That's a Spicy Pizza!

2

Area of Circles

MATERIALS

Scissors
Calculator with a π key

Lesson Overview

Students explore the area of a circle in terms of its circumference. They cut a circle into sectors and fit the sectors together to form a parallelogram. The parallelogram helps students see the area of a circle in relation to its circumference: $A = \left(\frac{1}{2}C\right)r$. Students derive the area for a circle and then solve problems using the formulas for the circumference and area of circles.

Grade 7

Proportionality

(4) The student applies mathematical process standards to represent and solve problems involving proportional relationships. The student is expected to:

(B) calculate unit rates from rates in mathematical and real-world problems.

Expressions, Equations, and Relationships

(8) The student applies mathematical process standards to develop geometric relationships with volume. The student is expected to:

(C) use models to determine the approximate formulas for the circumference and area of a circle and connect the models to the actual formulas.

(9) The student applies mathematical process standards to solve geometric problems. The student is expected to:

(B) determine the circumference and area of circles.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I 3.D, 3.E, 4.B, 4.C, 4.D, 4.D, 5.B, 5.F, 5.G

Essential Ideas

- If a circle is divided into equal parts, separated, and rearranged to resemble a parallelogram, the area of a circle can be approximated by using the formula for the area of a parallelogram with a base length equal to half the circumference and a height equal to the radius.

- The formula for calculating the area of a circle is $A = \pi r^2$ where A is the area of a circle, r is the length of the radius of the circle, and π is represented using the approximation 3.14.
- When solving problems involving circles, the circumference formula is used to determine the distance around a circle, while the area formula is used to determine the amount of space contained inside a circle.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: What Changed? What Stayed the Same?

Students review the idea that because a parallelogram can be decomposed and recomposed into a rectangle with the same dimensions, not only do both figures have the same area, but also that they can derive the formula for the area of a parallelogram to be the same formula for the area of a rectangle, $A = bh$. This concept of using the area of a figure with a known formula to generate another formula is the lead-in to deriving the formula for the area of a circle.

Develop

Activity 2.1: Deriving the Area Formula

Students cut sectors of a circle and arrange the sectors to form a parallelogram shape. They determine that the parallelogram and circle have the same area. Students determine that half of the circumference of the circle represents the base of the parallelogram and the radius represents its height. They then use this information to derive the formula for the area of a circle.

Day 2

Activity 2.2: Circumference or Area

Students investigate a variety of problem situations which require either a circumference or circle area measurement.

Activity 2.3: Unit Rates and Circle Area

Students are asked to recall unit rates and how to operate with them. They then solve a problem using unit rates of pizza area to dollar amount (or dollar amount to pizza area) in order to determine the best buy.

Demonstrate

Talk the Talk: Go With the Flow

Students solve a problem in which the area of a pipe's cross section is related to the amount of water it can deliver. They then compare two different pipe systems using what they know about circle area.

Getting Started: What Changed? What Stayed the Same?

ENGAGE

Facilitation Notes

In this activity, students review the idea that because a parallelogram can be decomposed and recomposed into a rectangle with the same dimensions, not only do both figures have the same area, but also that they can derive the formula for the area of a parallelogram to be the same formula for the area of a rectangle, $A = bh$. This concept of using the area of a figure with a known formula to generate another formula is the lead in to deriving the formula for the area of a circle.

Have students work with a partner to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- Did you calculate the area of both figures or did you take a shortcut? If you took a shortcut, what was it?
- If you didn't have a rectangle with the same dimensions as a parallelogram, how could you determine the area of the parallelogram?
- Which formula did you learn first, the area of a parallelogram or the area of a rectangle?
- Explain how knowing the area formula for a rectangle helped you derive the area formula for a parallelogram.

Summary

The formula for the area of a given figure can sometimes be derived by comparing it to the area of a figure with a known formula.

Activity 2.1
Deriving the Area Formula

DEVELOP

Facilitation Notes

In this activity, students derive the formula for the area of a circle using cut out sectors of a circle and previous knowledge regarding the area of a rectangle. Students also investigate the effect on circle area of doubling the radius.

Ask a student to read the introduction aloud. Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategies

- Have students work in groups of three, with each student making just one parallelogram. Have each student tape their parallelogram together so that they can compare all three figures.
- To scaffold support, have students label each part of the circle first before cutting it out; have them put an "*r*" next to each radius and a "*C*" on the inside of each curved part to demonstrate it is a part of the circumference. Once the parallelogram is composed, they will have an "*r*" on two sides to represent the height. They will be able to see that the other two sides are each composed of half of the circumference.

