3

Circular Reasoning

Solving Area and Circumference Problems

MATERIALS

None

Lesson Overview

Students use the area of a circle formula and the circumference formula to solve for unknown measurements in problem situations. Some of the situations are problems composed of more than one figure, and some of the situations include shaded and non-shaded regions. Students then determine whether to use the circumference or area formula to solve problems involving circles.

Grade 7
Expressions, Equations, and Relationships

(9) The student applies mathematical process standards to solve geometric problems. The student is expected to:

(B) determine the circumference and area of circles.

(C) determine the area of composite figures containing combinations of rectangles, squares, parallelograms, trapezoids, triangles, semicircles, and quarter circles.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Essential Ideas

- The formula to calculate the area of a circle is $A = \pi r^2$.
- The formula to calculate the circumference of a circle is $C = 2\pi r$.
- Composite figures that include circles are used to solve for unknowns.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: A Winning Formula

Students use the circumference of a circle to determine its area. This activity is designed to engage students in explaining and analyzing problem-solving strategies, as well as to investigate the relationship between a circle's circumference and its area.

Develop

Activity 3.1: A Maximum Area Problem

Students are given 120 feet of fencing and asked to construct a free-standing dog pen in such a way that the maximum amount of area is fenced in. The circumference formula is used to determine the radius, and then the radius can be used to determine the maximum area.

Day 2

Activity 3.2: Composite Figure Problems

Students solve a variety of mathematical problems involving figures composed of quadrilaterals, circles, semicircles, and trapezoids.

Activity 3.3: Shaded Region Problems

Students solve a variety of mathematical problems involving determining the area of shaded regions in figures containing two figures, such as a rectangle within a circle.

Demonstrate

Talk the Talk: Rupert's Leash

Students solve a real-world problem by determining the diameter, circumference, and area of a dog's play area, given its leash length.

Getting Started: A Winning Formula

ENGAGE

Facilitation Notes

In this activity, given the circumference of a circle, students develop a strategy and use it to solve for the area.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategies

To scaffold support for students as they get started, ask questions such as,

- What do you know?
- Can you draw a diagram to represent that?
- What numbers can you label in your diagram?
- Are there other measurements you could figure out?
- Are there any formulas that would be helpful?

Questions to ask

- What is the formula for calculating the area of a circle?
- What information is needed to calculate the area of a circle?
- What is the formula for calculating the circumference of a circle?
- Knowing the circumference, how can the length of the radius be determined?
- Knowing the circumference, how can the length of the diameter be determined?
- What is the length of the radius in this situation?
- What is the length of the diameter in this situation?

Summary

Given the circumference of a circle, the radius or diameter can be determined and used to calculate the area.

Activity 3.1
A Maximum Area Problem

DEVELOP

Facilitation Notes

In this activity, students are given 120 feet of fencing and asked to construct a free-standing dog pen in such a way that the maximum amount of area is fenced in.

Have students work with a partner or in a group to complete this activity. Share responses as a class.

Questions to ask

- Does 120 feet of fencing represent the circumference or area in this situation?
- What does maximizing the area mean in terms of this situation?
- What shape is usually associated with maximizing area?
- If you build the pen in the shape of a square, what will be length of each side?
- What is the area of a square dog pen?
- If you build the pen in the shape of a circle, what will be the circumference of the circle?
- What information is needed to determine the area of a circular dog pen?
- How can you get the information needed to determine the area of a circular dog pen?
- Given the circumference of a circle, how do you determine the length of the radius?
- What formula is used to determine the length of the radius in this situation?
- What formula is used to determine the area in this situation?
- What is the length of the radius?

Summary

Circular shapes are used to maximize area in real-world problem situations.

Activity 3.2

Composite Figure Problems

Facilitation Notes

In this activity, students calculate the area of composite figures composed of triangles, rectangles, trapezoids, and circles.

Differentiation strategies

To scaffold support for students to visualize the problem, help them make sense of the diagram and organize the process.

- Have them use colored pencils to demonstrate the different pieces. For example, if they are combining two semicircles to make a circle, they should be one color.
- Then, have them redraw the separate pieces with the appropriate dimensions and calculate the individual areas.

