rotation
- congruent line segments
- congruent angles

LESSON 1

Patty Paper, Patty Paper

Figures that have the same size and shape are **congruent figures**. If two figures are congruent, all corresponding sides and all corresponding angles have the same measures. **Corresponding sides** are sides that have the same relative position in geometric figures. **Corresponding angles** are angles that have the same relative position in geometric figures.

If two figures are congruent, you can obtain one figure by a combination of sliding, flipping, and spinning the figure until it lies on the other figure.

For example, Figure A is congruent to Figure C, but it is not congruent to Figure B or Figure D.
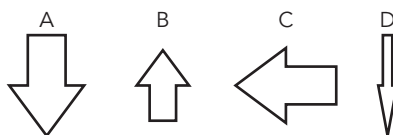
Slides, Flips, and Spins

A **plane** extends infinitely in all directions in two dimensions and has no thickness. A **transformation** is the mapping, or movement, of a plane and all the points of a figure on a plane according to a common action or operation. A **rigid motion** is a special type of transformation that preserves the size and shape of each figure.

The original figure on the plane is called the **pre-image**, and the new figure that results from a transformation is called the **image**. The labels for the vertices of an image use the symbol ('), which is read as "prime."

A **translation** is a rigid motion transformation that slides each point of a figure the same distance and direction along a line. A figure can be translated in any direction. Two special translations are vertical and horizontal translations. Sliding a figure left or right is a horizontal translation, and sliding it up or down is a vertical translation.

A **reflection** is a rigid motion transformation that flips a figure across a line of reflection. A **line of reflection** is a line that acts as a mirror so that corresponding points are the same distance from the line.

A **rotation** is a rigid motion transformation that turns a figure on a plane about a fixed point, called the **center of rotation**, through a given angle, called the **angle of rotation**. The center of rotation can be a point outside of the figure, inside of the figure, or on the figure itself. Rotation can be clockwise or counterclockwise.

translation   reflection   rotation

Lateral Moves
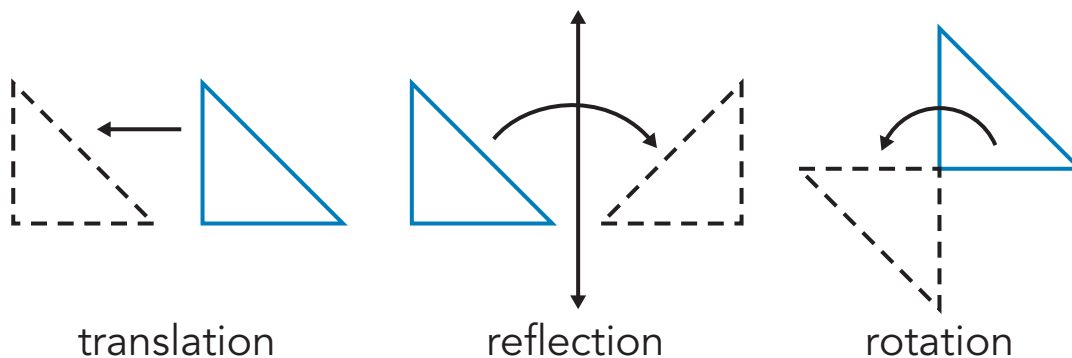
A translation slides an image on the coordinate plane. When an image is horizontally translated *c* units on the coordinate plane, the value of the *x*-coordinates change by *c* units. When an image is vertically translated *c* units on the coordinate plane, the value of the *y*-coordinate changes by *c*-units. The coordinates of an image after a translation are summarized in the table.

Original Point	Horizontal Translation to the Left	Horizontal Translation to the Right	Vertical Translation Up	Vertical Translation Down
(x, y)	$(x - c, y)$	$(x + c, y)$	$(x, y + c)$	$(x, y - c)$

For example, the coordinates of ΔABC are $A\ (0, 2)$, $B\ (2, 6)$, and $C\ (3, 3)$.

When ΔABC is translated down 8 units, the coordinates of the image are $A'\ (0, -6)$, $B'\ (2, -2)$, and $C'\ (3, -5)$.

When ΔABC is translated right 6 units, the coordinates of the image are $A''\ (6, 2)$, $B''\ (8, 6)$, and $C''\ (9, 3)$.

LESSON 4
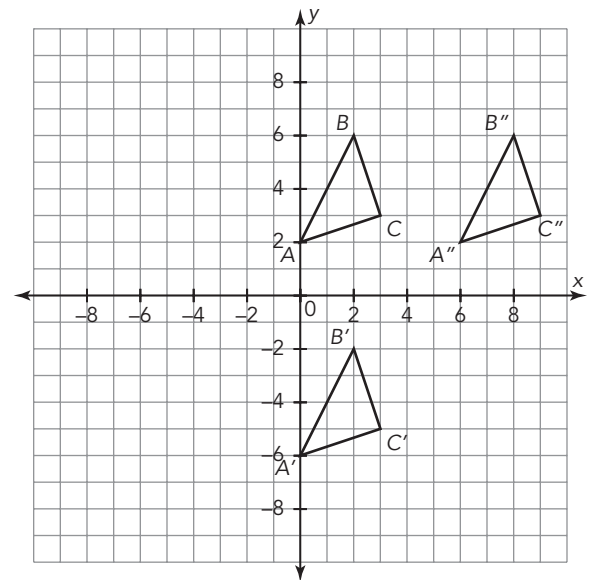
Mirror, Mirror

A reflection flips an image across a line of reflection. When an image on the coordinate plane is reflected across the *y*-axis, the value of the *x*-coordinate of the image is opposite the *x*-coordinate of the pre-image. When an image on the coordinate plane is reflected across the *x*-axis, the value of the *y*-coordinate of the image is opposite the *y*-coordinate of the pre-image. The coordinates of an image after a reflection on the coordinate plane are summarized in the table.

