

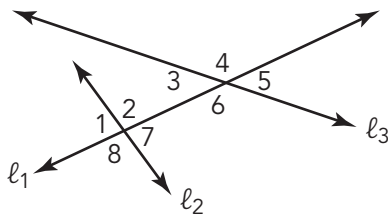
# Crisscrossed Applesauce

# 3

## Angle Relationships Formed by Lines Intersected by a Transversal

### WARM UP

Use the numbered angles in the diagram to answer each question.



1. Which angles form vertical angles?
2. Which angles are congruent?

### LEARNING GOALS

- Explore the angles determined by two lines that are intersected by a transversal.
- Use informal arguments to establish facts about the angles created when parallel lines are cut by a transversal.
- Identify corresponding angles, alternate interior angles, alternate exterior angles, same-side interior angles, and same-side exterior angles.
- Determine the measure of alternate interior angles, alternate exterior angles, same-side interior angles, same-side exterior angles, and corresponding angles.

### KEY TERMS

- transversal
- alternate interior angles
- alternate exterior angles
- same-side interior angles
- same-side exterior angles

When two lines intersect, special angle pair relationships are formed. What special angle pair relationships are formed when three lines intersect?

# Getting Started

## Euclid's Fifth Postulate

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A *postulate* is a statement that is accepted to be true without proof.

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Euclid is known as the father of geometry, and he stated five postulates upon which every other geometric relationship can be based. The fifth postulate is known as the *Parallel Postulate*. Consider one of the equivalent forms of this postulate:

"Given any straight line and a point not on the line, there exists one and only one straight line that passes through the point and never intersects the line."

**1. Draw a picture that shows your interpretation of this statement of the postulate.**

**2. Why do you think this postulate is called the Parallel Postulate?**

A common definition of parallel lines is co-planar lines that are always equidistant, or the same distance apart.

**3. Explain what is meant by this definition and demonstrate it on your diagram.**

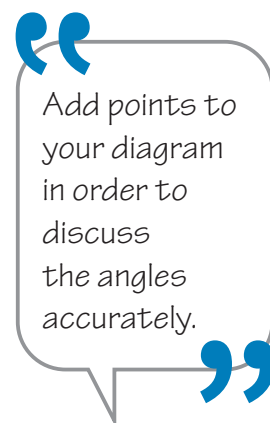
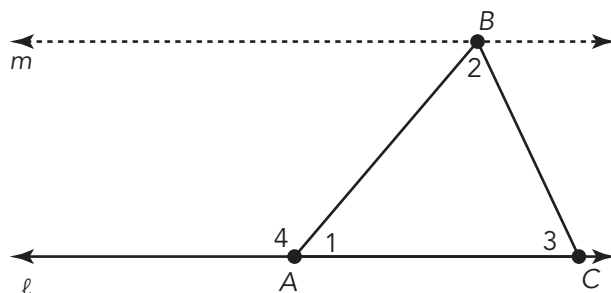
ACTIVITY  
**3.1**

# Creating New Angles from Triangles



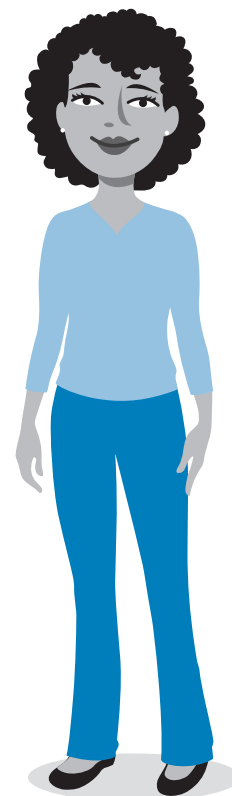
In the previous lesson, you determined measures of interior and exterior angles of triangles.

Consider the diagram shown. Lines  $m$  and  $\ell$  are parallel. This is notated as  $m \parallel \ell$ .



1. Explain the relationships between the numbered angles in the diagram.

2. Trace the diagram onto two sheets of patty paper and extend  $\overline{AB}$  to create a line that contains the side of the triangle. Align the triangles on your patty paper and translate the bottom triangle along  $\overline{AB}$  until  $\overline{AC}$  lies on line  $m$ . Trace your translated triangle on the top sheet of patty paper. Label the translated triangle  $A'B'C'$ .



3. Angle 1 in  $\triangle A'B'C'$  is a translation of Angle 1 in  $\triangle ABC$ .  
How are the measures of these angles related to each other?  
Explain your reasoning.

