Eggzactly!

Solving Problems with Ratios of Fractions

2

WARM UP

Determine each product or quotient.

1. $\frac{1}{2} \times \frac{3}{5}$
2. $\frac{5}{8} \times \frac{8}{5}$
3. $\frac{2}{3} \div \frac{3}{8}$
4. $\frac{3}{4} \div 1\frac{1}{2}$

LEARNING GOALS

- Compute unit rates from ratios of fractions, including ratios of lengths and areas.
- Interpret complex rates to solve real-world problems involving lengths and areas.

KEY TERM

- complex ratio

You have learned about rates and unit rates. You have written unit rates from ratios of whole numbers. How can you write ratios of fractions as unit rates in order to solve problems?

Getting Started

A Different Form, But Still the Same

Ratios can be written using any numbers. A ratio in which one or both of the quantities being compared are written as fractions is called a **complex ratio**.

For example, traveling $\frac{1}{3}$ mile in $\frac{1}{2}$ hour represents a ratio of fractions, or a complex ratio. It is also an example of a rate, since the units being compared are different.

You can write this ratio in fractional form: $\frac{\frac{1}{3}\text{ mi}}{\frac{1}{2}\text{ h}}$

1. Rewrite each given rate as an equivalent ratio of fractions, or complex ratio, by converting one or both units of measure.

a. $\frac{1}{2}$ inch of rain fell in 15 minutes.

b. Sam ran 3520 feet in 20 minutes.

c. The baby gained 6 ounces every week.

d. Gas costs $2.50 per gallon.

Think about equivalent relationships. Fifteen minutes is what fraction of an hour?

ACTIVITY 2.1

Comparing Ratios of Fractions

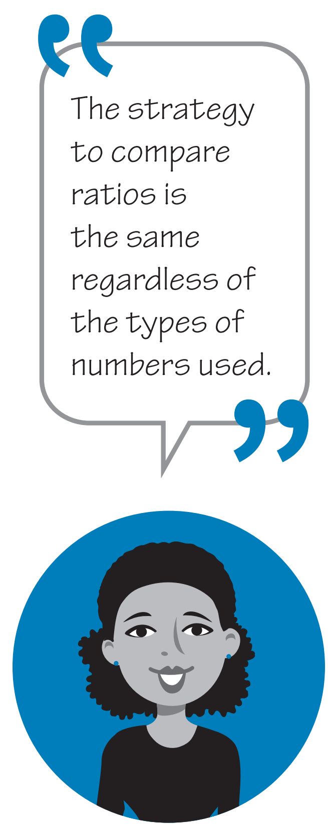
The table shows the weights of four different adult birds and the weights of their eggs.

	Mother's Weight (oz)	Egg Weight (oz)
Pigeon	10	$\frac{3}{4}$
Chicken	80	2
Swan	352	11
Robin	$2\frac{1}{2}$	$\frac{1}{10}$

1. Compare the weights of the eggs. List the birds in order from the bird with the heaviest egg to the bird with the lightest egg.

2. Determine the ratio of egg weight to mother's weight for each bird.

3. Compare the ratios of egg weight to mother's weight. List the birds in order from the greatest to the least ratio.



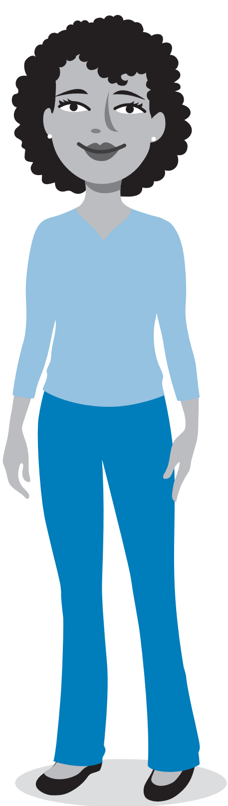
ACTIVITY 2.2

Determining Unit Rates from Ratios of Fractions

Although the ostrich is the largest living bird, it is also the fastest runner. The table shows distances that four birds ran, and the amount of time it took each bird to run that distance.

