Crisscrossed Applesauce

3

Angle Relationships Formed by Lines Intersected by a Transversal

MATERIALS

Straightedge
Patty paper
Protractor

Lesson Overview

Students explore the angles formed when two lines are intersected by a transversal. They use the Parallel Postulate and transformations to begin exploring and identifying the angles. The terms *transversal, alternate interior angles, alternate exterior angles, same-side interior angles,* and *same-side exterior angles* are introduced. Students are given a street map and asked to identify transversals and special pairs of angles. After measuring several angles, they conclude that when two parallel lines are intersected by a transversal, the alternate interior, alternate exterior, and corresponding angles are congruent. Students also conclude that same-side interior and same-side exterior angles are supplementary. When the lines are not parallel, these relationships do not hold true. Finally, students solve problems using parallel line and angle relationships.

Grade 8

Expressions, Equations, and Relationships

(8) The student applies mathematical process standards to use one-variable equations or inequalities in problem situations. The student is expected to:

> (D) use informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Essential Ideas

- A transversal is a line that intersects two or more lines.
- When two parallel lines are intersected by a transversal, corresponding angles are congruent.
- When two parallel lines are intersected by a transversal, alternate interior angles are congruent.

- When two parallel lines are intersected by a transversal, alternate exterior angles are congruent.
- When two parallel lines are intersected by a transversal, same-side interior angles are supplementary.
- When two parallel lines are intersected by a transversal, same-side exterior angles are supplementary.
- Parallel line and angle relationships can be proven using transformations.

Lesson Structure and Pacing: 3 Days

Day 1

Engage

Getting Started: Euclid's Fifth Postulate

Students analyze a version of Euclid's fifth postulate: the Parallel Postulate. They draw their interpretations of the postulate, explain why it is called the Parallel Postulate, and relate the drawing to the definition of parallel lines as lines that are always equidistant from each other.

Develop

Activity 3.1: Creating New Angles from Triangles

Students use the Parallel Postulate and a translation of a triangle along an extension of one side of the triangle to begin exploring angles formed by parallel lines cut by a transversal. They extend two sides of a triangle and reason about which angles are congruent to the angles in the original triangle. Students revisit this activity throughout the lesson.

Activity 3.2: Angles Formed by Three Lines

Students learn about angle pairs formed when two lines are intersected by a transversal: *alternate interior angles, alternate exterior angles, same-side interior angles, same-side exterior angles*, and *corresponding angles*. They identify examples of these special pairs of angles in diagrams, including the diagram from Activity 2.1.

Day 2

Activity 3.3: Analyzing Special Angle Pairs

Students are given a street map of Washington, D.C., and use the map to identify and measure examples of transversals, alternate interior angles, alternate exterior angles, same-side interior angles, same-side exterior angles, and corresponding angles. They begin making comparisons about relationships between the measures of the angle pairs.

Activity 3.4: Line Relationships and Angle Pairs

Students refer back to Activity 2.3 to draw conclusions about special pairs of angles formed by a transversal intersecting two non-parallel lines and a transversal intersecting two parallel lines. They conclude that when two parallel lines are intersected by a transversal, the alternate interior, alternate exterior, and corresponding angles are congruent and the same-side interior and same-side exterior angles are supplementary. Students use transformations to explain why these relationships exist for parallel lines.

Day 3

Activity 3.5: Solving for Unknown Angle Measures

Students solve problems using the relationships between angle pairs formed when parallel lines are intersected by a transversal. Problems focus on understanding the angle pairs based on multiple transversals and determining unknown measures of angles in different situations. Students then write and solve equations. The last situation requires students to extend a transversal so that it intersects both parallel lines, forming congruent alternate interior angles and a triangle.

Demonstrate

Talk the Talk: What's So Special?

Students summarize the situations in which special pairs of angles are either congruent or supplementary. They conclude that these situations exist when a transversal has intersected two parallel lines in most cases. However, vertical angles are always congruent, and linear pairs are always supplementary. Students also prove the Triangle Sum Theorem using parallel line relationships.

Facilitation Notes

In this activity, students interpret Euclid's fifth postulate: the Parallel Postulate. The definition of *parallel lines* is introduced.

Students already know the word *parallel*. Beginning in grade 4, students drew and identified points, lines, angles, and parallel and perpendicular lines. They also classified figures based on the parallel relationships.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- Are all lines straight lines?
- How long is your line?
- Did you draw arrowheads on the ends of your line? What do they represent?
- Did you draw your point above or below the given line? Does it matter?
- What is a postulate?
- What is a theorem?
- What is the difference between a postulate and a theorem?
- What does coplanar mean?
- Are any two lines coplanar lines?
- Are any three lines coplanar lines?
- Are any two points coplanar points?
- Are any three points coplanar points?

Summary

There exists only one line parallel to a given line passing through a point not on the line.

DEVELOP

Activity 3.1
Creating New Angles from Triangles

Facilitation Notes

In this activity, students explore the measures of angles formed by parallel lines cut by a third line.