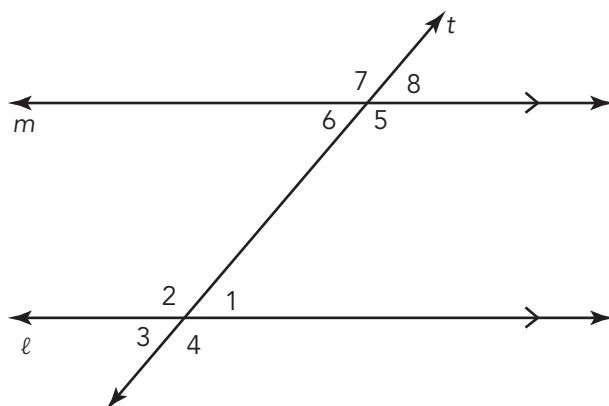
4. Extend  $\overline{CB}$  to create a line. Use what you know about special angle pairs to label all six angles at point  $B$  as congruent to  $\angle 1$ ,  $\angle 2$ , or  $\angle 3$ . Explain your reasoning. Sketch your patty paper drawing.

ACTIVITY  
**3.2**

## Angles Formed by Three Lines



Consider your diagram from the previous activity. If you remove  $\overline{BC}$  and the line containing  $\overline{BC}$ , your diagram might look similar to the diagram shown.



Arrowheads on lines in diagrams indicate parallel lines. Lines or segments with the same number of arrowheads are parallel.

In this diagram, the two parallel lines,  $m$  and  $\ell$ , are intersected by a transversal,  $t$ . A **transversal** is a line that intersects two or more lines.

Recall that corresponding angles are angles that have the same relative positions in geometric figures. In the previous activity, when you translated  $\triangle ABC$  to create  $\triangle A'B'C'$  you created three sets of corresponding angles. You can also refer to corresponding angles in relation to lines intersected by a transversal.

The transversal,  $t$ , in this diagram corresponds to the line that contained side  $AB$  in your patty paper diagram.

1. Use the diagram to name all pairs of corresponding angles.

2. Analyze each angle pair:  $\angle 1$  with  $\angle 6$  and  $\angle 2$  with  $\angle 5$ .

a. Are the angles between (on the *interior* of) lines  $m$  and  $\ell$ , or are they outside (on the *exterior* of) lines  $m$  and  $\ell$ ?

b. Are the angles on the same side of the transversal, or are they on opposite (*alternating*) sides of the transversal?

There is a special relationship between angles like  $\angle 1$  and  $\angle 6$  or  $\angle 2$  and  $\angle 5$ . **Alternate interior angles** are angles formed when a transversal intersects two other lines. These angle pairs are on opposite sides of the transversal and are between the two other lines.

**Alternate exterior angles** are also formed when a transversal intersects two lines. These angle pairs are on opposite sides of the transversal and are outside the other two lines.

**3. Use your diagram to name all pairs of alternate exterior angles.**

Two additional angle pairs are *same-side interior angles* and *same-side exterior angles*.

**4. Use the names to write a definition for each type of angle pair. Identify all pairs of each type of angle pair from the diagram.**

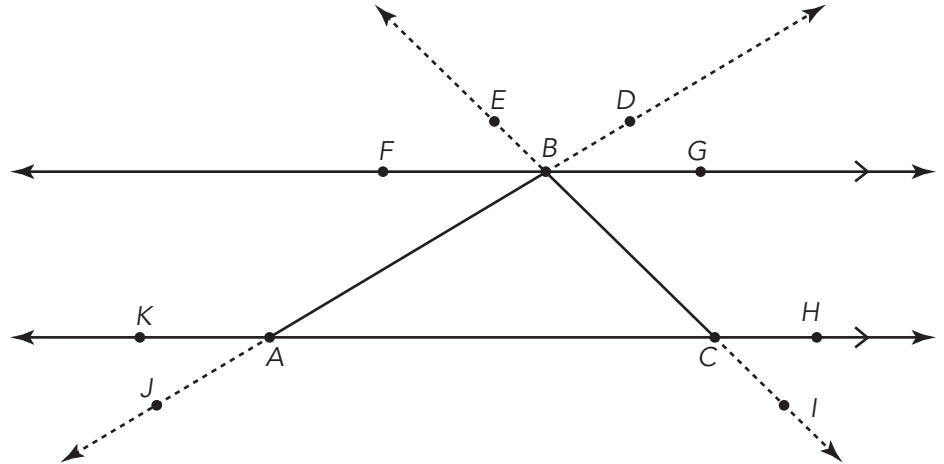
**a. same-side interior angles**

**b. same-side exterior angles**

5. In the diagram from the previous activity, each time you extended a side of the triangle, you created a transversal. Identify the angle pairs described by each statement.