Bird	Distance Covered	Time
Ostrich	22 miles	$\frac{1}{2}$ hour
Greater Roadrunner	300 yards	$\frac{1}{2}$ minute
Quail	20 yards	$2\frac{1}{2}$ seconds
Pheasant	200 yards	$\frac{5}{6}$ minute

"Remember, a rate is a ratio that compares two quantities that are measured in different units."



Each row in the table shows a rate. The rate for each bird in this situation is the distance covered per the amount of time.

1. Write the rate for each bird as a complex rate.

a. Ostrich

b. Greater Roadrunner

c. Quail

d. Pheasant

NOTES

The rates you wrote in Question 1 are each represented using different units of measure. In order to compare speeds let's determine the unit rate in miles per hour for each bird. Consider the numbers and units of the original rate to choose a strategy. Analyze each Worked Example.

You know that the ostrich ran 22 miles in $\frac{1}{2}$ hour.

WORKED EXAMPLE

The rate of the ostrich is already measured in miles and hours. You can set up a proportion and scale the original rate up to 1 hour.

$$\frac{\text{distance}}{\text{time}} \rightarrow \frac{22 \text{ mi}}{\frac{1}{2} \text{ h}} \underset{\times 2}{\overset{\times 2}{=}} \frac{44 \text{ mi}}{1 \text{ h}}$$

$$= \frac{44 \text{ mi}}{1 \text{ h}}$$

The ostrich's speed is 44 miles per hour.

2. Why was the scale factor of 2 used in this Worked Example?

There are 1760 yards in 1 mile.

You know that the Greater Roadrunner ran 300 yards in $\frac{1}{2}$ minute.

WORKED EXAMPLE

The rate of the Greater Roadrunner is written in yards per minute. You can use conversion rates to rewrite the rate in miles per hour.

$$\frac{300 \text{ yd}}{\frac{1}{2} \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{1 \text{ mi}}{1760 \text{ yd}}$$

$$\frac{300 \cancel{\text{yd}}}{\frac{1}{2} \cancel{\text{min}}} \cdot \frac{60 \cancel{\text{min}}}{1 \text{ hr}} \cdot \frac{1 \text{ mi}}{1760 \cancel{\text{yd}}}$$

$$\frac{300}{\frac{1}{2}} \cdot \frac{60}{1} \cdot \frac{1}{1760} \frac{\text{mi}}{\text{hr}}$$

$$600 \cdot 60 \cdot \frac{1}{1760} \frac{\text{mi}}{\text{hr}} \approx \frac{20.5 \text{ mi}}{1 \text{ hr}}$$

3. Why is the fractional representation of each conversion rate important?

4. Determine the quail's and pheasant's speeds in miles per hour.

a. quail's speed:

b. pheasant's speed:

5. Write the birds in order from the fastest rate to the slowest rate.

ACTIVITY
2.3 Converting Between Systems

In this activity, you will use the common conversions shown in the table to convert between customary and metric measurements.

Length	Mass	Capacity
1 in. = 2.54 cm	1 oz = 28.35 g	1 pt = 0.47 L
1 cm = 0.39 in.	1 g = 0.035 oz	1 L = 2.11 pint
1 ft = 30.48 cm	1 lb = 0.45 kg	1 qt = 0.95 L
1 m = 3.28 ft	1 kg = 2.2 lb	1 L = 1.06 qt
1 mi = 1.61 km		1 gal = 3.79 L
1 km = 0.62 mi		1 L = 0.26 gal
1 m = 39.37 in.		
1 in. = 0.0254 m		
1 m = 1.09 yd		

WORKED EXAMPLE

To convert between systems, you can scale up or scale down using ratios. Two methods are shown to determine how many kilograms are in 2.5 pounds.

$$\times 2.5 \quad 1 \text{ lb} = 0.45 \text{ kg} \quad \times 2.5$$
$$2.5 \text{ lb} = 1.125 \text{ kg}$$

Use the information from the chart.

Multiply to calculate the number of kilograms in 2.5 pounds.

$$\frac{1 \text{ lb}}{0.45 \text{ kg}} \overset{\times 2.5}{=} \frac{2.5 \text{ lb}}{1.125 \text{ kg}} \quad (\times 2.5)$$

Write a ratio using the information from the chart.