Provide students with patty paper for this activity.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- Is the sum of the measures of Angles 1, 2, and 3 equal to 180°?
- Why is the sum of the measures of Angles 1, 2, and 3 equal to 180°?
- Is Angle 4 an exterior angle of the triangle?
- Is the sum of the measures of Angles 2 and 3 equal to the measure of Angle 4?
- Why is the sum of the measures of Angles 2 and 3 equal to the measure of Angle 4?
- What is the relationship between Angles 1 and 4?
- Do Angles 1 and 4 form a linear pair?
- Do translations preserve angle measures?
- Which angle(s) is/are congruent to $\angle 1$?
- Which angle(s) is/are congruent to $\angle 2$?
- Which angle(s) is/are congruent to $\angle 3$?
- Which angles form a vertical pair?
- Why can you place $\angle 3$ as two of the six angles at point *B*?

Summary

Parallel lines intersected by a third line create corresponding angles that are congruent.

Activity 3.2
Angles Formed by Three Lines

Facilitation Notes

In this activity, students explore special angle pairs formed when two lines are intersected by a transversal, such as alternate interior angles, alternate exterior angles, same-side interior angles, same-side exterior angles, and corresponding angles.

Ask a student to read the introduction and definition aloud. Discuss as a class. Explain that the angle numbers from the previous activity are not referenced in this diagram; the diagram is just meant to show how 8 angles are formed.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- Does a transversal always intersect two parallel lines?
- When a transversal intersects two lines, how many pairs of corresponding angles are formed?
- What is the difference between alternate interior and alternate exterior angles?
- What do alternate interior and alternate exterior angles have in common?
- What is the difference between same-side interior and same-side exterior angles?
- What do same-side interior and same-side exterior angles have in common?

Differentiation strategies

- To scaffold support with keeping track of all of the angle relationships, provide five copies of the parallel line diagram on one side of a handout. Each time a new angle relationship is introduced, have students label a diagram with that name and then color each angle pair that relates to that explanation with a different color.
- To extend the activity,
  - Place the parallel lines in a vertical position and have students identify the same angle relationships.
  - Provide students with angle pairs and have them identify the angle relationship.

Have students work with a partner or in a group to complete Question 5. Share responses as a class.

Differentiation strategies

To scaffold support with making sense of the diagram,

- Suggest that students outline the referenced parallel lines with the same color and the referenced transversal with a different color when referring to complicated diagrams.
- Provide two copies of the diagram to allow for color-coding when a second transversal is referenced.

As students work, look for

Confusion in naming angles. Because of the many points in the diagram, there is more than one way to accurately identify each angle.

Questions to ask

- When line *BC* is the transversal, how many pairs of corresponding angles are formed?
- How many alternate interior angles are formed?
- How many alternate exterior angles are formed?
- How many same-side interior angles are formed?
- How many same-side exterior angles are formed?
- When line *AB* is the transversal, how many pairs of corresponding angles are formed?
- How many alternate interior angles are formed?
- How many alternate exterior angles are formed?
- How many same-side interior angles are formed?
- How many same-side exterior angles are formed?

Summary

Parallel lines intersected by a transversal create pairs of corresponding angles, alternate interior angles, alternate exterior angles, same-side interior angles, and same-side exterior angles.

Activity 3.3
Analyzing Special Angle Pairs

Facilitation Notes

In this activity, students use a city map to identify and measure examples of transversals, alternate interior angles, alternate exterior angles, same-side interior angles, same-side exterior angles, and corresponding angles.

Provide students with a protractor for this activity.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Differentiation strategies

- As an alternative grouping, assign half the class or half of a group/partner pair to complete Question 3 and the others to complete Question 4. Share with the other half of the class.
- To scaffold support, have students turn their paper so that the parallel lines are horizontal in order to reference their notes from the previous activity.

Questions to ask

- How did you determine which street was a transversal?
- How did you locate the pairs of alternate interior angles?
- How did you locate the pairs of alternate exterior angles?
- How did you locate the corresponding angles?
- How did you locate the same-side interior angles?
- How did you locate the same-side exterior angles?
- Does 6th Street intersect 7th Street?
- How did you determine which of the three streets were transversals?

Summary

Parallel lines intersected by a transversal create congruent corresponding angles, congruent alternate interior angles, and congruent alternate exterior angles. The same-side interior pairs of angles and the same-side exterior pairs of angles formed are supplementary.

Activity 3.4
Line Relationships and Angle Pairs

Facilitation Notes

In this activity, students use information from the previous activities to draw conclusions about the measures of special pairs of angles formed by a transversal intersecting two non-parallel lines and a transversal intersecting two parallel lines. Transformations are used to explain why these relationships exist for parallel lines.

Differentiation strategy

Before starting the activity, have students draw two new diagrams for reference.

- Draw a transversal intersecting two non-parallel lines and number each angle.
- Draw a transversal intersecting two parallel lines and number each angle.
- Have students use a protractor to measure and label each angle.

Have students work with a partner or in a group to complete Questions 1 through 9. Share responses as a class.

Questions to ask

- What did you notice about the measures of each pair of vertical angles?
- Were any adjacent angles equal in measure?
- Under what conditions are the alternate interior angles congruent?
- Under what conditions are the alternate exterior angles congruent?
- Under what conditions are the corresponding angles congruent?
- Under what conditions are the same-side interior angles supplementary?
- Under what conditions are the same-side exterior angles supplementary?

