Similarity Summary

KEY TERMS

- dilation
- center of dilation
- scale factor
- enlargement
- reduction
- similar

LESSON 1

Pinch-Zoom Geometry

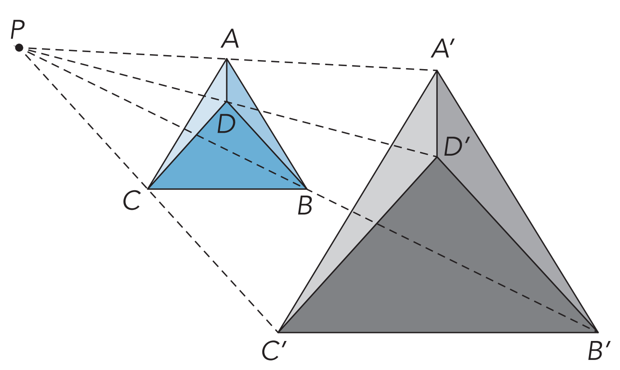
A **dilation** is a transformation that produces figures that are the same shape as the original figure, but not necessarily the same size. Each point on the original figure is moved along a straight line, and the straight line is drawn from a fixed point known as the **center of dilation**. The distance each point moves is determined by the scale factor used. The **scale factor** is the ratio of the *distance of the new figure from the center of dilation* to the *distance of the original figure from the center of dilation.*

When the scale factor is greater than 1, the new figure is called an **enlargement**.

This image of a logo was dilated to produce an enlargement using point P as the center of dilation.

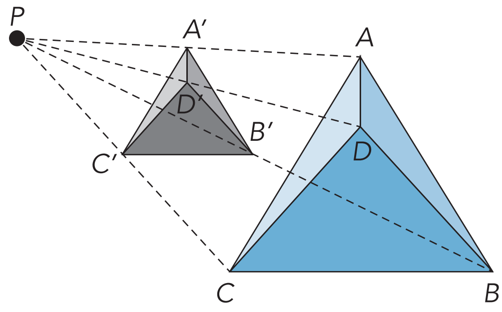
The scale factor can be expressed as

$$\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = \frac{PD'}{PD}.$$



When the scale factor is less than 1, the new figure is called a **reduction**.

For example, the original logo was dilated to produce a reduction using point *P* as the center of dilation.



The scale factor can be expressed as

$$\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = \frac{PD'}{PD}.$$

When you dilate a figure, you create a similar figure. When two figures are **similar**, the ratios of their corresponding side lengths are equal. This means that you can create a similar figure by multiplying or dividing all of the side lengths of a figure by the same scale factor (except 0). You can multiply or divide by 1 to create a similar figure, too. In that case, the similar figures are congruent figures. Corresponding angles in similar figures are congruent.

LESSON 2

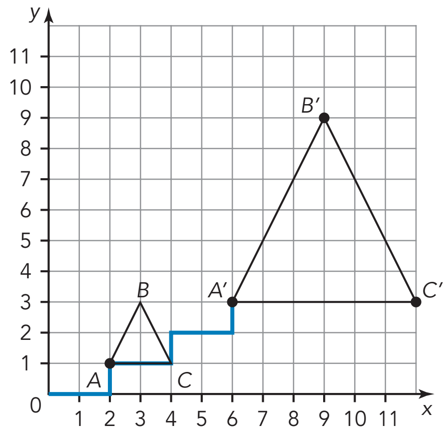
Running, Rising, Stepping, Scaling

If the dilation of a figure is centered at the origin, you can multiply the coordinates of the points of the original figure by the scale factor to determine the coordinates of the new figure. For scale factor *k*, the algebraic representation of the dilation is $(x, y) \rightarrow (kx, ky)$.

For example, to dilate △*ABC* by a scale factor of 3 using the origin as the center of dilation, repeatedly translate point *A* at (2, 1) by multiplying each of the point's coordinates by 3.

$$A'\ (2 \cdot 3, 1 \cdot 3) \rightarrow A'\ (6, 3)$$

Repeat for points *B* and *C*.



LESSON 3

From Here to There

When two figures are similar, the same scale factor can be applied to all side lengths to map one figure onto the other. You can compare the ratios of corresponding side lengths of figures to determine similarity. If the ratio, or scale factor, is the same for all corresponding sides, then the figures are similar.

Sometimes you may need a combination of translations, reflections, rotations, and dilations to map a figure onto a similar figure.

For example, $\triangle MAP$ is similar to $\triangle QRN$. The ratio of corresponding sides is equal to 2, or $\frac{1}{2}$. A possible sequence of transformations to map $\triangle QRN$ onto $\triangle MAP$ is a rotation of 180° about the origin and a dilation by a scale factor of 2. Images created from the same pre-image are always similar figures.