Questions to ask

- What part of the circle makes each side of your parallelogram?
- What part of the circle makes the top and bottom of your parallelogram?
- Where is the radius of the circle represented in the parallelogram?
- Where is the circumference of the circle represented in the parallelogram?
- What portion of the circle's circumference is used on each side?
- Demonstrate how you used substitution to derive the formula?
- How does this activity compare to the Getting Started part of the lesson?

Note

With the limited number of sectors used, the side of the parallelogram is the radius, but it does not technically represent the height because it is not perpendicular with the base. If an infinite number of sectors were used, a rectangle would be formed, and the height would be the radius. This information is beyond what is required for the course, but is provided in case a student asks a question about the height.

Have students work with a partner or in a group to complete Question 3. Share responses as a class.

Questions to ask

- Which answer is exact?
- How do your calculations using 3.14 compare to those calculations using the π key on a calculator?

- How do your calculations using $\frac{22}{7}$ compare to those calculations using the π key on a calculator?
- What representation of π do you prefer to use?
- Why do we tend to round our answer when calculating the area or circumference of a circle?

Have students work with a partner to complete Question 4. Share responses as a class.

Differentiation Strategy

To scaffold support, have students first draw a circle with a radius of 1 unit and a second circle with a radius of 2 units, and then calculate and compare the areas. Next, have them draw a third circle with a radius of 4 units. Ask them to calculate the area and compare it to the area of the circle with a radius of 2 units.

Questions to ask

- What will be the numerator of the ratio when you compare the circle areas?
- What will be the denominator of the ratio when you compare the circle areas?
- If you multiply the r in r^2 by 2, how will r^2 change?
- If the radius of a circle is cut in half, what effect will this have on the area?

Summary

The area of a circle can be calculated using the formula $A = \pi r^2$ where r is the radius of the circle.

Activity 2.2

Circumference or Area

Facilitation Notes

In this activity, students solve real-world problem situations involving either circumference or area.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- Is the word *area* mentioned in the problem situation?
- Is the phrase "distance around" mentioned in the problem situation?

- What clues tell you whether you need to find the area or the circumference?
- What unit was used to describe circumference in this situation?
- What unit was used to describe area in this situation?
- Why are different units used for circumference and area?

Summary

Circumference ($C = 2\pi r$) and area of a circle ($A = \pi r^2$) formulas are applied to solve real-world problem situations.

Activity 2.3

Unit Rates and Circle Area

Facilitation Notes

In this activity, students use unit rates and the area of a circle formula to determine the best buy in a real-world problem situation.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

As students work, look for

- Confusion between square inches per dollar and cents per square inch.
- Decimal errors in calculations.
- Rounding errors in decimals.
- Reversing numerator and denominator when determining square inches per dollar.
- Reversing numerator and denominator when determining cents per square inch.
- Misinterpreting unit rates to determine the better buy.
- Errors due to not using parentheses correctly on the calculator.

Questions to ask

- Who determined the rate of the number of square inches for every dollar?
- Who determined the rate of the amount, in dollars, for every square inch?
- Is the greater or smaller value for square inches per dollar the better buy?
- Is the smaller or greater value for dollars per square inch the better buy?

Differentiation strategy

Extend this task by asking students to determine the cost per slice of pizza, as well as the amount (in slices) a customer gets for every dollar. Ask students to explain if they could use the slices-per-dollar or dollars-per-slice ratios to determine the best buy (this strategy gives the same correct answer, but it is misleading, since the amount of pizza in each slice would be unknown).

Small: $1.17 per slice; about 0.85 slice per dollar
Medium: $1.25 per slice; 0.8 slice per dollar
Large: $1.30 per slice; about 0.77 slice per dollar
X-Large: $1.25 per slice; 0.8 slice per dollar
Enorme: $1.15 per slice; 0.87 slice per dollar
Ginorme: $0.97 per slice; 1.03 slices per dollar
Colossale: $1.37 per slice; 0.73 slice per dollar

Summary

Unit rates can be used determine the best buy in real-world problem situations.

Talk the Talk: Go With the Flow

DEMONSTRATE

Facilitation Notes

In this activity, students use the area of a circle formula to solve a real-world problem situation.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- Does this problem situation involve determining the circumference or area? How do you know?
- How can you determine which has the greatest area without doing all of the actual calculations?
- What is the area of the 8-inch pipe?
- What is the area of one 4-inch pipe?
- If you double the area of the one 4-inch pipe, should it be equivalent to the area of an 8-inch pipe? Why or why not?
- Did you expect the area of two 4-inch pipes to equal the area of one 8-inch pipe? Why or why not?

Summary

The area of a circle ($A = \pi r^2$) formula can be used to solve real-world problem situations.

