1

Seeing it From a Different Angle

Special Angle Relationships

WARM UP

Solve each equation.

1. $90 = 5x + x$

2. $x + 3x = 180$

3. $(180 - x) + (90 - x) = 210$

LEARNING GOALS

- Calculate the supplement of an angle.
- Calculate the complement of an angle.
- Classify adjacent angles, linear pairs, and vertical angles.
- Use facts about supplementary, complementary, vertical, and adjacent angles and linear pairs in multi-step problems to write and solve simple equations for unknown angles.

KEY TERMS

- straight angle
- supplementary angles
- complementary angles
- perpendicular
- collinear
- adjacent angles
- linear pair
- vertical angles

You know how to classify individual angles based on their measure. Now consider how angles are related to each other. What types of special relationships exist between angles?

Separated at Birth

A **straight angle** is formed when the sides of the angle point in exactly opposite directions. The two legs form a straight line through the vertex.

Let's use a straightedge and a protractor to investigate some special relationships between angles.

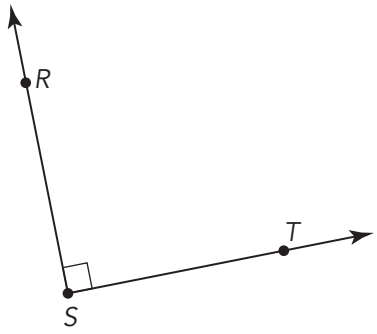
1. Draw a *straight angle*, $\angle ABC$, and label each point.

a. Draw a point *D* above line *AC* and use your straightedge to draw a ray, $\overrightarrow{BD}$. Next, use your protractor to measure the angles formed by $\overrightarrow{BD}$ and $\overleftrightarrow{AC}$.

b. Draw a point *E* different from point *D* below line *AC* and use your straightedge to draw another ray, $\overrightarrow{BE}$. Next, use your protractor to measure the angles formed by $\overrightarrow{BE}$ and $\overleftrightarrow{AC}$.

c. What do you notice about the measures of the angles formed by a single ray coming off of a line?

2. Use the right angle shown to answer each question.



a. Draw a point V in the interior of $\angle RST$ and use your straightedge to draw a ray, $\overrightarrow{SV}$. Next, use your protractor to measure the angles formed by $\overrightarrow{SV}$ and $\angle RST$.

b. Draw a point Z in the interior of $\angle RST$ and use your straightedge to draw another ray, $\overrightarrow{SZ}$. Next, use your protractor to measure the angles formed by $\overrightarrow{SZ}$ and $\angle RST$.

c. What do you notice about the measures of the angles formed by a single ray that divides a right angle into two angles?

ACTIVITY
1.1

Supplements and Complements

Two angles are **supplementary angles** if the sum of their angle measures is equal to 180°.

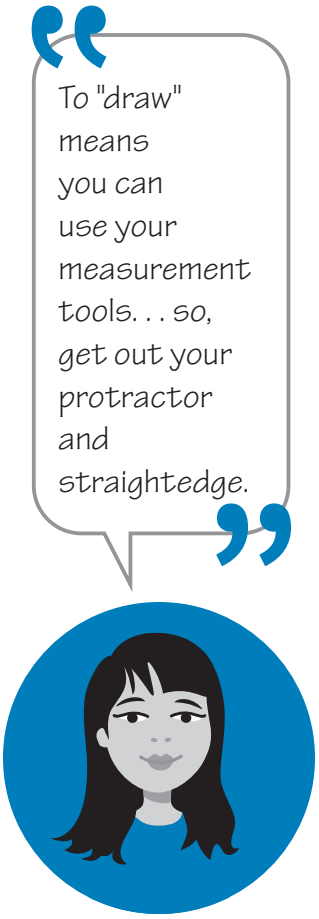
Two angles are **complementary angles** if the sum of their angle measures is equal to 90°.

In the previous activity, you created *supplementary angles* and *complementary angles*.

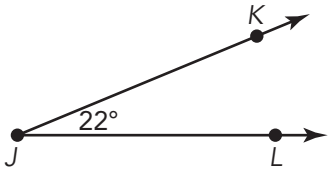
Let's create sets of supplementary angles.

