

Line and Angle Relationships

Topic 3 Overview



How is *Line and Angle Relationships* organized?

Students begin this topic exploring different pairs of angles including supplementary angles, complementary angles, vertical angles, and adjacent angles. They use patty paper to create angles and explore relationships between them. Then they write and solve equations involving angle pairs. Next, students use their knowledge of transformations, congruence, and similarity to establish the Triangle Sum Theorem, the Exterior Angle Theorem, the relationships formed between angles when parallel lines are cut by a transversal, and the Angle-Angle Similarity Theorem for similarity of triangles. Students use hands-on tools to make and justify conjectures about the sum of the interior angles of a triangle, the relationship between triangle side and angle measures, and the value of exterior angles of triangles. They then apply their results to new problems. Next, students use patty paper and translations to form parallel lines cut by a transversal and use their drawings and real-world scenarios to identify transversals and special pairs of angles. They determine and informally prove the relationships between the special angle pairs formed when parallel lines are cut by a transversal and use these relationships

to solve mathematical problems, including writing and solving equations. Finally, students use parallel line relationships and tools to establish the Angle-Angle (AA) Similarity Theorem and use the theorem to determine if triangles in complex diagrams are similar.



What is the entry point for students?

Students begin the topic exploring different pairs of angles and their relationships using patty paper. In elementary, students began measuring angles and naming them by their size relative to a 90 degree angle. They also learned how to classify triangles by their side lengths and their angle measures. In this topic, students continue to explore facts about triangles. They draw a triangle, rip it apart, and fit the angles together, noting that the angles form a line; therefore, the sum of the interior angles of a triangle is 180° . Students then use tools, including the Triangle Sum Theorem and rulers, to determine the relationship between the side lengths of a triangle and the measures of the angles. Students use the Triangle Sum Theorem and knowledge from supplementary angles to establish the Exterior Angle Theorem. Students rely on their knowledge of rigid motion transformations when exploring the angle

relationships formed when parallel lines are cut by a transversal.



How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic if they can:

- Recognize the special angle pairs formed when two lines intersect: supplementary angles, complementary angles, vertical angles, adjacent angles, and linear pairs.
- Determine complements and supplements of a given angle.
- Determine unknown angle measures by writing and solving algebraic equations based on relationships between angles.
- Informally prove that the sum of the interior angles of a triangle is 180° .
- Informally prove that the measure of an exterior angle of a triangle is equal to the sum of its two remote interior angles.
- Define and identify transversals.
- Identify the special angle pairs formed when parallel lines are intersected by a transversal.
- Informally prove the relationships and measurements of the angles created when two parallel lines are intersected by a transversal.
- Use transversals to explain why two sets of congruent corresponding angles are

sufficient for justifying the fact that two triangles are similar.

- Use the Angle-Angle Similarity Theorem to prove similarity among triangles.



Why is *Line and Angle Relationships* important?

In this topic, students establish important angle and triangle relationships and use what they know about transformations, congruence, and similarity to establish additional geometric facts. Throughout the topic, students are expected to follow lines of logic to reach conclusions, which is a foundation for formal proof in high school. The geometric results established in the topic via informal arguments will be formally proven in high school, but their experiences in this topic provide them with opportunities to build intuition and justify results.



How do the activities in *Line and Angle Relationships* promote student expertise in the mathematical process standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the mathematical process

standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

The standard for this topic specifically calls on students to use informal arguments to establish geometric facts. Therefore, throughout the topic, students are expected to reason about geometric objects and relationships and analyze presented arguments. They examine patterns to make generalizations about relationships in triangles and about the relationships between angle pairs when parallel lines

are cut by a transversal. Students must use precision as they define and identify special angles and angle pairs and solve problems using the theorems.

Materials Needed

- Centimeter ruler
- Patty paper
- Protractor
- Straightedge

New Notation in *Line and Angle Relationships*

The symbol \sim is used to indicate similar figures. For example, $\triangle ABC \sim \triangle DEF$ means that $\triangle ABC$ is similar to $\triangle DEF$.



Learning Together

ELPS: 1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 5.B, 5.F, 5.G

Lesson	Lesson Name	TEKS	Days	Highlights
1	Seeing It From a Different Angle: Special Angle Relationships	7.11C	2	Students explore the types of angles formed when two lines intersect. They learn the definitions of <i>complementary angles</i> , <i>supplementary angles</i> , <i>perpendicular lines</i> , <i>adjacent angles</i> , <i>linear pairs</i> of angles, and <i>vertical angles</i> . Throughout the lesson, students use patty paper to illustrate the special angle pairs and any special relationships between the measures of angle pairs. At the end of the lesson, students have a vocabulary study guide of patty paper. Students use these definitions to answer questions and then write and solve equations involving special angle pairs.
2	Pulling a One-Eighty!: Triangle Sum and Exterior Angle Theorems	7.11C 8.8D	2	Students explore and justify the relationships between angles and sides in a triangle. They establish the Triangle Sum Theorem and use the theorem as they explore the relationship between interior angle measures and the side lengths of triangles. Students identify exterior angles and remote interior angles of triangles and explore the relationship between these angles to establish the Exterior Angle Theorem. They then practice applying both theorems to demonstrate their knowledge of triangle relationships.

Lesson	Lesson Name	TEKS	Days	Highlights
3	Crisscrossed Applesauce: Angle Relationships Formed by Lines Intersected by a Transversal	8.8D	2	Students explore the angles formed when two lines are intersected by a transversal. They use the Parallel Postulate and transformations to begin exploring and identifying the angles. The terms <i>transversal</i> , <i>alternate interior angles</i> , <i>alternate exterior angles</i> , <i>same-side interior angles</i> , and <i>same-side exterior angles</i> are introduced. Students are given a street map and asked to identify transversals and special pairs of angles. After measuring several angles, they conclude that when two parallel lines are intersected by a transversal, the alternate interior, alternate exterior, and corresponding angles are congruent. Students also conclude that same-side interior and same-side exterior angles are supplementary. When the lines are not parallel, these relationships do not hold true. Finally, students solve problems using the parallel line and angle relationships.
4	The Vanishing Point: The Angle-Angle Similarity Theorem	8.8D	1	The Angle-Angle Similarity Theorem can be used to show that two triangles are similar. From previous lessons, students should already recognize that two similar triangles have congruent corresponding angles and proportional corresponding sides. the Angle-Angle Similarity Theorem allows students to show that two triangles are similar without comparing the measures of the parts of each triangle.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 7.11C LESSON 1 Seeing It From a Different Angle GETTING STARTED ACTIVITY 1 ACTIVITY 2	LESSON 1 continued ACTIVITY 3 ACTIVITY 4 ACTIVITY 5 ACTIVITY 6 TALK THE TALK	TEKS: 7.11C, 8.8D LESSON 2 Pulling a One-Eighty! GETTING STARTED ACTIVITY 1	LESSON 2 continued ACTIVITY 2 TALK THE TALK	TEKS: 8.8D LESSON 3 Crisscrossed Applesauce GETTING STARTED ACTIVITY 1 ACTIVITY 2
Day 6	Day 7	Day 8		
LESSON 3 continued ACTIVITY 3 ACTIVITY 4 ACTIVITY 5 TALK THE TALK	TEKS: 8.8D LESSON 4 The Vanishing Point GETTING STARTED ACTIVITY 1 ACTIVITY 2 ACTIVITY 3 TALK THE TALK	END OF TOPIC ASSESSMENT		

Assessment

There is one assessment aligned to this topic: End of Topic Assessment.