Summary

When two parallel lines are intersected by a transversal, the alternate interior, alternate exterior, and corresponding angles are congruent and the same-side interior and same-side exterior angles are supplementary.

Activity 3.5

Solving for Unknown Angle Measures

Facilitation Notes

In this activity, students solve for the unknown measures of angles using the relationships between angle pairs formed when parallel lines are intersected by a transversal.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- How many transversals are in this diagram?
- Should you assume the lengths of $\overline{EA}$ and $\overline{EB}$ are equal?
- If the length of $\overline{EA}$ is not equal to the length of $\overline{EB}$, what does this tell you about the measure of $\angle EAB$ and the measure of $\angle EBA$?
- Is it possible to determine the measures of any angles formed at point B? Why or why not?

- Is it possible to determine the measures of any angles formed at point *D*? Why or why not?
- Which angle measurements can you determine?

Have students work with a partner or in a group to complete Questions 2 through 6. Share responses as a class.

Differentiation strategies

To scaffold support

- With the parallelogram diagram, suggest that students extend the sides so the diagram looks more familiar.
- With complicated diagrams, remind students to look for overlapping triangles in the diagrams.

Questions to ask

- What is the relationship between $\angle G$ and $\angle M$?
- What is the measure of $\angle G$?
- What is the relationship between $\angle M$ and $\angle O$?
- What is the measure of $\angle O$?
- What is the relationship between $\angle O$ and $\angle E$?
- What is the measure of $\angle E$?
- What is the relationship between $\angle E$ and $\angle G$?
- What is the measure of $\angle G$?
- Which angles in the diagram are equal to 34°?
- What is the measure of the angle adjacent to the 34° angle?
- Which angles in the diagram are equal to 140°?
- What is the measure of the angle adjacent to the 140° angle?
- If $\overleftrightarrow{CE} \perp \overleftrightarrow{DE}$, what is the measure of $\angle E$?
- How can you determine the measure of $\angle ACD$?
- How can you determine the measure of $\angle BDC$?
- Will any triangles in the diagram help you to determine the measure of an angle?
- What is the measure of the angle adjacent to the 46° angle?
- What do the little boxes in the diagram represent?
- How can you extend a transversal so it forms a triangle in this diagram?
- Is there a pair of alternate interior angles you could use to help solve this problem?

Summary

The congruent and supplementary relationships between angles formed by a transversal intersecting two or more parallel lines are useful in determining unknown angle measurements.

DEMONSTRATE

Talk the Talk: What's So Special?

Facilitation Notes

In this activity, students answer questions related to special pairs of angles formed by a transversal intersecting parallel lines. They informally prove the Triangle Sum Theorem using parallel line relationships.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- When are alternate interior angles not congruent?
- When are alternate exterior angles not congruent?
- When are corresponding angles not congruent?
- When are same-side interior angles not supplementary?
- When are same-side exterior angles not supplementary?
- When are adjacent angles not supplementary?
- When are vertical angles not congruent?

Summary

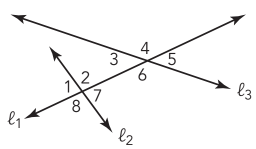
When two parallel lines are intersected by a transversal, the alternate interior, alternate exterior, and corresponding angles are congruent. The same-side interior and same-side exterior angles are supplementary.

Crisscrossed Applesauce 3

Angle Relationships Formed by Lines Intersected by a Transversal

WARM UP

Use the numbered angles in the diagram to answer each question.



1. Which angles form vertical angles?

2. Which angles are congruent?

LEARNING GOALS

- Explore the angles determined by two lines that are intersected by a transversal.
- Use informal arguments to establish facts about the angles created when parallel lines are cut by a transversal.
- Identify corresponding angles, alternate interior angles, alternate exterior angles, same-side interior angles, and same-side exterior angles.
- Determine the measure of alternate interior angles, alternate exterior angles, same-side interior angles, same-side exterior angles, and corresponding angles.

KEY TERMS

- transversal
- alternate interior angles
- alternate exterior angles
- same-side interior angles
- same-side exterior angles

When two lines intersect, special angle pair relationships are formed. What special angle pair relationships are formed when three lines intersect?

Warm Up Answers

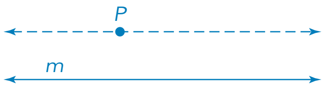
1. The pairs of vertical angles are:
   $\angle 1$ and $\angle 7$,
   $\angle 2$ and $\angle 8$,
   $\angle 3$ and $\angle 5$,
   $\angle 4$ and $\angle 6$.
2. $\angle 1 \cong \angle 7$, $\angle 2 \cong \angle 8$, $\angle 3 \cong \angle 5$, $\angle 4 \cong \angle 6$

ELL Tip

Have students create a vocabulary list with the Key Terms at the beginning of the lesson. When the term is covered throughout the lesson, have students write down the definition in their notebooks. For an exit slip, have students use each term in a sentence. Look over the exit slips to make sure the words are used correctly.