Questions to ask

- What shapes did you decompose this figure into?
- Is there another strategy that could have been used?
- What operation did you use once you calculated the individual areas?

Questions to ask for Question 1

- What two dimensions does the measurement 6.5 represent in the diagram?
- Did you calculate each semicircle separately or did you put them together to make one full circle?
- When would you not be able to put two semicircles together to make a full circle?

Questions to ask for Question 2

- A trapezoid can always be divided into a rectangle and two triangles. Why is that not an efficient strategy for this particular question?

Questions to ask for Question 3

- Into how many shapes did you decompose this figure?
- How did you deal with the three semicircles?
- Did you use one large triangle or two small triangles? Why?

Summary

Decomposing shapes into smaller, familiar regions is a strategy used to determine the area of a composite figure.

Activity 3.3

Shaded Region Problems

Facilitation Notes

In this activity, students use different strategies to determine the area of shaded regions inside geometric figures.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Differentiation strategies

- To scaffold support, help students understand the dual-use of the radius drawn in each figure in Question 1, which may not be obvious to them. Have students draw additional radii in each figure to help them make sense of the diagrams. To scaffold support for students to visualize the strategy used to calculate the area of the shaded region: provide them with a real-size drawing of the figure, have them cut out the larger circle and then cut out the smaller circle, and finally discuss how removing the smaller circle models the subtraction necessary to calculate the area of the shaded region.

- For Question 2, two radii drawn to make a diameter parallel to one of the sides would help students see the dimensions of the square.
- For Question 5, part (c), you may want to provide the hint that drawing lines from the center will show that a regular hexagon is made up of 6 triangles which all have the same area.

Misconception

When calculating the area of an annulus, students sometimes think they can subtract the radii lengths first, then calculate the area once. If this happens, allow students to follow through with their calculations and then make a mathematical comparison to the correct work so that students can see why their thinking is incorrect.

Questions to ask

- What shapes are in this figure?
- How do you determine the area of the circle?
- How do you determine the area of the square?
- What strategy can be used to determine the area of the shaded region?
- What unit is used to describe this area?
- Are Jimmy and Matthew both correct?
- Whose strategy involved solving for the area in terms of pi?
- If the two circles inside the large circle were different sizes, would Jimmy's method still work?
- If the two circles inside the large circle were different sizes, would Matthew's method still work?
- Could this situation be solved using Jimmy's method from the previous question?
- Could this situation be solved using Matthew's method from the previous question?

Have students work with a partner to complete Question 5. Share responses as a class.

Differentiation strategy

To scaffold support for students to visualize the problem, help them make sense of the diagram and organize the process.

- What strategy can be used to determine the area of the shaded region?
- What shapes are in this figure?

- What information do you know about that figure?

For part (a),

- What does the dotted line represent with respect to the circle?

For part (b),

- What does the dotted line represent with respect to the rectangle?
- What does the dotted line represent with respect to the right triangle?
- How do you determine the area of the rectangle?

For part (c),

- What shapes are in this figure?
- How do you determine the area of the regular hexagon?
- What strategy can be used to determine the area of the shaded region?

Questions to ask

- What was your general strategy you used to calculate the area of the shaded region?
- Is there more than one way to calculate this area?

Summary

Strategies involving familiar area formulas can be used to determine the area of shaded regions inside geometric figures.

Talk the Talk: Rupert's Leash

DEMONSTRATE

Facilitation Notes

In this activity, students solve for the length of the diameter, circumference, and area in a real-world problem situation.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- What shape is Rupert's play area?
- Explain how Rupert pulling on his leash for the maximum length is similar to where you investigated how a circle is formed.
- Is Rupert's play area circular?
- Does the length of the leash represent the length of the radius or the length of the diameter?
- What formula is used to determine the circumference?

- What formula is used to determine the area?

Misconceptions

For Question 2, some students will try to save steps by using this incorrect strategy: $\pi(15 - 12)^2$, while the correct strategy is $\pi 15^2 - \pi 9^2$; if that is the case, have them make to-scale diagrams using centimeters for each strategy to visualize the difference in the resulting areas.

Summary

Area formulas can be used to solve real-world problem situations.