1. **Use a protractor to draw a pair of supplementary angles that share a side. What is the measure of each angle?**

2. **Use a protractor to draw a pair of supplementary angles that do not share a side. What is the measure of each angle?**

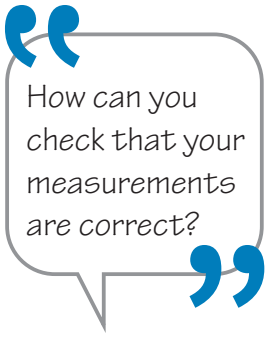


3. **Calculate the measure of an angle that is supplementary to $\angle KJL$.**



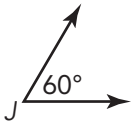
Now, let's create sets of complementary angles.

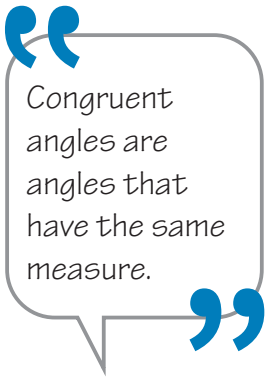
4. Use a protractor to draw a pair of complementary angles that share a side. What is the measure of each angle?



5. Use a protractor to draw a pair of complementary angles that do not share a side. What is the measure of each angle?

6. Calculate the measure of an angle that is complementary to $\angle J$.





7. Given each statement, write and solve an equation to determine the measure of each angle in the angle pair.

a. Two angles are both congruent and supplementary.

b. Two angles are both congruent and complementary.

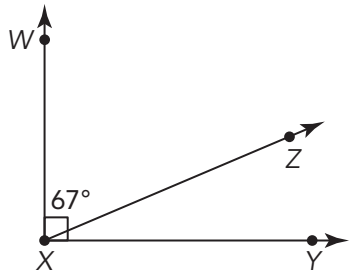
c. The supplement of an angle is half the measure of the angle itself.

d. The supplement of an angle is 20° more than the measure of the angle itself.

e. Angles 1 and 2 are complementary. The measure of Angle 2 is 10° larger than the measure of Angle 1.

f. Angles 1 and 2 are supplementary. The measure of Angle 1 is three degrees less than twice the measure of Angle 2.

8. Use the figure to determine the measure of $\angle ZXY$.



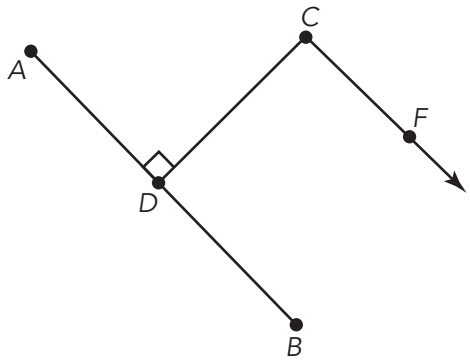
ACTIVITY
1.2 Perpendicular Lines

Let's explore angles formed by *perpendicular* lines.

1. Draw and label $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ at point *E*. How many right angles are formed?

2. Draw and label $\overleftrightarrow{BC} \perp \overleftrightarrow{AB}$ at point *B*. How many right angles are formed?

3. Name all angles that you know are right angles in the figure shown. Note: Points *A*, *D*, and *B* lie on the same line segment.



Two lines, line segments, or rays are **perpendicular** if they intersect to form 90° angles. The symbol for perpendicular is ⊥.

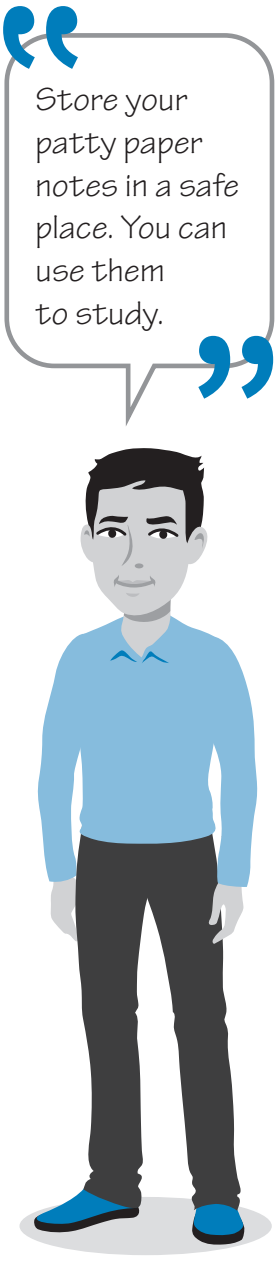
When points lie on the same line or line segment, they are said to be **collinear**.

