4

The Vanishing Point

The Angle-Angle Similarity Theorem

WARM UP

Suppose $\triangle BHX$ is similar to $\triangle KRC$.

1. List the corresponding angles.

2. Write the ratios to identify the proportional side lengths.

LEARNING GOALS

- Develop the minimum criteria to show that two triangles are similar.
- Use informal arguments to establish facts about the angle-angle criterion for similarity of triangles.
- Use the Angle-Angle Similarity Theorem to identify similar triangles.

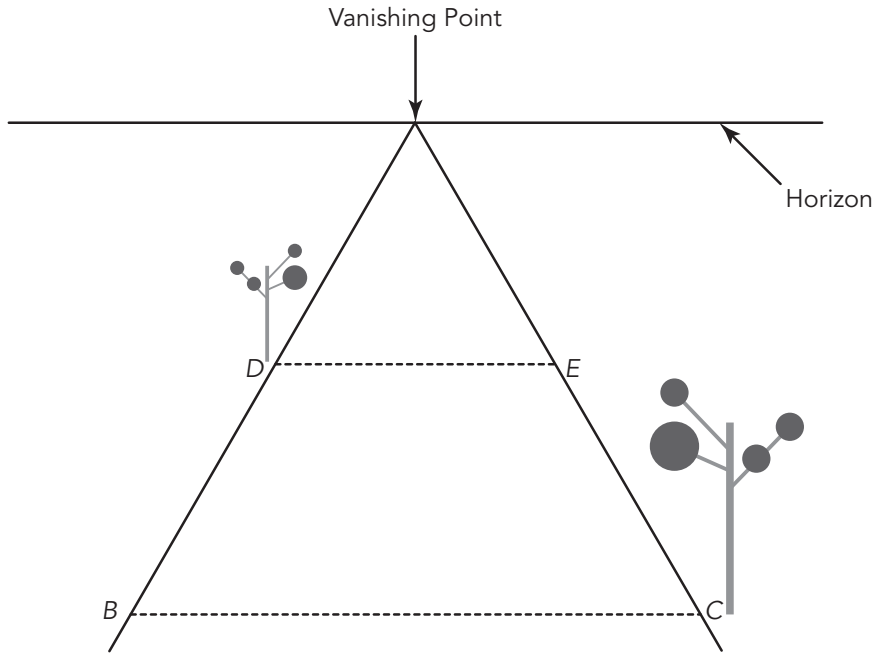
KEY TERM

- Angle-Angle (AA) Similarity Theorem

You have determined that when two triangles are similar, the corresponding angles are congruent and the corresponding sides are proportional. How can you show that two triangles are similar without measuring all of the angles and side lengths?

Vanishing Point

Graphic artists use knowledge about similarity to create realistic perspective drawings. Choose where the horizon should be and a vanishing point—a point where all the parallel lines in the drawing should appear to meet—and you too can create a perspective drawing.



The symbol ~ means "is similar to."

1. Suppose the vanishing point is point *A* and that $\overline{DE} \parallel \overline{BC}$. How could you demonstrate that $\triangle ABC \sim \triangle ADE$?

2. Draw a horizontal line in the path to create another similar triangle. Then sketch a tree at that line using the appropriate scale factor.

ACTIVITY
4.1

Exploring the Angle-Angle Similarity Theorem

NOTES

You have determined that when two triangles are similar, the corresponding angles are congruent and the corresponding sides are proportional. To show that two triangles are similar, do you need to show that all of the corresponding sides are proportional and all of the corresponding angles are congruent?

Let's explore an efficient method to determine if two triangles are similar.

1. If the measures of two angles of a triangle are known, is that enough information to draw a similar triangle? Let's explore this possibility.

a. Use a straightedge to draw $\triangle ABC$ in the space provided.

b. Use a protractor to measure $\angle A$ and $\angle B$ of $\triangle ABC$ and record the measurements.

$m\angle A =$ _____ $m\angle B =$ _____

c. Use the Triangle Sum Theorem to determine $m\angle C$.

NOTES

d. Draw a second triangle, $\triangle DEF$, in the space provided using the angle measurements from part (b).

e. Based on your knowledge, what other information is needed to determine if the two triangles are similar, and how can you acquire that information?

f. Determine the measurements to get the additional information needed and decide if the two triangles are similar.

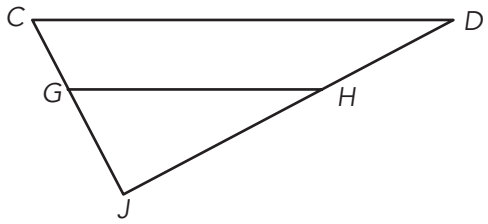
You have just shown that given the measures of two pairs of congruent corresponding angles of two triangles, it is possible to determine that two triangles are similar. In the study of geometry, this is expressed as a theorem.

The **Angle-Angle (AA) Similarity Theorem** states that if two angles of one triangle are congruent to the corresponding angles of another triangle, then the triangles are similar.

ACTIVITY
4.2 Using the Angle-Angle Similarity Theorem

Identify the triangles that are similar by the AA Similarity Theorem. Explain how you know that the triangles are similar.