Circular Reasoning

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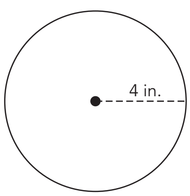
Solving Area and Circumference Problems

WARM UP

Determine the area of each circle. Use 3.14 for π.

1.



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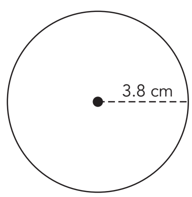

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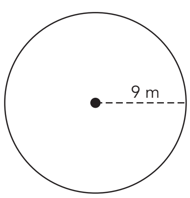

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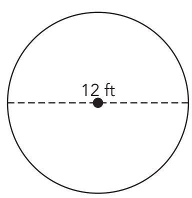


LEARNING GOALS

- Solve problems using the area and circumference formulas for a circle.
- Calculate the areas of composite figures.

You encounter circles regularly in life. Now that you know how to calculate the circumference and area of circles, what kind of problems can you solve?

Warm Up Answers

1. 50.24 sq in.
2. 45.34 sq cm
3. 254.34 sq m
4. 113.04 sq ft

Answers

1. Sample answer. Use the circumference to figure out the radius. Then, use the radius to calculate the area.
2. $157 = \pi d$, so $d \approx 50$ and $r \approx 25$

   Thus, the area of the circle is $\pi(25^2)$, or approximately 1962.5 sq cm.

Getting Started

A Winning Formula

Suppose that the circumference of a circle is approximately 157 centimeters.

1. Describe a strategy you can use to solve for the area of the circle.

2. Solve for the area of the circle. Use 3.14 for π.

ACTIVITY 3.1

A Maximum Area Problem

A friend gave you 120 feet of fencing. You decide to fence in a portion of the backyard for your dog. You want to maximize the amount of fenced land.

1. **Draw a diagram, label the dimensions, and compute the maximum fenced area. Assume the fence is free-standing and you are not using any existing structure.**

Answers

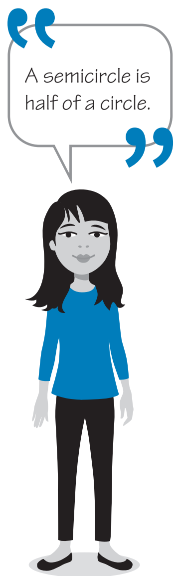
1. A circle with an approximate radius of 19.1 ft will give the maximum area of approximately 1145.5 sq ft.

Answers

1. 117.66625 sq cm
2. 91.13 sq ft
3. 644 sq in.

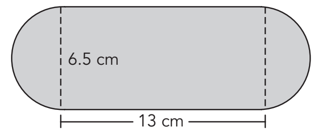
ACTIVITY 3.2

Composite Figure Problems

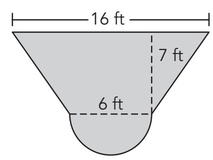


In previous grades you worked with composite figures made up of triangles and various quadrilaterals. Now that you know the area of a circle, you can calculate the area of more interesting composite figures.

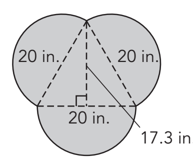
1. A figure is composed of a rectangle and two semicircles. Determine the area of the figure.



2. A figure is composed of a trapezoid and a semicircle. Determine the area of the figure.



3. A figure is composed of a triangle and three semicircles. Determine the area of the figure.



Answers

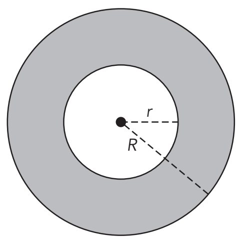
The answers provided are approximations.

1. 172.7 sq cm
2. 193.5 sq m

ACTIVITY **3.3** Shaded Region Problems

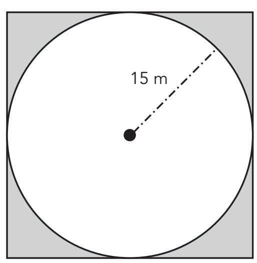
You have worked with composite figures by adding on areas. Now let's think about subtracting areas.