NOTES

That's a Spicy Pizza!

2

Area of Circles

WARM UP

Determine a unit rate for each situation.

1. $38.40 for 16 gallons of gas
2. 15 miles jogged in 3.75 hours
3. $26.99 for 15 pounds

LEARNING GOALS

- Describe the relationship between the circumference and area of a circle and use the area formula to solve problems.
- Decide whether circumference or area is an appropriate measure for a problem situation.
- Calculate unit rates associated with circle areas.

KEY TERM

- unit rate

You have learned about the different parts and measures of a circle, including radius, diameter, and circumference. How can you use the parts of a circle to determine the area of a circle?

Warm Up Answers

1. $2.40 per gallon or approximately 0.42 gallon per dollar
2. 4 miles per hour or $\frac{1}{4}$ hour per mile
3. Approximately $1.80 per pound or approximately 0.56 pound per dollar

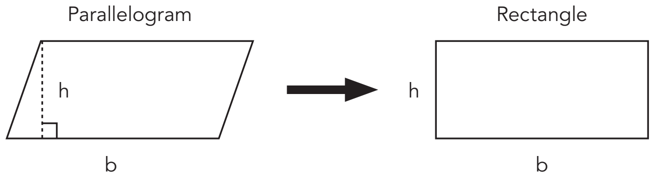
Answers

1. The triangle on the left of the parallelogram could be moved horizontally to the other side of the parallelogram.
2. The area of each figure is *bh*.

Getting Started

What Changed? What Stayed the Same?

The length of the base and height are the same in the parallelogram and rectangle shown.



1. How could you rearrange the parallelogram to create the rectangle?

2. What is the area of each figure?

ACTIVITY
2.1

Deriving the Area Formula

In the last lesson you derived formulas for the distance around a circle. In this lesson, you will investigate the space within a circle. Use the circle at the end of the lesson that is divided into 4, 8, and 16 equal parts.

1. Follow the steps to decompose the circle and then compose it into a new figure.

a. First, cut the circle into fourths and arrange the parts side by side so that they form a shape that looks like a parallelogram.

b. Then cut the circle into eighths and then sixteenths. Each time, arrange the parts to form a parallelogram.

2. Analyze the parallelogram you made each time.

a. How did the parallelogram change as you arranged it with the smaller equal parts of the same circle?

b. What would be the result if you built the parallelogram out of 40 equal circle sections? What about 100 equal circle sections?

c. Represent the approximate base length and height of the parallelogram in terms of the radius and circumference of the circle.

Answers

1. Check students' circles for parts (a) and (b).

2a. As more parts are used, it looks more like a parallelogram.

2b. Sample answer. It would look more like a parallelogram. The curved sides would start to look straighter.

2c. The length of the base b is approximately equal to half of the circumference of the circle, C, or $b \approx \frac{1}{2}C$. The height h is approximately equal to the radius of the circle r, or $h \approx r$.

Answers

2d. From substitution into $A = bh$, $A = \frac{1}{2}C \times r$.

2e. The area of the parallelogram is the same as the area of the circle.

2f. From substitution of the circumference formula for C, $A = \frac{1}{2} \times 2\pi r \times r$, so $A = \pi r^2$.

3a. See table below.

3b. The calculations that leave π in the answer represent the exact area. The calculations using the π key on the calculator are the best area approximations. The calculations using 3.14 for π are slightly less than those approximations using the π key. The calculations using $\frac{22}{7}$ for π are slightly greater than those approximations using the π key.

NOTES

d. Use your answers to part (c) to determine the formula for the area of the parallelogram.

e. How does the area of the parallelogram compare to the area of the circle?

f. Write a formula for the area of a circle.

3. Use different representations for π to calculate the area of a circle.

a. Calculate the area of each circle with the given radius. Round your answers to the nearest ten-thousandths, if necessary.

Value for π	r = 6 units	r = 1.5 units	$r = \frac{1}{2}$ unit
π			
Use the π key on a calculator			
Use 3.14 for π			
Use $\frac{22}{7}$ for π			

b. Compare your area calculations for each circle. How do the different values of π affect your calculations?

3a.

Value for π	$r = 6$ units	$r = 1.5$ units	$r = \frac{1}{2}$ unit
π	36π	2.25π	$\frac{1}{4}\pi$
Use π key on a calculator	$\approx$ 113.0973 sq units	$\approx$ 7.0686 sq units	$\approx$ 0.7854 sq units
Use 3.14 for π	$\approx$ 113.04 sq units	$\approx$ 7.065 sq units	$\approx$ 0.785 sq units
Use $\frac{22}{7}$ for π	$\approx$ 113.1429 sq units	$\approx$ 7.0714 sq units	$\frac{11}{14}$ or $\approx$ 0.7857 sq units

4. Suppose the ratio of radius lengths of two circles is 1 unit to 2 units.

a. What is the ratio of areas of the circles? Experiment with various radius lengths to make a conclusion.

b. If the length of the radius of a circle is doubled, what effect will this have on the area?

