

Pinch-Zoom Geometry

Dilations of Figures

1

MATERIALS

Rulers
Protractors

Lesson Overview

Students explore dilations on the plane. The terms *dilation*, *center of dilation*, *scale factor* or *dilation factor*, *enlargement*, and *reduction* are defined. Students dilate a variety of objects and figures using scale factors greater than and less than 1. They use a model to determine side lengths and angle measures after enlargements and reductions in order to verify similarity. Students connect dilations to changing image sizes in word processing and graphics software.

Grade 8

Proportionality

(3) The student applies mathematical process standards to use proportional relationships to describe dilations. The student is expected to:

(A) generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation.

Two-Dimensional Shapes

(10) The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

(A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- Dilation is a transformation that produces images that are the same shape as the pre-image, but not the same size.
- When a figure is dilated with a scale factor greater than 1, the resulting figure is a similar figure that is an enlargement, because each side length is multiplied by a scale factor that is larger than the identity factor of 1.
- When a figure is dilated with a scale factor less than 1, the resulting figure is a similar figure that is a reduction, because each side length is multiplied by a scale factor that is smaller than the identity factor of 1.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Scale Drawing by Doing

Students access their prior knowledge of scale factors and scale drawings from the previous course to recreate a company logo on different-sized monitors. Students explain the steps they used to recreate the figure at different scales of $\frac{1}{2}$ and 2.

Develop

Activity 1.1: Dilating Figures with a Scale Factor Greater Than 1

The terms *dilation*, *center of dilation*, *scale factor*, and *enlargement* are defined. A logo is dilated using a point outside the figure as the center of dilation. Students use ratios to express the ratio of the distance of the image from the center of dilation to the distance of the pre-image from the center of dilation. They measure the image side lengths and the segments connecting the image to the center of dilation. Students determine that the ratios that compare the lengths of corresponding segments connecting the image to the center of dilation are equal. A protractor is then used to measure the angles in each triangle. Students conclude that since corresponding angles are congruent and the ratios of corresponding sides are equal, the triangles have the same shape but different sizes.

Day 2

Activity 1.2: Dilating Figures with a Scale Factor Less Than 1

The term *reduction* is defined. A logo is dilated using a point outside the figure as the center of dilation. Students use ratios to express the ratio of the distance of the image from the center of dilation to the distance of the pre-image from the center of dilation. They measure the image side lengths and the segments connecting the image to the center of dilation. Students determine that the ratios that compare the lengths of corresponding segments connecting the image to the center of dilation are equal. A protractor is then used to measure the angles in each triangle. Students conclude that, since corresponding angles are congruent and the ratios of corresponding sides are equal, the triangles have the same shape but different sizes.

Activity 1.3: Creating and Verifying Similar Figures

The term *similar* is defined in the context of similar figures. Students consider images displayed as they might be displayed on word processing software. Students analyze how the images have been dilated by looking at the dimensions and the percent enlargement or reduction. Students use the change in percents or dimensions to determine if two figures are similar.

Demonstrate

Talk the Talk: It's a Cloud

Students draw an enlargement and reduction of a curved figure using a given center of dilation and given scale factors. Students then generalize about similar figures and congruent figures.

Facilitation Notes

In this activity, students use a scale factor of $\frac{1}{2}$ to sketch a reduction of a company logo and a scale factor of 2 to sketch an enlargement of the logo.

Emphasize to students the difference between *draw* and *sketch*. Measuring tools are used to create a drawing and no tools are necessary to create a sketch.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- How is this question different from the rigid motion transformations?
- What did you do first?
- Did you sketch the triangle or the circle first? Does it matter?
- Is the height of the triangle in your sketch $\frac{1}{2}$ the height of the triangle in the original logo?
- Is the length of the base of the triangle in your sketch $\frac{1}{2}$ the length of the base of the triangle in the original logo?
- Is the length of the radii in the circle of the sketch $\frac{1}{2}$ the length of the radii in the circle of the original logo?
- Does the triangle appear to be a right triangle?
- Is a leg of the right triangle also the diameter of the circle?
- Is a leg of the right triangle drawn tangent to the circle?
- Is the height of the triangle in your sketch twice the height of the triangle in the original logo?
- Is the length of the base of the triangle in your sketch twice the length of the base of the triangle in the original logo?
- Is the length of the radii in the circle of the sketch twice the length of the radii in the circle of the original logo?
- What is the scale factor between the first and second logos you sketched?
- What is the scale factor between the second and first logos you sketched?