1. In the concentric circles shown, *R* represents the radius of the larger circle and *r* represents the radius of the smaller circle. Suppose that *R* = 8 centimeters and *r* = 3 centimeters. Calculate the area of the shaded region.



Concentric circles are circles with a common center. The region bounded by two concentric circles is called the **annulus**.

2. A circle is inscribed in a square. Determine the area of the shaded region.



When a circle is inscribed in a square, the diameter of the circle is equal to the side length of the square.

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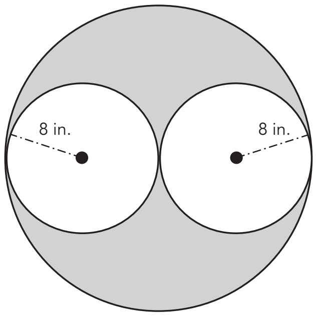
Answers

3. The area of the shaded region is approximately 402 square inches.

NOTES

3. Two small circles are drawn that touch each other, and both circles touch the large circle. Determine the area of the shaded region.



4. Jimmy and Matthew each said the area of the shaded region is about 402 square inches. Compare their strategies.

Jimmy

Area of 1 small circle
$A \approx 3.14(8)^2$
$A \approx 3.14(64)$
$A \approx 200.96$

Area of 2 small circles
$A \approx 2(200.96)$
$A \approx 401.92$

Area of large circle
$A \approx (3.14)(16)^2$
$A \approx (3.14)(256)$
$A \approx 803.84$

Area of shaded region
$803.84 - 401.92 \approx 401.92$

The area of the shaded region is about 402 sq in.

Matthew

Area of 1 small circle
$A = \pi(8)^2$
$A = 64\pi$

Area of 2 small circles
$A = 2(64\pi)$
$A = 128\pi$

Area of large circle
$A = \pi(16)^2$
$A = 256\pi$

Area of shaded region
$256\pi - 128\pi = 128\pi$
$A = 128\pi$
$A \approx 402.12$

This means the area of the shaded region is about 402 sq in.

6 • TOPIC 1: Circles and Ratio

ELL Tip

Group intermediate English Language Learners in pairs. Students can then collaborate to discuss the work of Jimmy and Matthew before answering parts (a) and (b). While students are in pairs, remind them to support one another using math vocabulary accurately or if a peer is struggling to name a word.

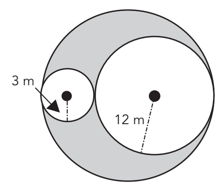
a. What did Jimmy and Matthew do the same?

b. What was different about their strategies?

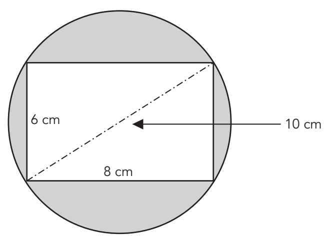
c. Which strategy do you prefer?

5. Determine the area of each shaded region.

a. One medium circle and one small circle touch each other, and each circle touches the large circle.



b. A rectangle is inscribed in a circle.



A rectangle is inscribed in a circle when all the vertices of the rectangle touch the circumference of the circle.

Answers

4a. Both Jimmy and Matthew found the area of 1 small circle first, then doubled it to find the area of 2 small circles. They both found the area of the large circle next. Finally, they both subtracted the area of the 2 small circles from the area of the large circle.

4b. Jimmy used 3.14 in place of pi throughout his equations. Matthew used π throughout his equations until the end when he replaced π with 3.14.

4c. Answers will vary.

5a. 226.08 sq m

5b. 30.5 sq cm

Answers

1. diameter = 24 ft;
   circumference = 75.36 ft;
   area = 452.16 sq ft
2. The area of Rupert's play area is 706.5 sq ft.

NOTES

TALK the TALK

Rupert's Leash

Jamal loves his dog, Rupert. On sunny days, Jamal keeps Rupert on a 12-foot leash in the backyard. The leash is secured to a stake in the ground.

1. **Determine the diameter, circumference, and area of Rupert's play area. Use 3.14 for π.**

2. **Suppose Jamal wants to give Rupert a little more room to play. He uses a 15-foot leash instead of the 12-foot leash. What is the area of Rupert's play area now? Use 3.14 for π.**