Answers

4a. The ratio is $\frac{1}{4}$.

4b. The area will quadruple.

Answers

1a. circumference;
Gina walks about 188.4 ft.

1b. area;
Jason will cover about 2826 square feet with plant food.

ACTIVITY 2.2

Circumference or Area

Circle Formulas
$C = \pi d$, or $2\pi r$
$A = \pi r^2$

The circumference of a circle is the distance around the circle, while the area of a circle is the amount of space contained inside the circle. When solving problems involving circles, it is important to think about what you are trying to determine.

1. A city park has a large circular garden with a path around it. The diameter of the garden is 60 feet.

a. Gina likes to walk along the circular path during her lunch breaks. How far does Gina walk if she completes one rotation around the path?

b. Jason works for the City Park Department. He needs to spread plant food all over the garden. What is the area of the park he will cover with plant food?

2. Samantha is making a vegetable pizza. First, she presses the dough so that it fills a circular pan with a 16-inch diameter and covers it with sauce. What is the area of the pizza Samantha will cover with sauce?

3. Members of a community center have decided to paint a large circular mural in the middle of the parking lot. The radius of the mural is to be 11 yards. Before they begin painting the mural, they use rope to form the outline. How much rope will they need?

Answers

2. area;
   Samantha will cover about 200.96 square inches with sauce.
3. circumference;
   They will need about 69.08 feet of rope.

ACTIVITY 2.3 Unit Rates and Circle Area

Talarico's Pizza has a large variety of pizza sizes.

	Small	Medium	Large	X-Large	Enorme	Ginorme	Colossale
Diameter	10 in.	13 in.	16 in.	18 in.	24 in.	28 in.	36 in.
Slices	6	8	10	12	20	30	40
Cost	$6.99	$9.99	$12.99	$14.99	$22.99	$28.99	$54.99

Recall that a **unit rate** is a ratio of two different measures in which either the numerator or denominator is 1.

Lina and Michael are trying to decide whether to get two pizzas or one Ginorme pizza. They ask themselves, "Which choice is the better buy?"

They each calculated a unit rate for the Ginorme pizza.

Lina

1 Ginorme: $\frac{\pi(14)^2}{28.99} = \frac{196\pi}{28.99} \approx$ 21.24 square inches per dollar

The Ginorme gives you approximately 21.24 square inches of pizza per dollar.

Michael

1 Ginorme: $\frac{28.99}{14^2\pi} = \frac{28.99}{196\pi} \approx \$0.05$ per square inch

The Ginorme costs approximately \$0.05 for each square inch of pizza.

1. **Consider Lina's and Michael's work.**

   a. **Explain why Lina's and Michael's unit rates are different but still both correct.**

   b. **How would you decide which pizza was the better buy if you calculated the unit rate for each pizza using Lina's method versus Michael's method.**

2. **Which of the seven sizes of pizza from Talarico's Pizza is the best buy? Explain your answer.**

Answers

1a. Lina determined the rate of the number of square inches of pizza for every \$1, and Michael determined the rate of the amount, in dollars, for every 1 square inch.

1b. Lina: The greater the number of square inches per dollar, the better the buy. Michael: The smaller the number of dollars per square inch of pizza, the better the buy.

2. Answers are approximates using 3.14 for π.

   Small: 8.9 cents per sq in.; Medium: 7.5 cents per sq in.; Large: 6.5 cents per sq in.; X-Large: 5.9 cents per sq in.; Enorme: 5.1 cents per sq in.; Ginorme: 4.7 cents per sq in.; Colossale: 5.4 cents per sq in.

   The Ginorme is the best deal.

Answers

1. The 8 cm pipe will allow more water to flow through the pipe. The area of the 8 cm pipe is 200.96 sq cm, and the area of the two 4 cm pipes is 100.48 sq cm. The 8 cm pipe is double the area of the two 4 cm pipes.

NOTES

TALK the TALK

Go With the Flow

1. Residents of a community are trying to determine which configuration would allow more water to flow through the pipe(s), one pipe with a radius of 8 cm or two pipes that each have a radius of 4 cm. Which configuration allows the most water to flow through the pipe(s), and what is the difference between the two configurations? Show your work and explain your reasoning.

Circle Area Cutouts