Summary

Scale factors are used to create reductions and enlargements.

Activity 1.1

Dilating Figures with a Scale Factor Greater Than 1



Facilitation Notes

In this activity, students use measuring tools and a given dilation with a scale factor greater than 1 to conclude that when corresponding angles are congruent and the ratios of corresponding sides are equal, the figures have the same shape but different sizes. The terms *dilation*, *center of dilation*, *scale factor*, and *enlargement* are introduced.

Ask a student to read the definitions and information aloud. Analyze the Worked Example as a class.

Differentiation strategies

- To scaffold support, compare and contrast these concepts.
 - The common use of the term *dilation* (e.g., having your eyes dilated for an eye exam) vs. the mathematical definition of the term *dilation*.
 - Dilations vs. rigid motion transformations.
 - The information needed to complete a dilation (center of dilation and scale factor) vs. the information needed to complete a translation (units in horizontal and/or vertical direction), a reflection (line of reflection), or a rotation (center of rotation, angle measure, direction).
- Have students interact with the Worked Example, demonstrating the steps in chronological order. Do not deal with measurements; students will deal with measurements in the questions that follow.
 - Use a colored pencil to outline the original figure and mark its vertices.
 - With the same color, mark the center of dilation and dotted lines extending to each of the vertices of the original figure.
 - Use a second color to mark the center of dilation and dotted lines extending to each of the vertices of the new figure.
 - Connect the vertices of the new figure to make the enlargement.
 - Discuss how the scale factors can be expressed and whether they are greater than or less than one.

Provide students with a rulers and protractors. Have students work with a partner or in a group to complete Questions 1 through 7. Share responses as a class.

Differentiation strategy

Have half of the class complete Questions 2, 3, and the bottom half of Question 5; have the other half of the class complete Question 4 and the top half of Question 5. Share answers as a class. Then complete Questions 6 and 7 together.

Questions to ask

- How many times larger is the new figure than the original figure?
- Is the scale factor in this situation $\frac{1}{2}$ or 2?
- How are the measures of the original figure and the new figure placed in the fraction in order to reflect the appropriate scale factor?
- When determining the scale factor, will you get the same result if you use the distances from the center of dilation or the measures of the sides of the new and original figures?
- If the numerator and denominator were switched, what would be the meaning of the ratio?
- If the scale factor of the dilation is greater than 1, is the new figure always an enlargement of the original figure?
- If the shape of the figure stayed the same, are all of the corresponding angles congruent?
- If the shape of the figure stayed the same, are all of the corresponding sides proportional?

Misconception

Students often think that an enlarged figure has larger angles. Use a right angle to clarify their misunderstanding. For example, draw two similar right triangles. Point out that although the sides are larger in one triangle, the 90° angle remained the same size.

Summary

An enlargement results when a dilation has a scale factor value that is greater than 1.

Activity 1.2

Dilating Figures with a Scale Factor Less Than 1



Facilitation Notes

In this activity, students use measuring tools and a given dilation with a scale factor less than 1 to conclude that when corresponding angles are congruent and the ratios of corresponding sides are

equal, the figures have the same shape but different sizes. The term *reduction* is introduced.

Ask a student to read the definition aloud. Review the Worked Example as a class.

Differentiation strategy

Have students interact with the Worked Example, as described in the previous activity.

Questions to ask

- How is this Worked Example different than the previous one?
- How is this Worked Example the same as the previous one?
- Why are the positions of the original figure and new figure switched?

Provide students with rulers and protractors. Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

Differentiation strategy

Have half of the class complete Questions 2, 3, and the bottom half of Question 5; have the other half of the class complete Question 4 and the top half of Question 5. Share answers as a class. Then complete Question 6 together. (If you used this differentiation strategy in the previous activity, alternate which half of the class completes which questions so that all students have a different experience this time.)

