Eggzactly!

2

Solving Problems with Ratios of Fractions

MATERIALS

None

Lesson Overview

In this lesson, students determine ratios and write rates, including complex ratios and rates. Students will write proportions and use rates to determine miles per hour. They use common conversions to convert between the customary and metric measurement systems using unit rates and proportions. They will scale up and scale down to determine unknown quantities.

Grade 7

Proportionality

(4) The student applies mathematical process standards to represent and solve problems involving proportional relationships. The student is expected to:

(B) calculate unit from rates in mathematical and real-world problems.

(E) convert between measurement systems, including the use of proportions and the use of unit rates.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Essential Ideas

- A complex ratio has a fractional numerator or denominator (or both).
- Complex ratios and rates can be used to solve problems.
- Unit rates and proportions can be used to convert between measurement systems.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: A Different Form, But Still the Same

Students write complex ratios, which are ratios in which one or both quantities being compared are written as fractions. This activity is designed to engage students in thinking about rates with whole numbers as complex rates, by interpreting one or both of the quantities as parts of a whole.

Develop

Activity 2.1: Comparing Ratios of Fractions

The weights of four different birds and the weights of their eggs are given in a table. Students compare the weights of the eggs of four different birds. They then will determine the ratio of egg weight to mother's weight for each bird and answer related questions.

Activity 2.2: Determining Unit Rates from Ratios of Fractions

The distance four different birds run and the amount of time it takes them to run the distance is given in a table. Students determine the rate at which each bird runs from the given table. They then use ratio reasoning and rates to determine bird speeds in terms of miles per hour.

Day 2

Activity 2.3: Converting Between Systems

Common conversions are listed in a table. An example of converting pounds to kilograms is provided. In this example, a ratio is formed using what is known and what is unknown, and this ratio is set equal to a second ratio, a form of one, using the related conversion facts. Questions focus students to convert metric lengths to customary lengths and customary lengths to metric lengths.

Activity 2.4: Solving Problems with Fractional Rates

Students solve a variety of problems involving time, area, and surface area. Students determine unit rates from complex ratios in order to solve.

Demonstrate

Talk the Talk: True, False, Example

In this activity, students generalize about what they have learned in the lesson by deciding whether statements are true or false and providing examples to justify their answers.

Getting Started: A Different Form, But Still the Same

ENGAGE

Facilitation Notes

In this activity, students rewrite rates as an equivalent ratios of fractions or complex ratios by converting one or both units of measure.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- How many minutes are in one hour?
- Fifteen minutes is what fraction of one hour?
- How many feet are in one mile?
- Three-thousand five-hundred twenty feet is what fraction of one mile?
- Twenty minutes is what fraction of one hour?
- How many ounces are in one pound?
- Six ounces is what fraction of one pound?
- How can $2.50 be rewritten as a fraction in terms of one dollar?

Summary

Complex ratios are ratios in which one or both of the quantities being compared are written as fractions.

Activity 2.1

Comparing Ratios of Fractions

DEVELOP

Facilitation Notes

In this activity, students use information given in a table to write ratios and answer questions related to the ratios.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- What ratios were complex ratios?
- How did you simplify the complex ratios?
- How did you compare ratios?
- Were some ratios easier to compare than others? Explain.

- How did you determine the greatest ratio?
- How did you determine the least ratio?
- How did you determine which bird lays the largest egg for its size?
- How did you determine which bird lays the smallest egg for its size?
- If there was a pigeon as big as the chicken, how big would you expect its egg to be?
- About how much does a dozen eggs weigh?
- Were your conclusions what you expected?

Summary

Complex ratios are used for comparisons in real-world situations.

Activity 2.2

Determining Unit Rates from Ratios of Fractions

Facilitation Notes

In this activity, students use the information in a table to write complex rates that use different units of measure. Then, through Worked Examples and for comparison purposes, they convert these rates into a common rate, such as miles per hour.

Ask a student to read the information before Question 1 aloud. Discuss the table as a class.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- How many yards does the Greater Roadrunner travel in $\frac{1}{2}$ minute?
- How many yards does the quail travel in $2\frac{1}{2}$ seconds?
- How many yards does the pheasant travel in $\frac{5}{6}$ minute?
- How can you compare the speeds at which these birds travel if the number of seconds is different for each bird?