Patty paper is traditionally a food service product, used to separate hamburger patties, meats, cheeses, and pastries.

You can use patty paper to create special angle pairs, just as you can use a protractor and straightedge.

4. **Draw a figure on a sheet of patty paper. What do you notice when you write on the paper? What about when you fold the paper?**

Read each question in its entirety before using your patty paper to illustrate the angles. You should use one sheet of patty paper for each question.



5. **Draw a straight angle on your patty paper. Then *fold* your patty paper to create a pair of supplementary angles that are not congruent. Label the patty paper "Supplementary Angles."**

6. **Draw a straight angle on your patty paper. Then *fold* your patty paper to create a pair of supplementary angles that are congruent. What do you know about the angles? What do you know about the straight angle and the line you created with your fold? Label the patty paper accordingly.**

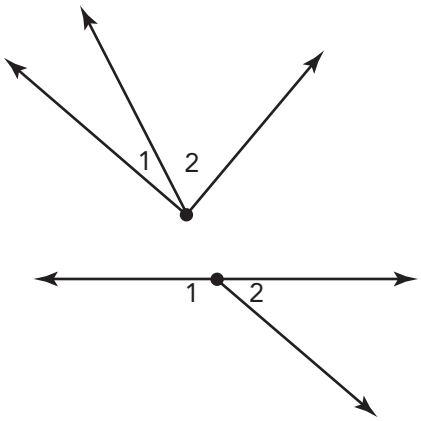
7. **Draw a right angle on your patty paper. Then *fold* your patty paper to create a pair of complementary angles that are not congruent. Label the patty paper "Complementary Angles."**

ACTIVITY
1.3 Adjacent Angles

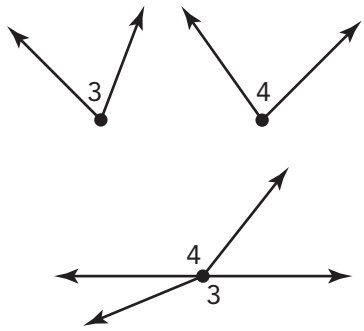
In each of the next three activities you will explore special angle pairs. Let's get started.

WORKED EXAMPLE

∠1 and ∠2 are adjacent angles.

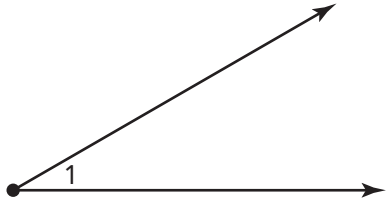


∠3 and ∠4 are *not* adjacent angles.

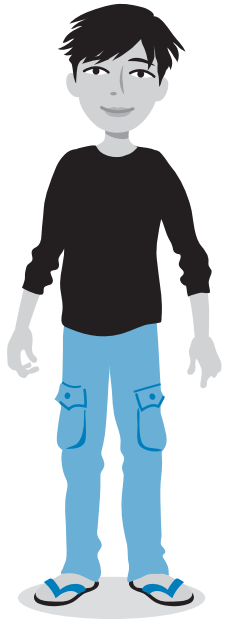


1. Use the figures in the Worked Example to define *adjacent angles* in your own words.

2. Draw and label ∠2 so that it is adjacent to ∠1.



"Can you think of other ways to draw ∠2 so that it is adjacent to ∠1?"



3. **Is it possible to draw two angles that share a vertex, but do not share a common side? If so, draw an example. Would the angles be adjacent angles? If not, explain your reasoning.**

Adjacent angles are two angles that share a common vertex and share a common side.

4. **Is it possible to draw two angles that share a side, but do not share a vertex? If so, draw an example. If not, explain your reasoning.**

5. **Use one sheet of patty paper to create a set of *congruent* adjacent angles. Describe your process. Label the patty paper "Congruent Adjacent Angles."**

6. **Use one sheet of patty paper to create a set of *non-congruent* adjacent angles. Describe your process. Label the patty paper "Adjacent Angles."**