Questions to ask

- How many times larger is the new figure than the original figure?
- Is the scale factor in this situation $\frac{1}{2}$ or 2?
- How are the measures of the original figure and the new figure placed in the fraction in order to reflect the appropriate scale factor?
- When determining the scale factor, will you get the same result if you use the distances from the center of dilation or the measures of the sides of the new and original figures?
- If the numerator and denominator were switched, what would be the meaning of the ratio?
- If the scale factor of the dilation is less than 1, is the new figure always a reduction of the original figure?
- If the shape of the figure stayed the same, are all of the corresponding angles congruent?
- If the shape of the figure stayed the same, are all of the corresponding sides proportional?

Summary

A reduction results when a dilation has a scale factor value that is less than 1.

Activity 1.3

Creating and Verifying Similar Figures



Facilitation Notes

In this activity, students analyze how images have been dilated by looking at the dimensions and the percent enlargement or reduction. Students use the change in percents or dimensions to determine if two figures are similar. The term *similar* is introduced.

Ask a student to read the introduction aloud and complete Question 1 as a class.

Questions to ask

- Which of the adjusted logos look like they are the same shape as the original logo?
- Which of the adjusted logos appear to have corresponding angles that are congruent to the original logo?
- If the adjusted logo appears to look skinnier than the original logo, is it the same shape?
- If the adjusted logo appears to look flatter than the original logo, is it the same shape?

Ask a student to read the definition and information aloud. Examine the Worked Example as a class.

Misconception

Students are familiar with the common use of the term *similar*, meaning *the same*. Contrast this meaning to the mathematical definition of *similar*.

Questions to ask

- Are similar figures always the same shape?
- Can two figures be both congruent and similar? Explain.
- What is the relationship among the terms *dilation*, *similar*, and *proportional*?
- What does *absolute* mean when referring to the height and width?
- What is the meaning of a 100% scale?
- What is the meaning of a 50% scale?

- How do you think the 50% scale was determined?
- What happens when you divide each dimension of the smaller image by the corresponding dimension of the larger image?

Differentiation strategy

To extend the activity, have students determine the center of dilation, measure the distances from the vertices of each figure to the center of dilation, and write proportions to prove the scale factor in the Worked Example is accurate.

Have students work with a partner or in a group to complete Questions 2 through 6. Share responses as a class.

Questions to ask

- What is the meaning of the percents given for the height and the width?
- What happens when the dimensions are not scaled with the same scale factor?
- If you are given an absolute height and width for a new figure, how do you determine if the figure is similar to the original figure?
- Does adding the same value to the length and width of the rectangle maintain the shape of the rectangle?
- Is a 2×4 rectangle the same shape as a 7×9 rectangle?

Differentiation strategy

To extend the activity, discuss the meaning of these phrases: a scale factor of 2, an image scaled to 200%, and a 100% increase.

Summary

Figures are similar when the ratios of their corresponding side lengths are equal and their corresponding angles are congruent.

DEMONSTRATE

Talk the Talk: It's a Cloud

Facilitation Notes

In this activity, students dilate a given figure creating an enlargement and a reduction and answer questions related to similar figures.

Provide the students with a ruler. Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

Differentiation strategies

- Have students select one of the two dilations to complete.
- Suggest students use a lot of points in order to make good dilations of the curved cloud figure.
- Have students label the points.
- Because both the enlargement and reduction will overlap with the original drawing, use colored pencils to aid in visualizing the dilations.