Original Point	Reflection Over *x*-Axis	Reflection Over *y*-Axis
(x, y)	(x, −y)	(−x, y)

For example, the coordinates of Quadrilateral *ABCD* are A (3, 2), B (2, 5), C (5, 7), and D (6, 1).

When Quadrilateral *ABCD* is reflected across the *x*-axis, the coordinates of the image are A′ (3, −2), B′ (2, −5), C′ (5, −7), and D′ (6, −1).

When Quadrilateral *ABCD* is reflected across the *y*-axis, the coordinates of the image are A″ (−3, 2), B″ (−2, 5), C″ (−5, 7), and D″ (−6, 1).
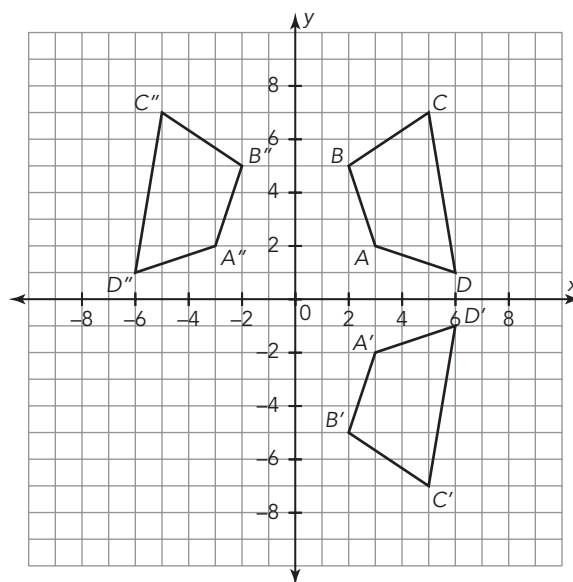
Half Turns and Quarter Turns

A rotation turns a figure about a point through an angle of rotation. When the center of rotation is at the origin (0, 0), and the angle of rotation is 90°, 180°, 270°, or 360° the coordinates of an image can be determined using the rules summarized in the table.

| Original Point and Rotation About the Origin 360°	Rotation About the Origin 90° Counterclockwise and 270° Clockwise	Rotation About the Origin 90° Clockwise and 270° Counterclockwise	Rotation About the Origin 180°
(x, y)	$(-y, x)$	$(y, -x)$	$(-x, -y)$

For example, the coordinates of $\triangle ABC$ are A (2, 1), B (5, 8), and C (6, 4).

When $\triangle ABC$ is rotated 90° counterclockwise or 270° clockwise about the origin, the coordinates of the image are A' (−1, 2), B' (−8, 5), and C' (−4, 6).

When $\triangle ABC$ is rotated 180° about the origin, the coordinates of the image are A'' (−2, −1), B'' (−5, −8), and C'' (−6, −4).

When $\triangle ABC$ is rotated 90° clockwise or 270° counterclockwise about the origin, the coordinates of the image are A'' (1, −2), B'' (8, −5), and C'' (4, −6).

When $\triangle ABC$ is rotated 360° about the origin, the coordinates are the same as the coordinates of the original triangle.
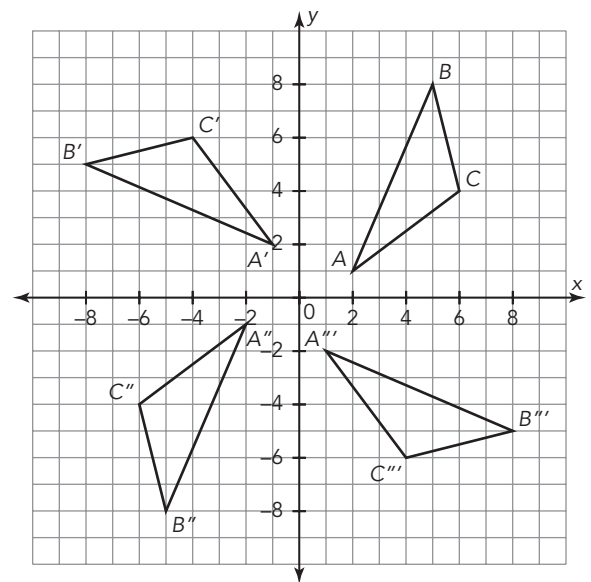
Every Which Way

Because rigid motions maintain the size and shape of an image, you can use a sequence of translations, reflections, and rotations to verify that two figures are congruent.

In congruent figures, the corresponding sides are congruent line segments. **Congruent line segments** are line segments that have the same length. Likewise, if corresponding angles have the same measure, they are congruent angles. **Congruent angles** are angles that are equal in measure.

For example, if the sides of two different figures are equal in length, such that the length of side *AB* in Triangle *ABC* is equal to the length of side *DE* in Triangle *DEF*, these sides are said to be congruent.

$\overline{AB} \cong \overline{DE}$ is read "line segment *AB* is congruent to line segment *DE*."

Likewise, if the angles of two different figures are equal in measure, such that the measure of angle *A* in Triangle *ABC* is equal to the measure of angle *D* in Triangle *DEF*, these angles are said to be congruent.

$\angle A \cong \angle D$ is read "angle *A* is congruent to angle *D*."

There is often more than one sequence of transformations that can be used to verify that two figures are congruent.