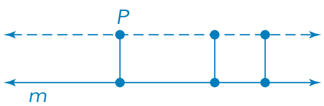
Answers

1. Sample drawing.



2. Sample answer.
   When I drew my new line, I created a set of parallel lines.
3. Sample answer.
   I can draw a perpendicular segment between the two parallel lines, and this segment is always the same length.

   Sample drawing.



Getting Started

Euclid's Fifth Postulate

A *postulate* is a statement that is accepted to be true without proof.

Euclid is known as the father of geometry, and he stated five postulates upon which every other geometric relationship can be based. The fifth postulate is known as the *Parallel Postulate*. Consider one of the equivalent forms of this postulate:

"Given any straight line and a point not on the line, there exists one and only one straight line that passes through the point and never intersects the line."

1. Draw a picture that shows your interpretation of this statement of the postulate.

2. Why do you think this postulate is called the Parallel Postulate?

A common definition of parallel lines is co-planar lines that are always equidistant, or the same distance apart.

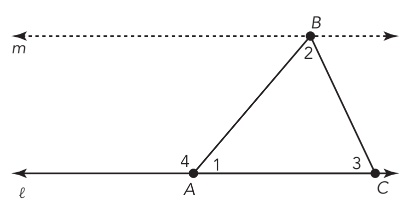
3. Explain what is meant by this definition and demonstrate it on your diagram.

ACTIVITY **3.1**

Creating New Angles from Triangles

In the previous lesson, you determined measures of interior and exterior angles of triangles.

Consider the diagram shown. Lines *m* and *ℓ* are parallel. This is notated as $m \parallel \ell$.



1. Explain the relationships between the numbered angles in the diagram.

2. Trace the diagram onto two sheets of patty paper and extend $\overline{AB}$ to create a line that contains the side of the triangle. Align the triangles on your patty paper and translate the bottom triangle along $\overline{AB}$ until $\overline{AC}$ lies on line *m*. Trace your translated triangle on the top sheet of patty paper. Label the translated triangle *A′B′C′*.

Answers

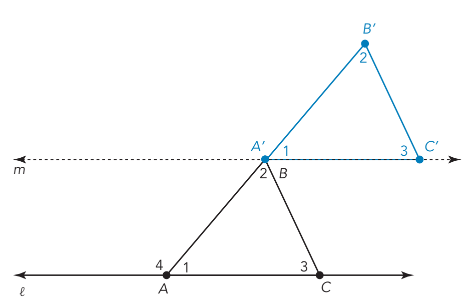
1. The sum of the measures of Angles 1, 2, and 3 is 180°. Angle 4 is an exterior angle with remote interior angles 2 and 3; therefore, the sum of the measures of Angles 2 and 3 is equal to the measure of Angle 4. Angles 1 and 4 are supplementary because they form a linear pair.

   $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

   $m\angle 1 + m\angle 4 = 180°$

   $m\angle 2 + m\angle 3 = m\angle 4$

2. See diagram below.

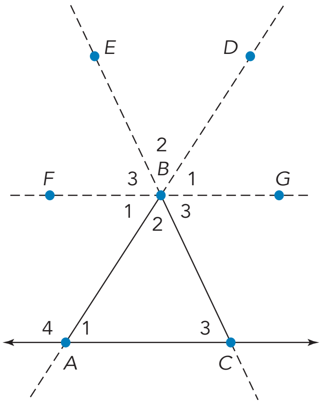


Answers

3. The angles are congruent. Translations preserve angle measure.

4. Answers may vary. Sample answers use accompanying diagram.

- $\angle DBG$ is congruent to $\angle 1$ because it is a translation of $\angle 1$. $\angle FBA$ is congruent to $\angle 1$ because the angles form a vertical pair, and vertical angles are congruent.
- $\angle EBD$ is congruent to $\angle 2$ because the angles form a vertical pair, and vertical angles are congruent.
- $\angle EBF$ is congruent to $\angle 3$. Because $\angle EBF$, $\angle EBD$, and $\angle DBG$ form a line, which is 180°, the sum of the measures of Angles 1, 2, and 3 is 180°. Since $\angle EBD$ and $\angle DBG$ are congruent to $\angle 2$ and to $\angle 1$, respectively, $m\angle EBF$ must be equal to $m\angle 3$. Because $\angle EBF$ and $\angle GBC$ are vertical angles, $\angle GBC$ is also congruent to $\angle 3$.



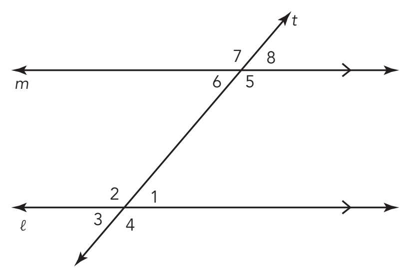
3. Angle 1 in $\triangle A'B'C'$ is a translation of Angle 1 in $\triangle ABC$. How are the measures of these angles related to each other? Explain your reasoning.

4. Extend $\overline{CB}$ to create a line. Use what you know about special angle pairs to label all six angles at point *B* as congruent to $\angle 1$, $\angle 2$, or $\angle 3$. Explain your reasoning. Sketch your patty paper drawing.

ACTIVITY 3.2

Angles Formed by Three Lines

Consider your diagram from the previous activity. If you remove $\overline{BC}$ and the line containing $\overline{BC}$, your diagram might look similar to the diagram shown.