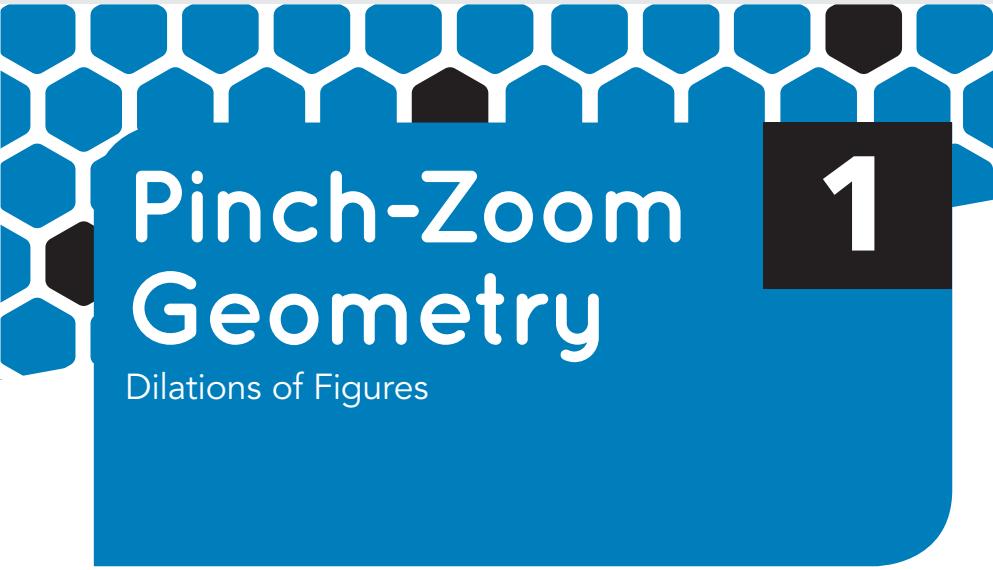
Questions to ask

- Does using the scale factor of $\frac{4}{3}$ result in an image that is a reduction or enlargement?
- Does using the scale factor of $\frac{3}{4}$ result in an image that is a reduction or enlargement?
- Are the corresponding sides congruent or proportional?
- Are the corresponding angles congruent or proportional?
- Why do the new figure and original figure overlap when completing your dilation?

Summary

All congruent figures are similar; however, not all similar figures are congruent.

NOTES



Pinch-Zoom Geometry

Dilations of Figures

1

Warm Up Answer

The scale factor is 100.

WARM UP

A billboard advertises a watch. The face of the watch is 2 meters wide on the billboard. The face of the actual watch is 2 centimeters wide. What scale factor was used to create the billboard?

LEARNING GOALS

- Dilate figures given a center of dilation and scale factor such that the resulting dilation is an enlargement or a reduction of the original figure.
- Identify the scale factor used in a dilation of a figure.
- Determine whether a two-dimensional figure is similar to another by obtaining one from the other using a sequence of dilations.
- Describe a sequence of dilations that demonstrates that two figures are similar.

KEY TERMS

- dilation
- center of dilation
- scale factor
- enlargement
- reduction
- similar

You have learned about geometric transformations that preserve the size and shape of figures. You also know how to use scale factors to produce scale drawings. Is there a geometric transformation that changes the scale of a figure?

LESSON 1: Pinch-Zoom Geometry • 1

ELL Tip

To help students become familiar with the Key Terms, say each term and then have students repeat the term out loud together. After going through the terms, use each word in a sentence to provide context. Ask students what they know about the terms, or if they have heard of them before.

Answers

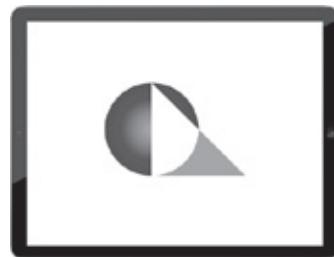
1. Check students' drawings.

Getting Started

Scale Drawing by Doing

Recall that a scale drawing is a representation of a real object or place that is in proportion to the real object or place it represents. The ratios of corresponding side lengths between the drawing and the object are all the same.

Consider the logo shown on the tablet screen.



1. When the logo on the tablet screen appears on the smartphone screen, it will be reduced by a scale factor of $\frac{1}{2}$. Sketch the logo on the smartphone screen and explain your process.



Answers

2. Check students' drawings.

2. When the logo on the tablet screen appears on the desktop screen, it will be enlarged by a scale factor of 2. Sketch the logo on the desktop screen and explain your process.



