Circular Reasoning

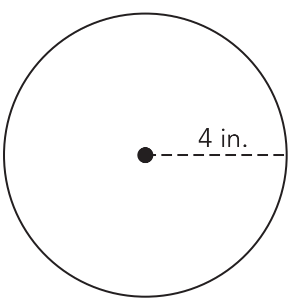
3

Solving Area and Circumference Problems

WARM UP

Determine the area of each circle. Use 3.14 for π.

1.



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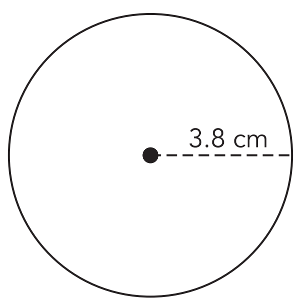

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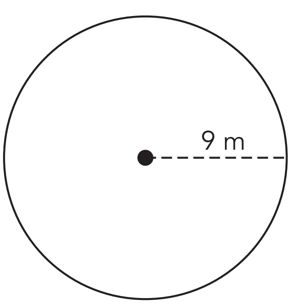

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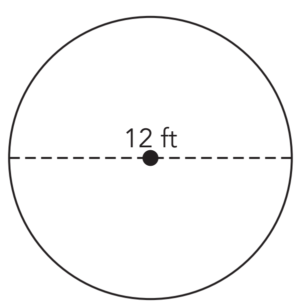


LEARNING GOALS

- Solve problems using the area and circumference formulas for a circle.
- Calculate the areas of composite figures.

You encounter circles regularly in life. Now that you know how to calculate the circumference and area of circles, what kind of problems can you solve?

A Winning Formula

Suppose that the circumference of a circle is approximately 157 centimeters.

1. Describe a strategy you can use to solve for the area of the circle.

2. Solve for the area of the circle. Use 3.14 for π.

ACTIVITY 3.1

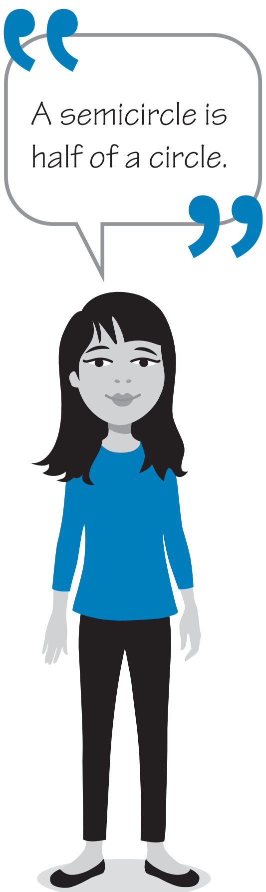
A Maximum Area Problem

A friend gave you 120 feet of fencing. You decide to fence in a portion of the backyard for your dog. You want to maximize the amount of fenced land.

1. **Draw a diagram, label the dimensions, and compute the maximum fenced area. Assume the fence is free-standing and you are not using any existing structure.**

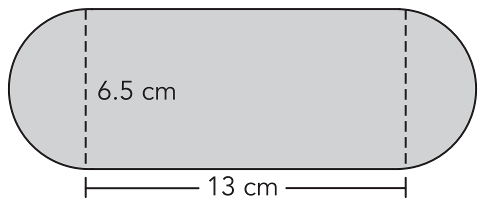
ACTIVITY
3.2

Composite Figure Problems

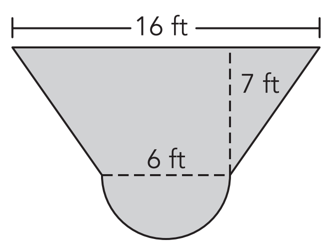


In previous grades you worked with composite figures made up of triangles and various quadrilaterals. Now that you know the area of a circle, you can calculate the area of more interesting composite figures.

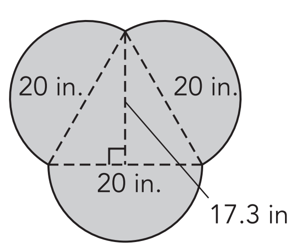
1. A figure is composed of a rectangle and two semicircles. Determine the area of the figure.



2. A figure is composed of a trapezoid and a semicircle. Determine the area of the figure.



3. A figure is composed of a triangle and three semicircles. Determine the area of the figure.



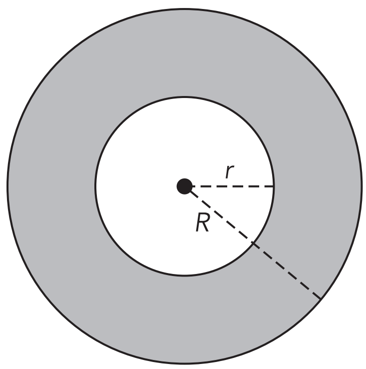
ACTIVITY
3.3

Shaded Region Problems

You have worked with composite figures by adding on areas. Now let's think about subtracting areas.