Arrowheads on lines in diagrams indicate parallel lines. Lines or segments with the same number of arrowheads are parallel.

In this diagram, the two parallel lines, *m* and *ℓ*, are intersected by a *transversal*, *t*. A **transversal** is a line that intersects two or more lines.

The transversal, *t*, in this diagram corresponds to the line that contained side *AB* in your patty paper diagram.

Recall that corresponding angles are angles that have the same relative positions in geometric figures. In the previous activity, when you translated $\triangle ABC$ to create $\triangle A'B'C'$ you created three sets of corresponding angles. You can also refer to corresponding angles in relation to lines intersected by a transversal.

1. Use the diagram to name all pairs of corresponding angles.

2. Analyze each angle pair: $\angle 1$ with $\angle 6$ and $\angle 2$ with $\angle 5$.

a. Are the angles between (on the *interior* of) lines *m* and *ℓ*, or are they outside (on the *exterior* of) lines *m* and *ℓ*?

b. Are the angles on the same side of the transversal, or are they on opposite (*alternating*) sides of the transversal?

Answers

1. The corresponding angles are Angles 1 and 8, 4 and 5, 2 and 7, and 3 and 6.

2a. The angles are on the interior of lines *m* and *ℓ*.

2b. The angles are on alternating sides of the transversal.

Answers

3. The alternate exterior angle pairs are ∠3 with ∠8 and ∠4 with ∠7.

4a. Same-side interior angles are angles formed when a transversal intersects two other lines; these angles are on the same side of the transversal and are between the other two lines. The same-side interior angles are ∠1 with ∠5 and ∠2 with ∠6.

4b. Same-side exterior angles are angles formed when a transversal intersects two other lines; these angles are on the same side of the transversal and are outside the other two lines. The same-side exterior angles are ∠3 with ∠7 and ∠4 with ∠8.

NOTES

There is a special relationship between angles like ∠1 and ∠6 or ∠2 and ∠5. **Alternate interior angles** are angles formed when a transversal intersects two other lines. These angle pairs are on opposite sides of the transversal and are between the two other lines.

Alternate exterior angles are also formed when a transversal intersects two lines. These angle pairs are on opposite sides of the transversal and are outside the other two lines.

3. Use your diagram to name all pairs of alternate exterior angles.

Two additional angle pairs are *same-side interior angles* and *same-side exterior angles*.

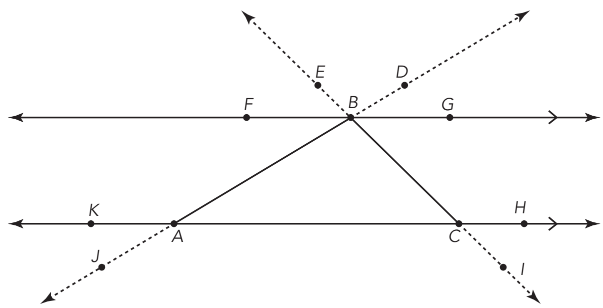
4. Use the names to write a definition for each type of angle pair. Identify all pairs of each type of angle pair from the diagram.

a. same-side interior angles

b. same-side exterior angles

5. In the diagram from the previous activity, each time you extended a side of the triangle, you created a transversal. Identify the angle pairs described by each statement.

a. corresponding angles if $\overleftrightarrow{BC}$ is the transversal



b. alternate interior angles if $\overleftrightarrow{BC}$ is the transversal

c. alternate exterior angles if $\overleftrightarrow{AB}$ is the transversal

d. same-side interior angles if $\overleftrightarrow{AB}$ is the transversal

Same-side interior angles are on the same side of the transversal and are between the other two lines.

e. same-side exterior angles if $\overleftrightarrow{AB}$ is the transversal

Same-side exterior angles are on the same side of the transversal and are outside the other two lines.

Answers

5a. $\angle ACB$ and $\angle FBE$, $\angle HCB$ and $\angle GBE$, $\angle ICA$ and $\angle CBF$, $\angle ICH$ and $\angle CBG$

5b. $\angle ACB$ and $\angle CBG$, $\angle HCB$ and $\angle FBC$

5c. $\angle JAC$ and $\angle FBD$, $\angle JAK$ and $\angle DBG$

5d. $\angle CAB$ and $\angle ABG$, $\angle KAB$ and $\angle ABF$

5e. $\angle JAK$ and $\angle FBD$, $\angle JAC$ and $\angle GBD$

Answers

1. All four of the streets are transversals. A transversal is a line that intersects two other lines. P St. and N St. both intersect Massachusetts Ave. and 6th St., so they are both transversals. Massachusetts Ave. and 6th St. both intersect P St. and N St., so they are both transversals.
2. $m\angle 1 = m\angle 4 = m\angle 5 = m\angle 8 = 100°$