Ask students to read the Worked Example aloud and complete Question 2 as a class.

Questions to ask

- Do you think you must always use multiplication to convert between units of measure?

- Do you think you could ever use addition or subtraction?
- Name the operation(s) you would use to scale up a ratio?
- Name the operation(s) you would use to scale down a ratio?

Ask students to read the Worked Example aloud and discuss Question 2 as a class. Then have the students read through the next Worked Example using conversion rates and answer Question 3 as a class.

Have students work with a partner or in a group to complete Questions 4 and 5. Share responses as a class.

Questions to ask

- How did you determine the quail's speed?
- How did you determine the pheasant's speed?
- Did you express the speeds in miles per hour?

Summary

For comparison purposes, the representation of each rate can be converted to a common rate.

Activity 2.3

Converting Between Systems

Facilitation Notes

In this activity, students use common conversions given in a table to solve real-world problems that require converting between customary and metric measurements.

Ask a student to read the information before Question 1 aloud. Discuss the table and Worked Example as a class.

Have students complete Questions 1 and 2 with a partner. Share responses as a class.

Questions to ask

- How did you determine the length of the room in inches?
- How did you determine the length of the room in feet?
- How did the example provided help you to determine the length of the room in inches?
- How did the example provided help you to determine the length of the room in feet?
- How was determining the length of the room in inches similar to determining the width of the room in inches?

- How was determining the length of the room in feet similar to determining the width of the room in feet?
- How many inches are in one foot?
- How many inches are in two feet?
- How many inches are in three feet?

For Questions 3 through 5, three situations are described which involve converting from metric to customary measurements and vice versa. Students compare the values of measurements to determine which is greater than, less than, or the same amount.

Have students complete Questions 3 through 5 with a partner. Share responses as a class.

Questions to ask

- How did you convert kilometers into yards?
- How did you convert meters into feet?
- When setting up the proportion to convert pounds into kilograms, what ratio equivalent of 1 is used?

Summary

Sometimes it is necessary to convert between systems in order to compare measurements in both metric and customary units.

Activity 2.4

Solving Problems with Fractional Rates

Facilitation Notes

In this activity, students solve real-world problem situations that use fractional rates.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Differentiation

Each group can be given only one of the four questions. Allow time for each group to do a classroom presentation of their solution. Encourage students to ask their classmates questions about their solution path.

Questions to ask

- How did you determine the charge for $7\frac{1}{2}$ hours of tutoring?
- How can the table values be used to determine the charge for $7\frac{1}{2}$ hours of tutoring?

- If Tony charged $21.25, did he tutor for more or less than one hour?
- How much do you save on a single order of wings if you take advantage of the new deal?
- What operation was used to determine the amount of drink mix needed to make 1 cup of drink?
- How many small squares are contained in the large square?
- How do you determine the area of each small square?
- What is the area of each small square?

Summary

Fractional rates are used in real-world problem situations.

Talk the Talk: True, False, Example

DEMONSTRATE

Facilitation Notes

In this activity, students determine if statements about rates are true or false. They also explain their reasoning when possible.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