7. **Are all adjacent angles supplementary? Explain your reasoning.**

8. **Are all supplementary angles adjacent? Explain your reasoning.**

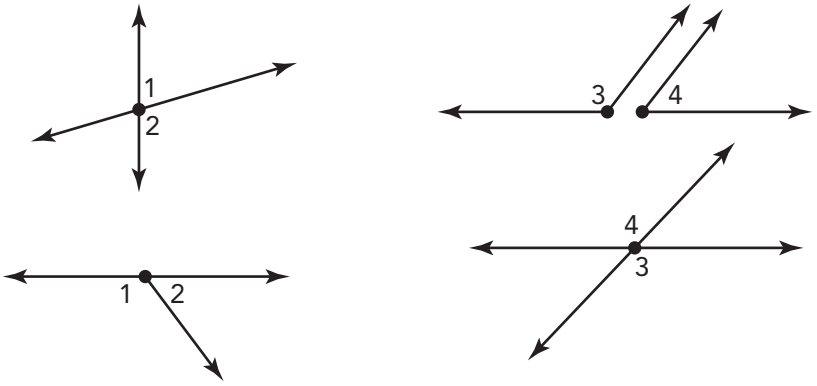
NOTES

ACTIVITY
1.4 Linear Pairs

Let's explore a different angle relationship.

WORKED EXAMPLE

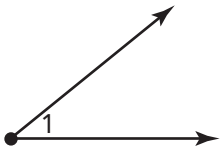
$\angle 1$ and $\angle 2$ form a linear pair. $\angle 3$ and $\angle 4$ are do *not* form a linear pair.



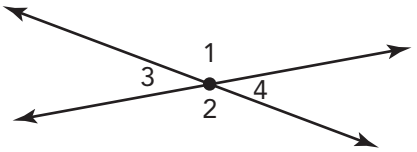
1. **Use the figures in the Worked Example to define a *linear pair of angles* in your own words.**

A **linear pair** of angles is formed by two adjacent angles that have noncommon sides that form a line.

2. Draw ∠2 so that it forms a linear pair with ∠1. Use one sheet of patty paper and record your response. Label your patty paper "Linear Pair."



3. Name all linear pairs in the figure shown.



4. If the angles that form a linear pair are congruent, what can you conclude?

5. What is the difference between a linear pair of angles and supplementary angles that share a common side?

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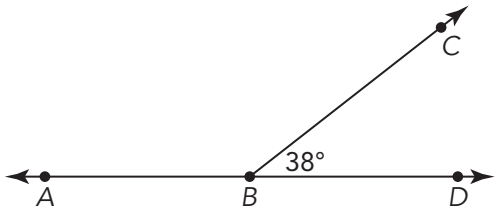
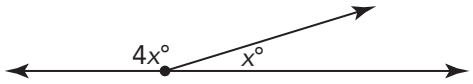
6. What is the difference between a linear pair of angles and supplementary angles that do not share a common side?

7. Angle *ABC* and angle *CBD* form a linear pair. Write and solve an equation to determine the measure of $\angle ABC$.



8. The angles shown are a linear pair of angles. Solve for *x*.



9. Write and complete the sentence on your Linear Pairs patty paper.

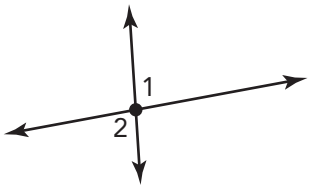
   If two angles form a linear pair, then the sum of the measures of the linear pair of angles is _____.

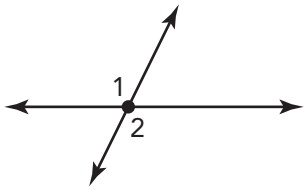
ACTIVITY
1.5 Vertical Angles

Let's explore one more special angle relationship.

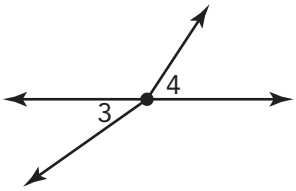
WORKED EXAMPLE

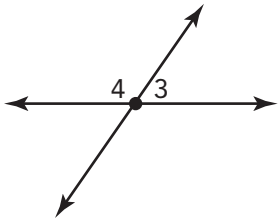
∠1 and ∠2 are vertical angles.





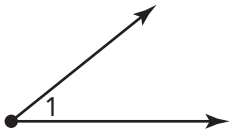
∠3 and ∠4 are *not* vertical angles.



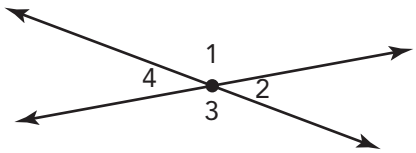


1. Use the figures in the Worked Example to define *vertical angles* in your own words.

2. Draw ∠2 so that it forms a vertical angle pair with ∠1.



3. Name all vertical angle pairs in the diagram shown.



Vertical angles are two nonadjacent angles that are formed by two intersecting lines.