   $m\angle 2 = m\angle 3 = m\angle 6 = m\angle 7 = 80°$

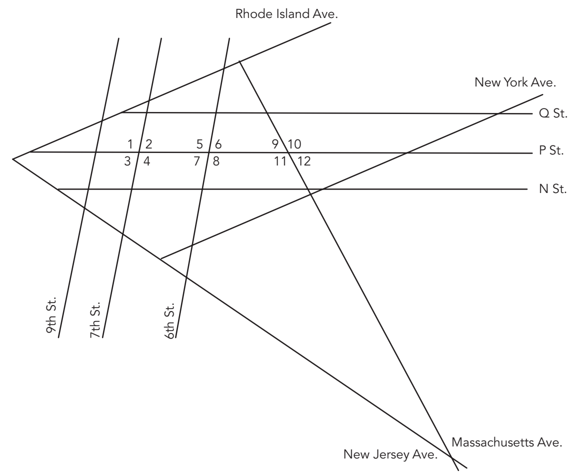
   $m\angle 9 = m\angle 12 = 62°$

   $m\angle 10 = m\angle 11 = 118°$

ACTIVITY 3.3

Analyzing Special Angle Pairs

Consider the map of Washington, D.C., shown. Assume that all line segments that appear to be parallel are parallel.



1. Consider only P St., N St., Massachusetts Ave., and 6th St. Which of these streets, if any, are transversals? Explain your reasoning.

Let's explore the relationships between the angles formed from lines cut by transversals.

2. Use a protractor to measure all 12 angles labeled on the diagram.

8 • TOPIC 3: Line and Angle Relationships

NOTES

3. Consider only 6th St., 7th St., and P St.

   a. Which of these streets, if any, are transversals? Explain your reasoning.

   b. What is the relationship between 6th St. and 7th St.?

   c. Name the pairs of alternate interior angles. What do you notice about their angle measures?

   d. Name the pairs of alternate exterior angles. What do you notice about their angle measures?

   e. Name the pairs of corresponding angles. What do you notice about their angle measures?

   f. Name the pairs of same-side interior angles. What do you notice about their angle measures?

   g. Name the pairs of same-side exterior angles. What do you notice about their angle measures?

Answers

3a. P St. is a transversal because it intersects 6th St. and 7th St.

3b. 6th St. is parallel to 7th St.

3c. $\angle 2$ and $\angle 7$, $\angle 4$ and $\angle 5$

The alternate interior angles in each pair have equal measures.

3d. $\angle 3$ and $\angle 6$, $\angle 1$ and $\angle 8$

The alternate exterior angles in each pair have equal measures.

3e. $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$

The corresponding angles in each pair have equal measures.

3f. $\angle 2$ and $\angle 5$, $\angle 4$ and $\angle 7$

The same-side interior angles in each pair are supplementary.

3g. $\angle 1$ and $\angle 6$, $\angle 3$ and $\angle 8$

The same-side exterior angles in each pair are supplementary.

Answers

4a. All three streets are transversals. Each street intersects the other two streets.

4b. 6th St. and Massachusetts Ave. are intersecting lines that are not parallel.

4c. $\angle 6$ and $\angle 11$, $\angle 8$ and $\angle 9$

The alternate interior angles in each pair do not have equal measures.

4d. $\angle 5$ and $\angle 12$, $\angle 7$ and $\angle 10$

The alternate exterior angles in each pair do not have equal measures.

4e. $\angle 5$ and $\angle 9$, $\angle 6$ and $\angle 10$, $\angle 7$ and $\angle 11$, $\angle 8$ and $\angle 12$

The corresponding angles in each pair do not have equal measures.

4f. $\angle 6$ and $\angle 9$, $\angle 8$ and $\angle 11$

The same-side interior angles do not appear to have a special relationship.

4g. $\angle 5$ and $\angle 10$, $\angle 7$ and $\angle 12$

The same-side exterior angles in each pair do not appear to have a special relationship.

4. Consider only 6th St., Massachusetts Ave., and P St.

a. Which of these streets, if any, are transversals?

b. What is the relationship between 6th St. and Massachusetts Ave.?

c. Name the pairs of alternate interior angles. What do you notice about their angle measures?

d. Name the pairs of alternate exterior angles. What do you notice about their angle measures?

e. Name the pairs of corresponding angles. What do you notice about their angle measures?

f. Name the pairs of same-side interior angles. What do you notice about their angle measures?

g. Name the pairs of same-side exterior angles. What do you notice about their angle measures?

ACTIVITY 3.4

Line Relationships and Angle Pairs

NOTES

In the previous activity, you explored angle pairs formed by a transversal intersecting two non-parallel lines and a transversal intersecting two parallel lines.

1. Make a conjecture about the types of lines cut by a transversal and the measures of the special angle pairs.

Refer back to the measurements of the labeled angles on the diagram of Washington, D.C.

2. What do you notice about the measures of each pair of alternate interior angles when the lines are

a. non-parallel?

b. parallel?

3. What do you notice about the measures of each pair of alternate exterior angles when the lines are

a. non-parallel?

b. parallel?

Answers

1. Conjectures will vary.

2a. The alternate interior angles do not have equal measures.

2b. The alternate interior angles have equal measures.

3a. The alternate exterior angles do not have equal measures.

3b. The alternate exterior angles have equal measures.

Answers

4a. The corresponding angles do not have equal measures.

4b. The corresponding angles have equal measures.

5a. The same-side interior angles are not supplementary.

5b. The same-side interior angles are supplementary.