Scale up to calculate the number of kilograms in 2.5 pounds.

The local zoo hosted a marathon to raise money to remodel the aviary. An aviary is a large enclosure for birds, which gives them more living space where they can fly, unlike confining them to birdcages.

1. To train for a marathon, which is 26.2 miles or approximately 42.2 km, runners build up their endurance by running shorter distances. Complete the table shown by writing the unknown measurements. Round to the nearest tenth.

Race	Kilometers	Miles
Short Distance	5	
Medium Distance	10	
Medium Distance	20	
Half Marathon		13.1
Ultramarathon	100	
Ironman Triathlon Swim		2.4
Ironman Triathlon Bike		112

2. The zoo earned the money that they needed to remodel the aviary! To figure out the amount of supplies needed, they will need to measure the space. The zookeeper realizes that she only has a meter stick, not a ruler or a yardstick. She measures the aviary but needs to know the dimensions in inches and feet in order to purchase the materials. She records the following measurements:

   - The length of the room is 5 meters.
   - The width of the room is 4 meters.
   - The height of the room is 2.5 meters.

   a. What is the length of the room in inches? In feet? Round to the nearest hundredth.

   b. What is the width of the room in inches? In feet? Round to the nearest hundredth.

   c. What is the height of the room in inches? In feet? Round to the nearest hundredth.

   d. There are 39.37 inches in a meter. Explain to a classmate how many feet are in a meter.

3. **During cold conditions, pheasants fly 60 yards before taking to the ground for cover. In other parts of the year, they fly about 2 kilometers. Is the length of a pheasant's flight longer during cold conditions or other parts of the year? How do you know?**

4. **Molly and Shawna volunteer at the zoo and are each taking care of a growing ostrich. Molly says that her ostrich is 1.5 meters tall. Shawna's ostrich is 5 feet tall. Molly says that her ostrich is taller, but Shawna disagrees. Who is correct? Explain your reasoning.**

5. **Larry, Casey, Shaun, and Jamal are also raising ostriches. Larry's ostrich weighs 110 pounds, Casey's weighs 98 pounds, Shaun's weighs 42 kg, and Jamal's weighs 52 kg. Place the boys in order from the lowest weight of their ostrich to the highest weight using pounds and kilograms. Round to the nearest hundredth.**

ACTIVITY 2.4

Solving Problems with Fractional Rates

1. Tony needs a rate table for his tutoring jobs so that he can look up the charge quickly.

   a. Complete the rate table.

Time (Hours)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	3	$3\frac{1}{2}$	4
Charge ($)			37.50				

   b. How much would Tony charge for $7\frac{1}{2}$ hours of tutoring?

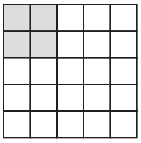
   c. Tony made $212.50 last weekend. How long did he tutor? Explain how you solved the problem.

2. At Pepe's Pizzas, a new deal gives you $1\frac{1}{2}$ orders of wings for half the price of a single order. Without the deal, a single order of wings costs $12. What is the cost of a single order of wings with the deal?

3. Abby uses $3\frac{3}{4}$ scoops of drink mix to make 10 cups of drink.

   a. How much drink mix would she need to use to make 1 cup of drink?

   b. She only has $11\frac{1}{4}$ scoops of drink mix remaining. How many cups of drink can she make?



4. The square shown is composed of smaller equally-sized squares. The shaded section has an area of $\frac{9}{25}$ square inches. What is the area of the large square?

NOTES

TALK the TALK

True, False, Example

Determine whether each statement is true or false. Provide one or more examples and an explanation to justify your answer.

1. To compute a unit rate associated with a ratio of fractions, multiply both the numerator and denominator by the reciprocal of the denominator. True False

2. Any ratio can be written as a complex ratio. True False

3. You never scale down to write a complex rate as a unit rate. True False

4. A statement with the word "per" is always a unit rate. True False

5. Dividing the numerator by the denominator is one way to convert a rate to a unit rate. True False