- a. corresponding angles if  $\overleftrightarrow{BC}$  is the transversal

- b. alternate interior angles if  $\overleftrightarrow{BC}$  is the transversal



- c. alternate exterior angles if  $\overleftrightarrow{AB}$  is the transversal

- d. same-side interior angles if  $\overleftrightarrow{AB}$  is the transversal

- e. same-side exterior angles if  $\overleftrightarrow{AB}$  is the transversal

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**Same-side interior angles** are on the same side of the transversal and are between the other two lines.

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**Same-side exterior angles** are on the same side of the transversal and are outside the other two lines.

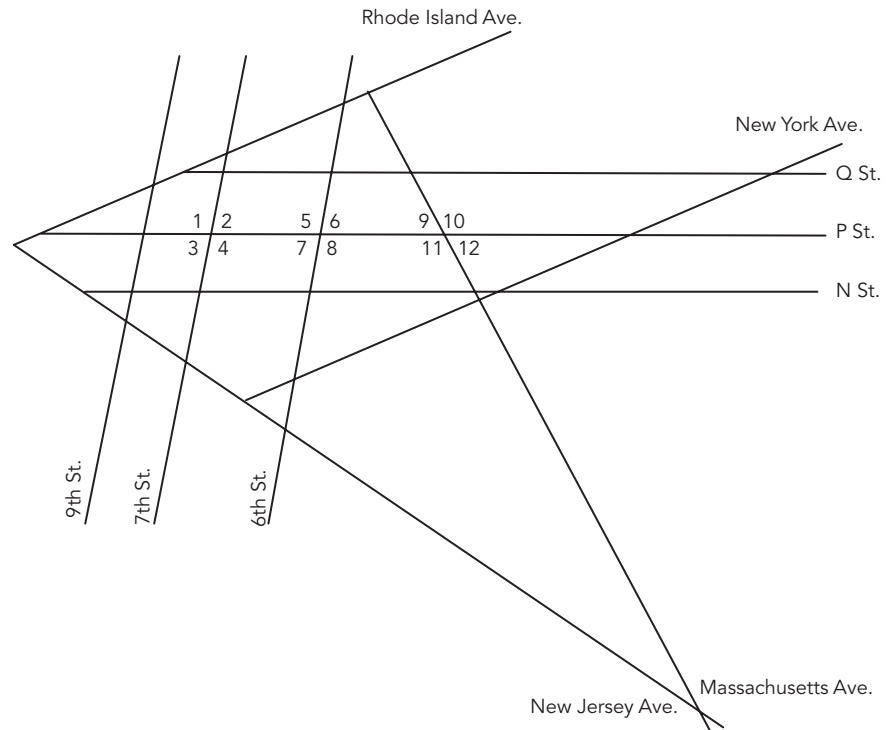
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ACTIVITY  
**3.3**

## Analyzing Special Angle Pairs



Consider the map of Washington, D.C., shown. Assume that all line segments that appear to be parallel are parallel.



1. Consider only P St., N St., Massachusetts Ave., and 6th St. Which of these streets, if any, are transversals? Explain your reasoning.

Let's explore the relationships between the angles formed from lines cut by transversals.

2. Use a protractor to measure all 12 angles labeled on the diagram.



3. Consider only 6th St., 7th St., and P St.

a. Which of these streets, if any, are transversals? Explain your reasoning.

b. What is the relationship between 6th St. and 7th St.?

c. Name the pairs of alternate interior angles. What do you notice about their angle measures?

d. Name the pairs of alternate exterior angles. What do you notice about their angle measures?

e. Name the pairs of corresponding angles. What do you notice about their angle measures?

f. Name the pairs of same-side interior angles. What do you notice about their angle measures?

g. Name the pairs of same-side exterior angles. What do you notice about their angle measures?

4. Consider only 6th St., Massachusetts Ave., and P St.

a. Which of these streets, if any, are transversals?

b. What is the relationship between 6th St. and Massachusetts Ave.?