6a. The same-side exterior angles are not supplementary.

6b. The same-side exterior angles are supplementary.

NOTES

4. What do you notice about the measures of each pair of corresponding angles when the lines are

a. non-parallel?

b. parallel?

5. What do you notice about the measures of the same-side interior angles when the lines are

a. non-parallel?

b. parallel?

6. What do you notice about the measures of the same-side exterior angles when the lines are

a. non-parallel?

b. parallel?

7. **Summarize your conclusions in the table by writing the relationships of the measures of the angles. The relationships are either congruent or not congruent, supplementary or not supplementary.**

Angles	Two Parallel Lines Intersected by a Transversal	Two Non-Parallel Lines Intersected by a Transversal
Alternate Interior Angles		
Alternate Exterior Angles		
Corresponding Angles		
Same-Side Interior Angles		
Same-Side Exterior Angles		

8. **Use transformations to explain how to map the angle pairs that are congruent.**

9. **Use transformations to explain why certain angle pairs are supplementary.**

Answers

7. See table below.

8. Sample answer.
When lines are parallel, I can translate the angles the same distance up and over to map corresponding angles onto each other. I know that translations preserve the size of angles. When lines are parallel, I can create a center of rotation at the midpoint of the segment of the transversal between the two parallel lines. Then I can rotate 180 degrees to map the alternate interior angles onto each other and to map the alternate exterior angles onto each other.

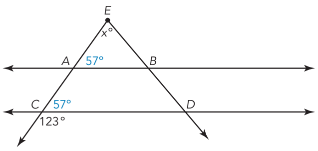
9. Sample answer.
When lines are parallel, I can create a center of rotation at the midpoint of the segment of the transversal between the two parallel lines and then rotate 180 degrees. After the rotation, same-side interior angles become a linear pair and are supplementary. Also, same-side exterior angles become a linear pair and are supplementary.

7.

Angles	Two Parallel Lines Intersected by a Transversal	Two Non-Parallel Lines Intersected by a Transversal
Alternate Interior Angles	congruent	not congruent
Alternate Exterior Angles	congruent	not congruent
Corresponding Angles	congruent	not congruent
Same-Side Interior Angles	supplementary	not supplementary
Same-Side Exterior Angles	supplementary	not supplementary

Answers

1.



1a. Sylvia assumed that the corresponding angles on ray *ED* were congruent to the angles formed on ray *EC*, so she solved for *x* by using the triangle at the top of the figure: $180° - 57° - 57° = 66°$.

1b. Scott could redraw ray *ED* several different ways so that the measures of the angles located at points *B* and *D* change. This would show Sylvia that the angles formed on ray *EC* are not congruent to the angles formed on ray *ED*.

1c. Scott is correct. There is not enough information to solve for *x*.

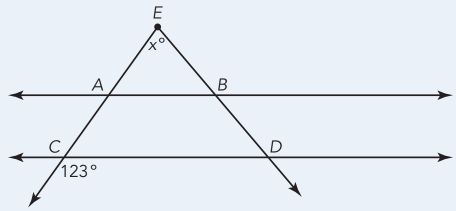
ACTIVITY **3.5**

Solving for Unknown Angle Measures

Use what you know about angle pairs to answer each question.

1. Sylvia and Scott were working together to solve the problem shown.

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. Solve for *x*. Show all your work.

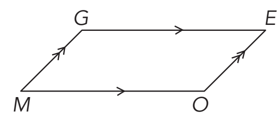


a. Sylvia concluded that $x = 66°$. How did Sylvia get her answer?

b. Scott does not agree with Sylvia's answer. He thinks there is not enough information to solve the problem. How could Scott alter the figure to show why he disagrees with Sylvia's answer?

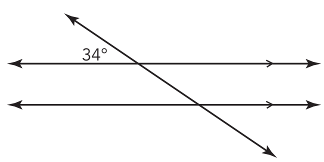
c. Who is correct?

2. **Opposite sides of the figure shown are parallel. Suppose that the measure of Angle *M* is equal to 30°. Solve for the measures of Angles *G*, *E*, and *O*. Explain your reasoning.**

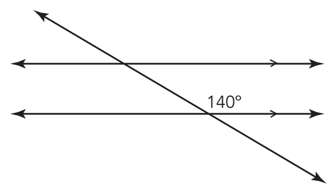


3. **Determine the measure of each unknown angle.**

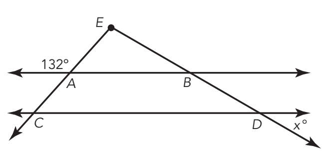
a.



b.



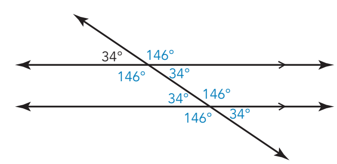
4. **In this figure, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overrightarrow{EC} \perp \overrightarrow{ED}$. Solve for *x*. Show all your work.**



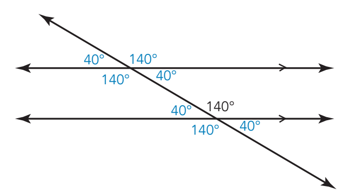
Answers

2. The measure of Angle *G* is equal to 150° because Angle *M* and Angle *G* are same-side interior angles, so they are supplementary. The measure of Angle *E* is 30° because Angle *G* and Angle *E* are same-side interior angles, so they are supplementary. The measure of Angle *O* is 150° because Angle *E* and Angle *O* are same-side interior angles, so they are supplementary.