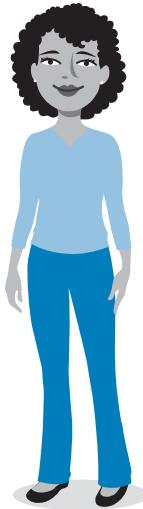
LESSON 1: Pinch-Zoom Geometry • 3

ACTIVITY
1.1

Dilating Figures with a Scale Factor Greater Than 1



“
The image of a dilation can also be called a scale drawing.
”

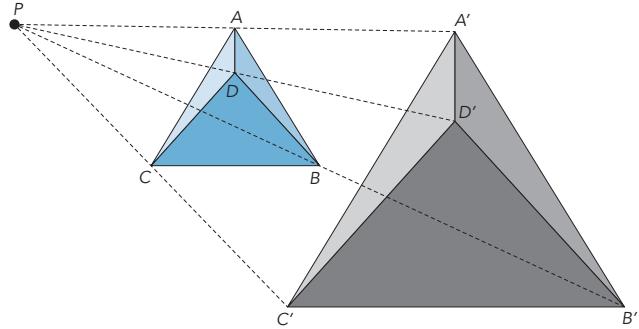


Dilations are transformations that produce figures that are the same shape as the original figure, but not necessarily the same size. Each point on the original figure is moved along a straight line, and the straight line is drawn from a fixed point known as the **center of dilation**. The distance each point moves is determined by the scale factor used.

The **scale factor** is the ratio of the distance of the new figure from the center of dilation to the distance of the original figure from the center of dilation. When the scale factor is greater than 1, the new figure is called an **enlargement**.

WORKED EXAMPLE

This image of a logo was dilated to produce an enlargement using point P as the center of dilation.



The scale factor can be expressed as $\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = \frac{PD'}{PD}$.

1. In the Worked Example, the scale factor is represented by 4 equivalent ratios. What distances are represented by each part of those ratios? Is the scale factor less than 1, equal to 1, or greater than 1? Explain your reasoning.

2. Measure the segment lengths of the original logo in millimeters.

$$m\overline{AB} = \underline{\hspace{2cm}} \quad m\overline{AC} = \underline{\hspace{2cm}}$$

$$m\overline{BC} = \underline{\hspace{2cm}} \quad m\overline{AD} = \underline{\hspace{2cm}}$$

3. Measure the segment lengths of the new logo in millimeters.

$$m\overline{A'B'} = \underline{\hspace{2cm}} \quad m\overline{A'C'} = \underline{\hspace{2cm}}$$

$$m\overline{B'C'} = \underline{\hspace{2cm}} \quad m\overline{A'D'} = \underline{\hspace{2cm}}$$

4. Measure each line segment in millimeters.

$$m\overline{AP} = \underline{\hspace{2cm}} \quad m\overline{AP} = \underline{\hspace{2cm}}$$

$$m\overline{BP} = \underline{\hspace{2cm}} \quad m\overline{BP} = \underline{\hspace{2cm}}$$

$$m\overline{CP} = \underline{\hspace{2cm}} \quad m\overline{CP} = \underline{\hspace{2cm}}$$

$$m\overline{DP} = \underline{\hspace{2cm}} \quad m\overline{DP} = \underline{\hspace{2cm}}$$

5. Determine each ratio.

$$\frac{A'P}{AP} = \underline{\hspace{2cm}} \quad \frac{B'P}{BP} = \underline{\hspace{2cm}}$$

$$\frac{C'P}{CP} = \underline{\hspace{2cm}} \quad \frac{D'P}{DP} = \underline{\hspace{2cm}}$$

$$\frac{B'C'}{BC} = \underline{\hspace{2cm}} \quad \frac{A'B'}{AB} = \underline{\hspace{2cm}}$$

$$\frac{A'D'}{AD} = \underline{\hspace{2cm}} \quad \frac{A'C'}{AC} = \underline{\hspace{2cm}}$$

Answers

Measurements are approximations.

1. The scale factor represents the ratio of the *distance of the new figure from the center of dilation : distance of the original figure from the center of dilation*. The scale factor is greater than 1.