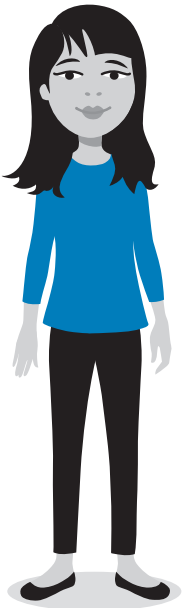
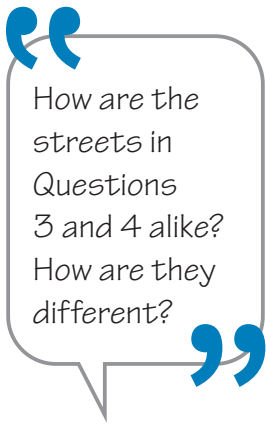
c. Name the pairs of alternate interior angles. What do you notice about their angle measures?

d. Name the pairs of alternate exterior angles. What do you notice about their angle measures?

e. Name the pairs of corresponding angles. What do you notice about their angle measures?

f. Name the pairs of same-side interior angles. What do you notice about their angle measures?

g. Name the pairs of same-side exterior angles. What do you notice about their angle measures?



ACTIVITY  
**3.4**

## Line Relationships and Angle Pairs



NOTES

In the previous activity, you explored angle pairs formed by a transversal intersecting two non-parallel lines and a transversal intersecting two parallel lines.

1. **Make a conjecture about the types of lines cut by a transversal and the measures of the special angle pairs.**

Refer back to the measurements of the labeled angles on the diagram of Washington, D.C.

2. **What do you notice about the measures of each pair of alternate interior angles when the lines are**

- a. **non-parallel?**

- b. **parallel?**

3. **What do you notice about the measures of each pair of alternate exterior angles when the lines are**

- a. **non-parallel?**

- b. **parallel?**

4. What do you notice about the measures of each pair of corresponding angles when the lines are

a. non-parallel?

b. parallel?

5. What do you notice about the measures of the same-side interior angles when the lines are

a. non-parallel?

b. parallel?

6. What do you notice about the measures of the same-side exterior angles when the lines are

a. non-parallel?

b. parallel?

7. Summarize your conclusions in the table by writing the relationships of the measures of the angles. The relationships are either congruent or not congruent, supplementary or not supplementary.

Angles	Two Parallel Lines Intersected by a Transversal	Two Non-Parallel Lines Intersected by a Transversal
Alternate Interior Angles		
Alternate Exterior Angles		
Corresponding Angles		
Same-Side Interior Angles		
Same-Side Exterior Angles		

8. Use transformations to explain how to map the angle pairs that are congruent.

9. Use transformations to explain why certain angle pairs are supplementary.

ACTIVITY  
3.5

# Solving for Unknown Angle Measures

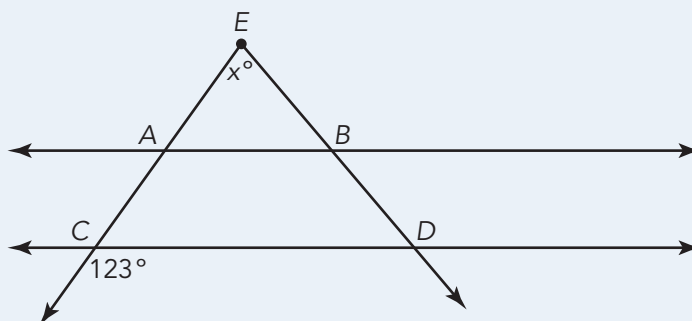


Use what you know about angle pairs to answer each question.



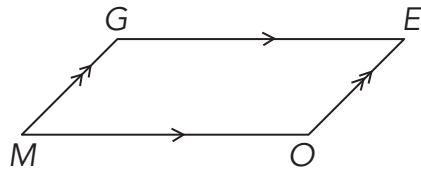
1. Sylvia and Scott were working together to solve the problem shown.

Given:  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ . Solve for  $x$ . Show all your work.



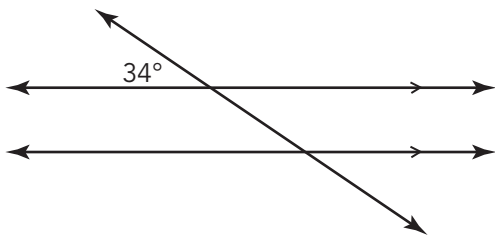
- Sylvia concluded that  $x = 66^\circ$ . How did Sylvia get her answer?
- Scott does not agree with Sylvia's answer. He thinks there is not enough information to solve the problem. How could Scott alter the figure to show why he disagrees with Sylvia's answer?
- Who is correct?

2. Opposite sides of the figure shown are parallel. Suppose that the measure of Angle  $M$  is equal to  $30^\circ$ . Solve for the measures of Angles  $G$ ,  $E$ , and  $O$ . Explain your reasoning.