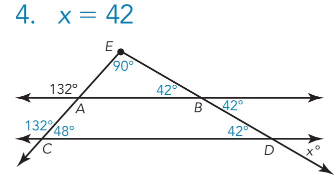
3a.



3b.

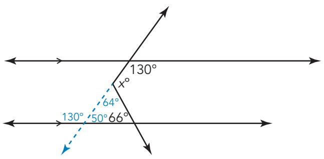


4. $x = 42$

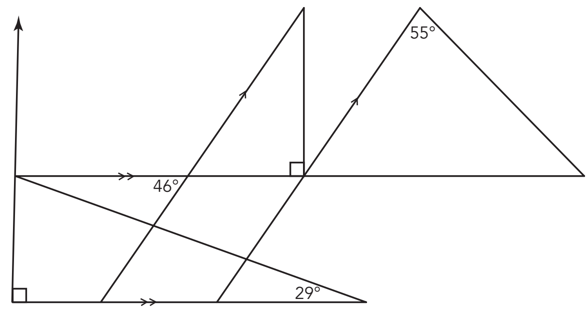


Answers

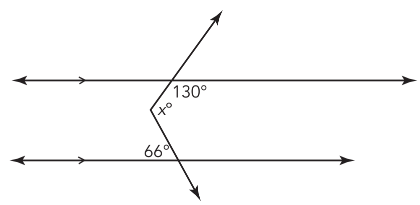
5. See image below.
6. $x = 116$



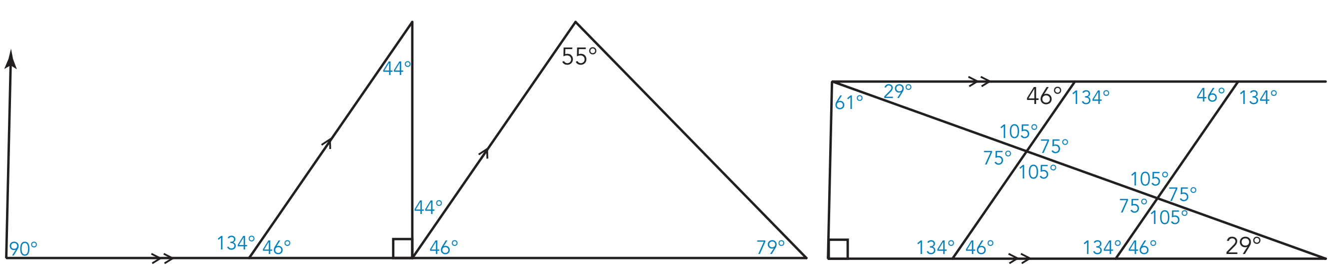
5. Determine the measure of each angle in this figure.



6. Solve for *x*. Show all your work.



5.



NOTES

TALK the TALK

What's So Special?

1. If two lines are intersected by a transversal, when are

   a. alternate interior angles congruent?

   b. alternate exterior angles congruent?

   c. vertical angles congruent?

   d. corresponding angles congruent?

   e. same-side interior angles supplementary?

   f. same-side exterior angles supplementary?

   g. linear pairs of angles supplementary?

Answers

1a. When two parallel lines are intersected by a transversal, alternate interior angles are congruent.

1b. When two parallel lines are intersected by a transversal, alternate exterior angles are congruent.

1c. Vertical angles are always congruent.

1d. When two parallel lines are intersected by a transversal, corresponding angles are congruent.

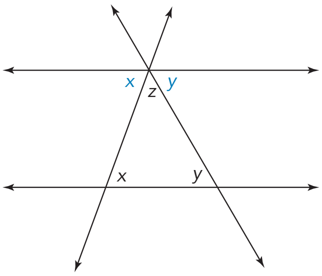
1e. When two parallel lines are intersected by a transversal, same-side interior angles are supplementary.

1f. When two parallel lines are intersected by a transversal, same-side exterior angles are supplementary.

1g. Linear pairs are always supplementary.

Answers

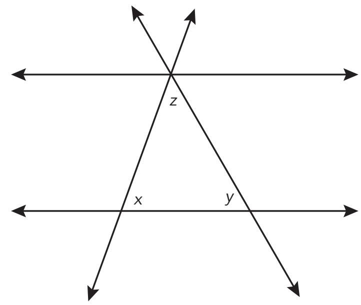
2.



Sample answer.
If two parallel lines are cut by a transversal, then I know that alternate interior angles are congruent. So, I can label another angle as *x* and *y*. At the top of the figure, I know that the measures of Angles *x*, *y*, and *z* sum to 180 degrees because they form a straight line. These are the three angles of the triangle so I know that the measures of the three angles of the triangle also sum to 180 degrees.

NOTES

2. Briana says that she can use what she learned about parallel lines cut by a transversal to show that the measures of the angles of a triangle sum to 180°. She drew the figure shown.



Explain what Briana discovered.