2. $m\overline{AB} = 34 \text{ mm}$
 $m\overline{AC} = 34 \text{ mm}$
 $m\overline{BC} = 34 \text{ mm}$
 $m\overline{AD} = 8 \text{ mm}$

3. $m\overline{A'B'} = 68 \text{ mm}$
 $m\overline{A'C'} = 68 \text{ mm}$
 $m\overline{B'C'} = 68 \text{ mm}$
 $m\overline{A'D'} = 16 \text{ mm}$

4. $m\overline{AP} = 92 \text{ mm}$
 $m\overline{AP} = 46 \text{ mm}$
 $m\overline{B'P} = 140 \text{ mm}$
 $m\overline{BP} = 70 \text{ mm}$
 $m\overline{C'P} = 84 \text{ mm}$
 $m\overline{CP} = 42 \text{ mm}$
 $m\overline{D'P} = 96 \text{ mm}$
 $m\overline{DP} = 48 \text{ mm}$

5. $\frac{A'P}{AP} = \frac{92}{46} = 2$
 $\frac{B'P}{BP} = \frac{140}{70} = 2$
 $\frac{C'P}{CP} = \frac{84}{42} = 2$
 $\frac{D'P}{DP} = \frac{96}{48} = 2$
 $\frac{B'C'}{BC} = \frac{68}{34} = 2$
 $\frac{A'B'}{AB} = \frac{68}{34} = 2$
 $\frac{A'D'}{AD} = \frac{16}{8} = 2$
 $\frac{A'C'}{AC} = \frac{68}{34} = 2$

The notation \overline{AB} means "segment AB." The notation AB means "the length of segment AB ."

To indicate the measure of the segment, you can write AB or $m\overline{AB}$.

Answers

6. Conjectures will vary.

$$\angle A = \angle A' = 60^\circ$$

$$\angle B = \angle B' = 60^\circ$$

$$\angle C = \angle C' = 60^\circ$$

$$\angle ADB = \angle A'D'B' = 120^\circ$$

$$\angle ADC = \angle A'D'C' = 120^\circ$$

$$\angle CDB = \angle C'D'B' = 120^\circ$$

All measures of corresponding angles are congruent.

7. Sample answer.

The shape is exactly the same, but the new logo is twice as large as the original logo.

6. How do you think the angle measures of the new logo will compare with those of the old logo? Make a conjecture. Then, test your conjecture by measuring various angles in the original and new logos. Describe your conclusion.

7. Compare the original logo and the new logo. What do you notice?

ACTIVITY 1.2

Dilating Figures with a Scale Factor Less Than 1

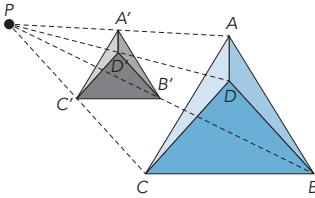


When the scale factor is less than 1, the new figure is called a **reduction**.

The size of the logo and its distance from point P are the same as the Worked Example showing an enlargement of the logo.

WORKED EXAMPLE

The original logo was dilated to produce a reduction using point P as the center of dilation.



The scale factor can be expressed as $\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = \frac{PD'}{PD}$.

Answers

Measurements are approximations.

1. In the Worked Example, the scale factor is represented by 4 equivalent ratios. What distances are represented by each part of those ratios? Is the scale factor less than 1, equal to 1, or greater than 1? Explain your reasoning.

2. Measure the segment lengths of the new logo in millimeters.

$$m\overline{A'B'} = \underline{\hspace{2cm}} \quad m\overline{A'C'} = \underline{\hspace{2cm}}$$

$$m\overline{B'C'} = \underline{\hspace{2cm}} \quad m\overline{A'D'} = \underline{\hspace{2cm}}$$

3. Measure each line segment in millimeters.

$$m\overline{A'P} = \underline{\hspace{2cm}} \quad m\overline{B'P} = \underline{\hspace{2cm}}$$

$$m\overline{C'P} = \underline{\hspace{2cm}} \quad m\overline{D'P} = \underline{\hspace{2cm}}$$