1. $\overline{CD} \parallel \overline{GH}$



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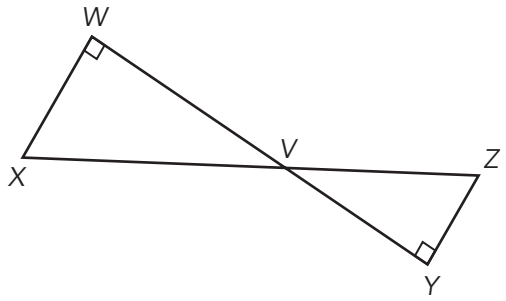

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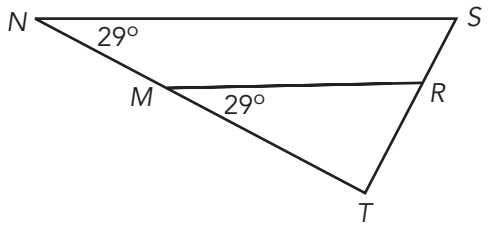

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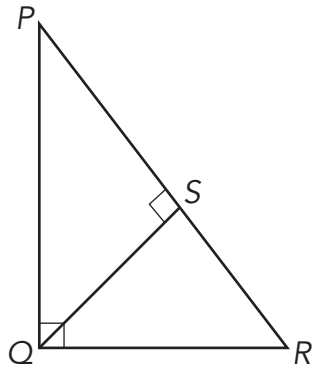
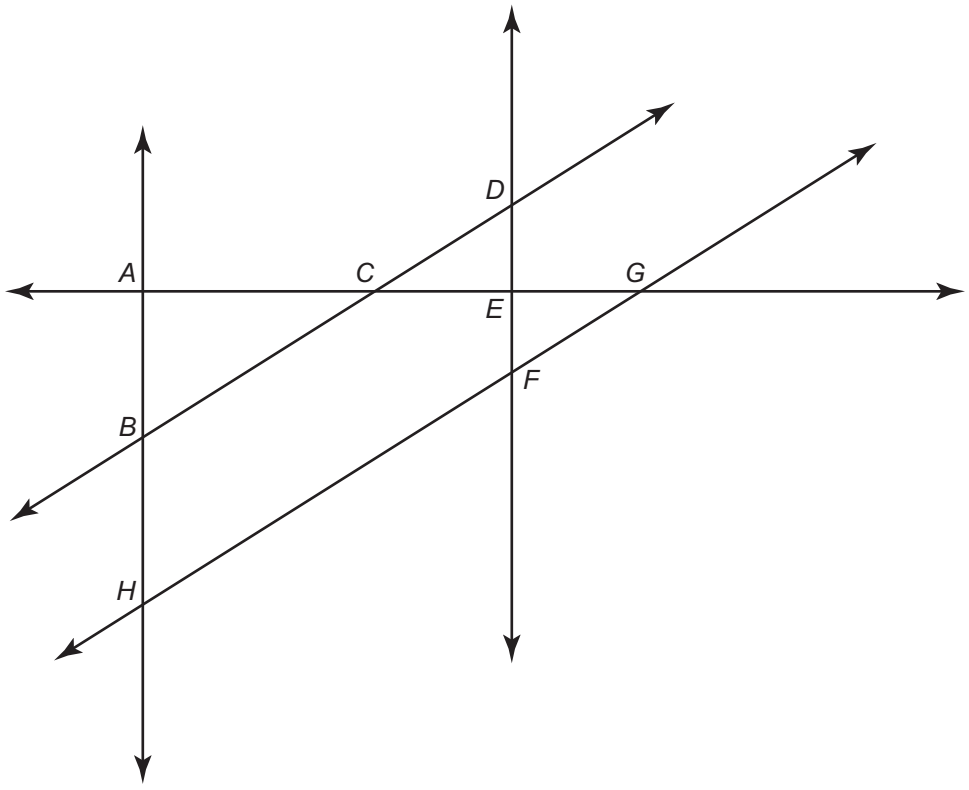


ACTIVITY
4.3

Reasoning with the Angle-Angle Similarity Theorem

Use what you have learned about triangle similarity to answer each question.

Given: $\overleftrightarrow{BD} \parallel \overleftrightarrow{HG}$, $\overleftrightarrow{AH} \parallel \overleftrightarrow{DF}$, $\overleftrightarrow{AH} \perp \overleftrightarrow{AG}$, $\overleftrightarrow{DF} \perp \overleftrightarrow{AG}$



1. Is $\triangle ABC \sim \triangle AHG$? Explain your reasoning.

2. Is $\triangle ABC \sim \triangle EDC$? Explain your reasoning.

3. Is $\triangle EDC \sim \triangle EFG$? Explain your reasoning.

4. Is $\triangle ABC \sim \triangle EFG$? Explain your reasoning.

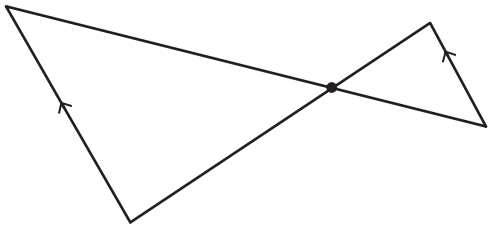
5. Is $\triangle AHG \sim \triangle EFG$? Explain your reasoning.

NOTES

TALK the TALK

Bow-Tie Triangles

You can draw special triangles known as bow-tie triangles. First, draw a pair of parallel line segements. Then, connect the pairs of endpoints with line segments so that the line segments intersect, like this:



1. **Are bow-tie triangles always similar? Show your work and explain your reasoning. Then, compare your work with your classmates' work.**