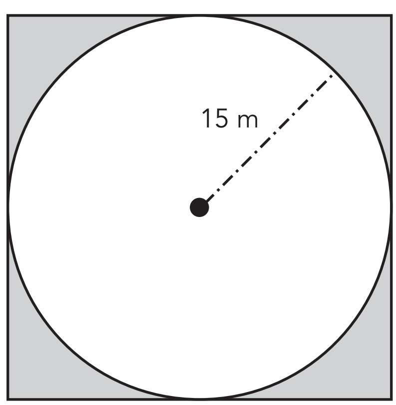
1. In the concentric circles shown, R represents the radius of the larger circle and r represents the radius of the smaller circle. Suppose that $R = 8$ centimeters and $r = 3$ centimeters. Calculate the area of the shaded region.

Concentric circles are circles with a common center. The region bounded by two concentric circles is called the annulus.



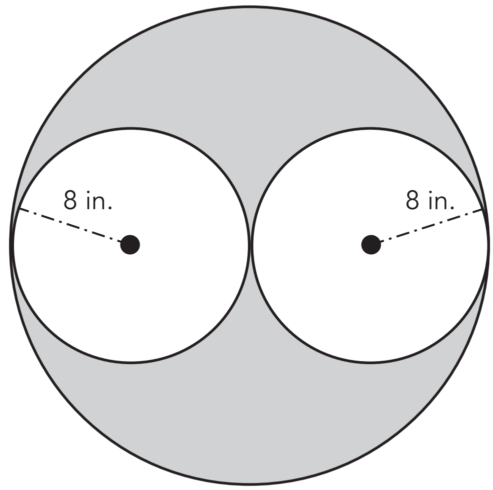
2. A circle is inscribed in a square. Determine the area of the shaded region.

When a circle is inscribed in a square, the diameter of the circle is equal to the side length of the square.



NOTES

3. **Two small circles are drawn that touch each other, and both circles touch the large circle. Determine the area of the shaded region.**



4. **Jimmy and Matthew each said the area of the shaded region is about 402 square inches. Compare their strategies.**

Jimmy

Area of 1 small circle
$A \approx 3.14(8)^2$
$A \approx 3.14(64)$
$A \approx 200.96$

Area of 2 small circles
$A \approx 2(200.96)$
$A \approx 401.92$

Area of large circle
$A \approx (3.14)(16)^2$
$A \approx (3.14)(256)$
$A \approx 803.84$

Area of shaded region
$803.84 - 401.92 \approx 401.92$

The area of the shaded region is about 402 sq in.

Matthew

Area of 1 small circle
$A = \pi(8)^2$
$A = 64\pi$

Area of 2 small circles
$A = 2(64\pi)$
$A = 128\pi$

Area of large circle
$A = \pi(16)^2$
$A = 256\pi$

Area of shaded region
$256\pi - 128\pi = 128\pi$
$A = 128\pi$
$A \approx 402.12$

This means the area of the shaded region is about 402 sq in.

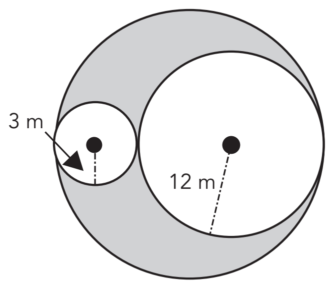
a. What did Jimmy and Matthew do the same?

b. What was different about their strategies?

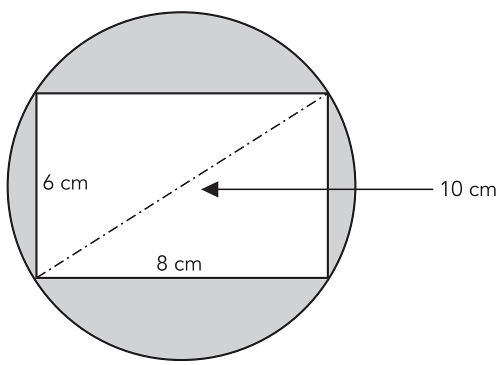
c. Which strategy do you prefer?

5. Determine the area of each shaded region.

a. One medium circle and one small circle touch each other, and each circle touches the large circle.



b. A rectangle is inscribed in a circle.



A rectangle is inscribed in a circle when all the vertices of the rectangle touch the circumference of the circle.

NOTES

TALK the TALK

Rupert's Leash

Jamal loves his dog, Rupert. On sunny days, Jamal keeps Rupert on a 12-foot leash in the backyard. The leash is secured to a stake in the ground.

1. **Determine the diameter, circumference, and area of Rupert's play area. Use 3.14 for π.**

2. **Suppose Jamal wants to give Rupert a little more room to play. He uses a 15-foot leash instead of the 12-foot leash. What is the area of Rupert's play area now? Use 3.14 for π.**