4. Determine each ratio.

$$\frac{A'P}{AP} = \underline{\hspace{2cm}} \quad \frac{B'P}{BP} = \underline{\hspace{2cm}}$$

$$\frac{C'P}{CP} = \underline{\hspace{2cm}} \quad \frac{D'P}{DP} = \underline{\hspace{2cm}}$$

$$\frac{B'C'}{BC} = \underline{\hspace{2cm}} \quad \frac{A'B'}{AB} = \underline{\hspace{2cm}}$$

$$\frac{A'D'}{AD} = \underline{\hspace{2cm}} \quad \frac{A'C'}{AC} = \underline{\hspace{2cm}}$$

1. The scale factor represents the ratio of the *distance of the new figure from the center of dilation : distance of the original figure from the center of dilation*.

The scale factor is less than 1.

2. $m\overline{A'B'} = 17 \text{ mm}$

$m\overline{A'C'} = 17 \text{ mm}$

$m\overline{B'C'} = 17 \text{ mm}$

$m\overline{A'D'} = 4 \text{ mm}$

3. $m\overline{A'P} = 23 \text{ mm}$

$m\overline{B'P} = 35 \text{ mm}$

$m\overline{C'P} = 21 \text{ mm}$

$m\overline{D'P} = 24 \text{ mm}$

4. $\frac{A'P}{AP} = \frac{23}{46} = \frac{1}{2}$

$\frac{B'P}{BP} = \frac{35}{70} = \frac{1}{2}$

$\frac{C'P}{CP} = \frac{21}{42} = \frac{1}{2}$

$\frac{D'P}{DP} = \frac{24}{48} = \frac{1}{2}$

$\frac{B'C'}{BC} = \frac{17}{34} = \frac{1}{2}$

$\frac{A'B'}{AB} = \frac{17}{34} = \frac{1}{2}$

$\frac{A'D'}{AD} = \frac{4}{8} = \frac{1}{2}$

$\frac{A'C'}{AC} = \frac{17}{34} = \frac{1}{2}$

Answers

5. Conjectures will vary.

$$\angle A = \angle A' = 60^\circ$$

$$\angle B = \angle B' = 60^\circ$$

$$\angle C = \angle C' = 60^\circ$$

$$\angle ADB = \angle A'D'B' = 120^\circ$$

$$\angle ADC = \angle A'D'C' = 120^\circ$$

$$\angle CDB = \angle C'D'B' = 120^\circ$$

All measures of corresponding angles are congruent.

6. Sample answer.

The shape is exactly the same, but the new logo is half as large as the original logo.

5. How do you think the angle measures of the new logo will compare with those of the old logo? Make a conjecture. Then, test your conjecture by measuring various angles in the original and new logos. Describe your conclusion.

6. Compare the original logo and the new logo. What do you notice?

ACTIVITY
1.3

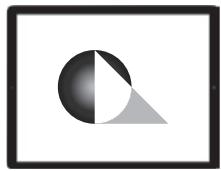
Creating and Verifying Similar Figures



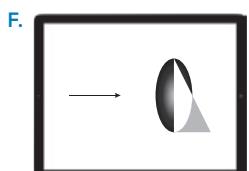
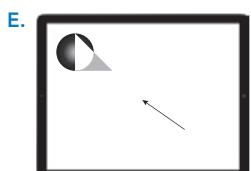
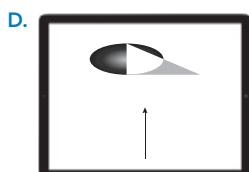
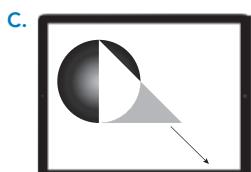
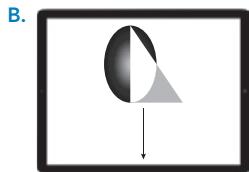
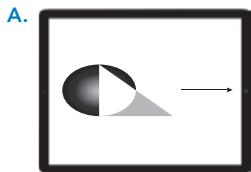
When working with images on a computer, the size of the images can be changed by dragging a corner or side of the image. How you drag the images determines whether or not the scale of the image is maintained.