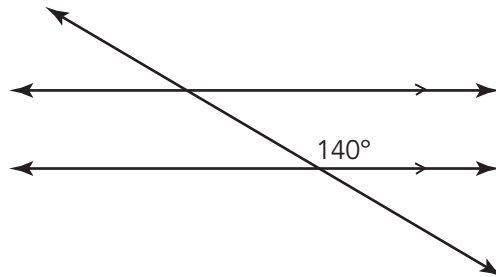


3. Determine the measure of each unknown angle.

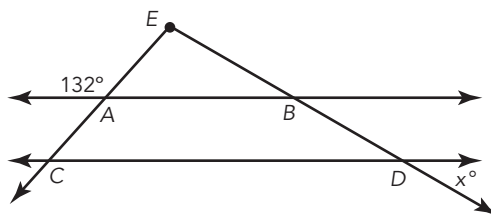
a.



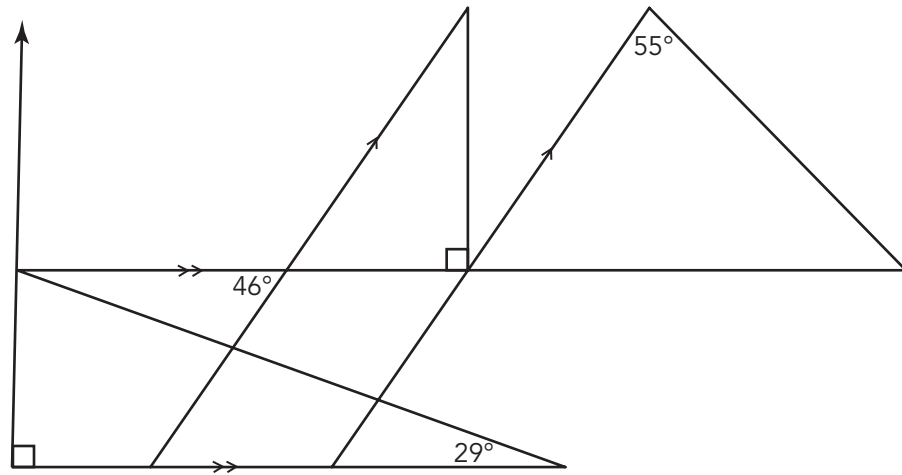
b.



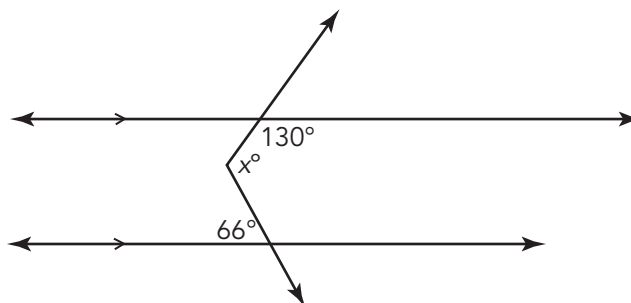
4. In this figure,  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$  and  $\overrightarrow{EC} \perp \overrightarrow{ED}$ . Solve for  $x$ . Show all your work.



5. Determine the measure of each angle in this figure.



6. Solve for  $x$ . Show all your work.





## TALK the TALK

### What's So Special?

1. If two lines are intersected by a transversal, when are

a. alternate interior angles congruent?

b. alternate exterior angles congruent?

c. vertical angles congruent?

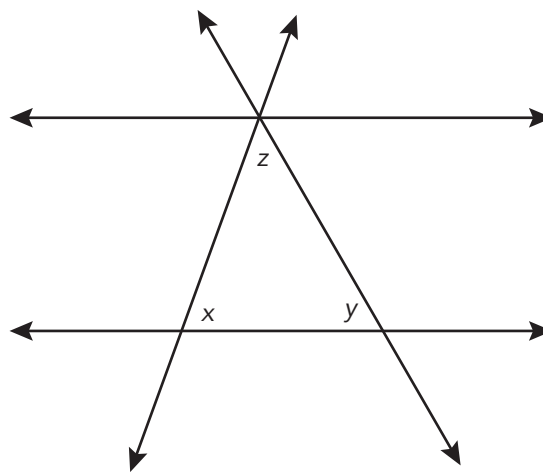
d. corresponding angles congruent?

e. same-side interior angles supplementary?

f. same-side exterior angles supplementary?

g. linear pairs of angles supplementary?

2. Briana says that she can use what she learned about parallel lines cut by a transversal to show that the measures of the angles of a triangle sum to  $180^\circ$ . She drew the figure shown.



Explain what Briana discovered.