4. Trace the figure in Question 3 on two different sheets of patty paper. Be sure you number the angles. Use the patty paper to investigate the measures of vertical angles. What do you notice?

5. Use your protractor to measure each angle in Question 3. What do you notice?

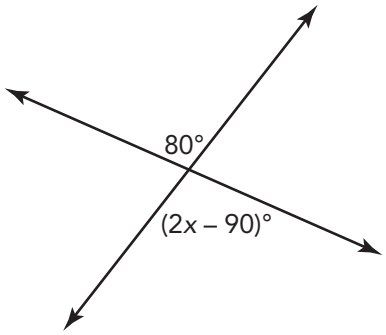
6. Use what you know about supplementary angles and linear pairs to justify your investigations in Questions 4 and 5.

7. Label one sheet of your patty paper "Vertical Angles" and write and complete the sentence.

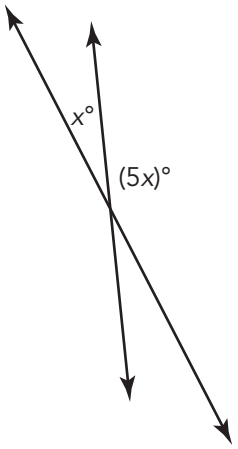
When two lines intersect to form vertical angles, each pair of vertical angles _____.

8. Write and solve an equation to determine the measures of all four angles in each diagram.

a.



b.

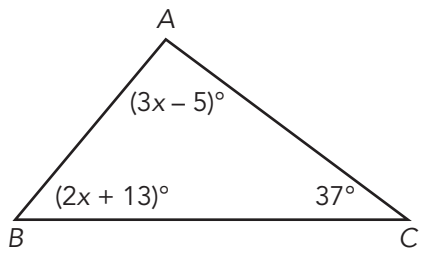


ACTIVITY
1.6 Applying the Triangle Sum Theorem

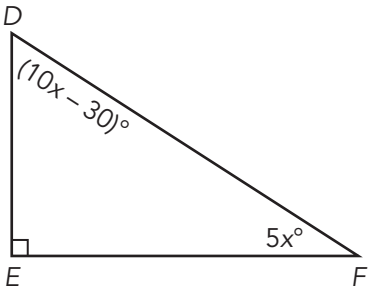
You have learned about the Triangle Sum Theorem, which states that the sum of the measures of the interior angles of a triangle is 180°. You can use that information to write equations and solve for an unknown value.

Determine the measure of each unknown angle.

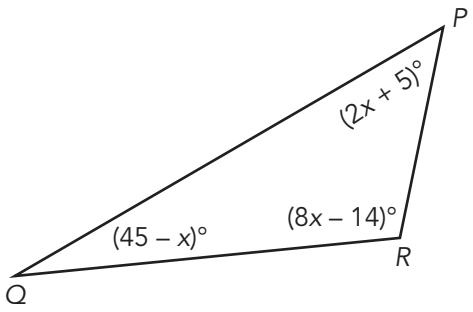
1.



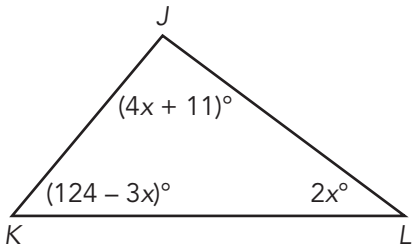
2.



3.



4.



NOTES

TALK the TALK

Perfect Pairs

Answer each question. Draw a figure to justify your response.

1. **Two intersecting lines form how many**

   a. **pairs of supplementary angles?**

   b. **pairs of complementary angles?**

   c. **pairs of adjacent angles?**

   d. **linear pairs of angles?**

   e. **pairs of vertical angles?**

2. **Suppose two lines intersect. If you are given the measure of one angle, can you determine the measures of the remaining angles without using a protractor? Explain your reasoning.**

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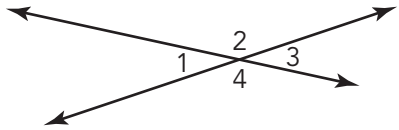
3. When two lines intersect, four different angles are formed as shown.



a. Describe the relationship between vertical angles.

b. Describe the relationship between adjacent angles.

4. Draw and label a diagram that includes at least one of each relationship. Then identify the angles that satisfy each description.

- complementary angles
- supplementary angles
- perpendicular lines
- adjacent angles
- linear pair
- vertical angles