Anne needs to adjust the original logo to use on different web pages. She plays around with the image to determine how she can adjust the logo and still maintain the same scale.

Each image contains an arrow that indicates how Anne adjusts the logo and the resulting logo.



1. Which of the adjusted logos do you think are dilations of the original? Which are not? Explain your thinking.



Answers

1. Logos C and E are dilations of the original; Logos A, B, D, and F are not dilations because they are not similar.

When you dilate a figure, you create a *similar* figure. When two figures are **similar**, the ratios of their corresponding side lengths are equal. This means that you can create a similar figure by multiplying or dividing all of the side lengths of a figure by the same scale factor (except 0). You can multiply or divide by 1 to create a similar figure, too. In that case, the similar figures are congruent figures. Corresponding angles in similar figures are congruent.

Answers

2. Yes. The dimensions of the larger image are reduced by a scale factor of $\frac{1}{2}$.
3. $\frac{1}{2}$; Divide each dimension of the smaller image by the corresponding dimension of the larger image, or notice that the percents given for the height and width are each 50%.
4. The percents show 100%, and a figure scaled 100% is congruent to the original.

NOTES

Many word processing and graphics software programs allow users to change the sizes of images.

WORKED EXAMPLE

Consider the images shown. The height of the original image is 2.66 inches, and the width is 3.48 inches. The original image is then dilated to create a reduction.



2. Are the two images similar? Explain how you know.

3. What scale factor was used to reduce the image? Describe two different ways you can determine the scale factor.

4. How can you tell that a height of 2.66 in. and a width of 3.48 in. are the original dimensions of the image?

Answers

5. Consider each set of new dimensions or scale percents that show adjustments to this original image. Describe how the image changed and whether the new image is similar to the original. Show your work and explain your reasoning.



Height
<input checked="" type="radio"/> Absolute 2"
<input type="radio"/> Relative
Width
<input checked="" type="radio"/> Absolute 3"
<input type="radio"/> Relative

a. Scale

Height: 225% Width: 225%

b. Scale

Height: 90% Width: 110%

c. Height

Absolute 1.5"
 Relative

Width

Absolute 2.25"
 Relative

d. Height

Absolute 2"
 Relative

Width

Absolute 2"
 Relative

6. Explain why Jed's reasoning is not correct. Draw examples to illustrate your explanation.

Jed



I can dilate a rectangular figure by adding the same value to its length and width.

5a. The image is similar. The scale factor is the same for both dimensions. It is an enlargement with new dimensions that are 4.5 in. \times 6.75 in.

5b. The image is not similar. The dimensions are not scaled with the same scale factor.

5c. The image is similar. It is a reduction. The scale factor is $\frac{3}{4}$ or 75%.

5d. The image is not similar. The dimensions are not scaled with the same scale factor. The height stayed the same, and the width was reduced.

6. Sample answer. When dilating a rectangular figure, you must multiply each dimension by the same scale factor; adding does not work. For example, an original rectangle has a length of 1 in. and a width of 3 in. By adding 1 to each dimension, the new rectangle has a length of 2 in. and a width of 4 in. The ratios of side lengths are $\frac{1}{2}$ and $\frac{3}{4}$, which are not equivalent.



Answers

1. Check students' work.
2. The side lengths of a new figure compared with the side lengths of the original figure form equivalent ratios equal to the scale factor used to dilate the original figure.
3. The corresponding angles in similar figures are congruent.
4. False. A counterexample would be a large square and a small square.
5. True

NOTES

TALK the TALK

It's a Cloud

1. Dilate the figure shown using scale factors of $\frac{4}{3}$ and $\frac{3}{4}$ and point Q as the center of dilation.

Q



2. Describe the relationship between the corresponding sides in an original figure and the new figure resulting from a dilation.
3. Describe the relationship between the corresponding angles in an original figure and the new figure resulting from a dilation.

Determine if each statement is true or false. If a statement is false, include a counterexample. Explain your reasoning.

4. True False All similar figures are also congruent figures.
5. True False All congruent figures are also similar figures.