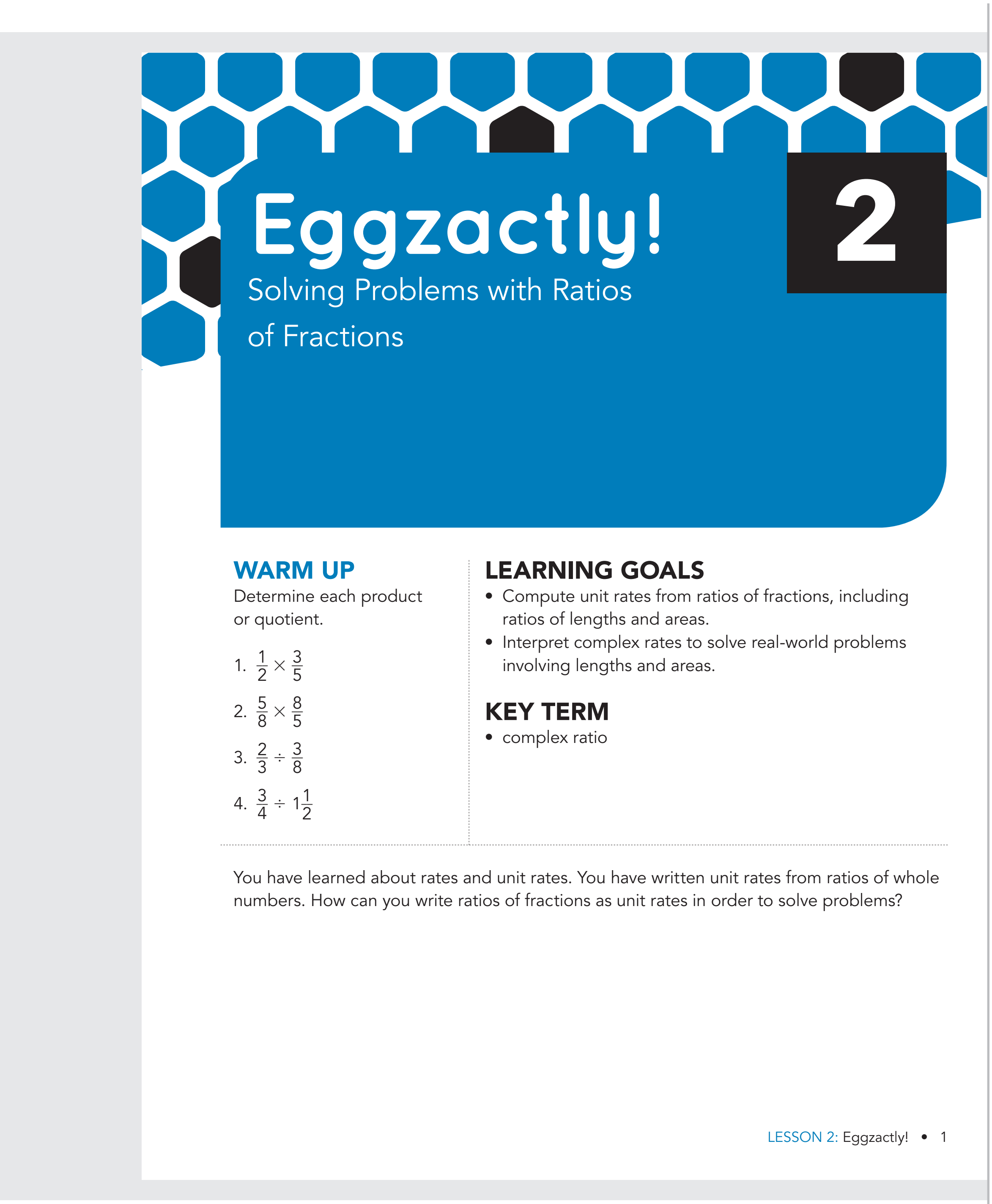
Questions to ask

- What is a rate?
- What is a unit rate?
- How is a unit rate the same and different than other rates?
- What is a ratio?
- What is a complex ratio?
- What is a complex rate?
- Can all rates be converted into unit rates?

Summary

All rates can be converted into unit rates. Any ratio can be written as a complex ratio.

NOTES



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2

Solving Problems with Ratios of Fractions

WARM UP

Determine each product or quotient.

1. $\frac{1}{2} \times \frac{3}{5}$
2. $\frac{5}{8} \times \frac{8}{5}$
3. $\frac{2}{3} \div \frac{3}{8}$
4. $\frac{3}{4} \div 1\frac{1}{2}$

LEARNING GOALS

- Compute unit rates from ratios of fractions, including ratios of lengths and areas.
- Interpret complex rates to solve real-world problems involving lengths and areas.

KEY TERM

- complex ratio

You have learned about rates and unit rates. You have written unit rates from ratios of whole numbers. How can you write ratios of fractions as unit rates in order to solve problems?

Warm Up Answers

1. $\frac{3}{10}$
2. 1
3. $1\frac{7}{9}$
4. $\frac{1}{2}$

Answers

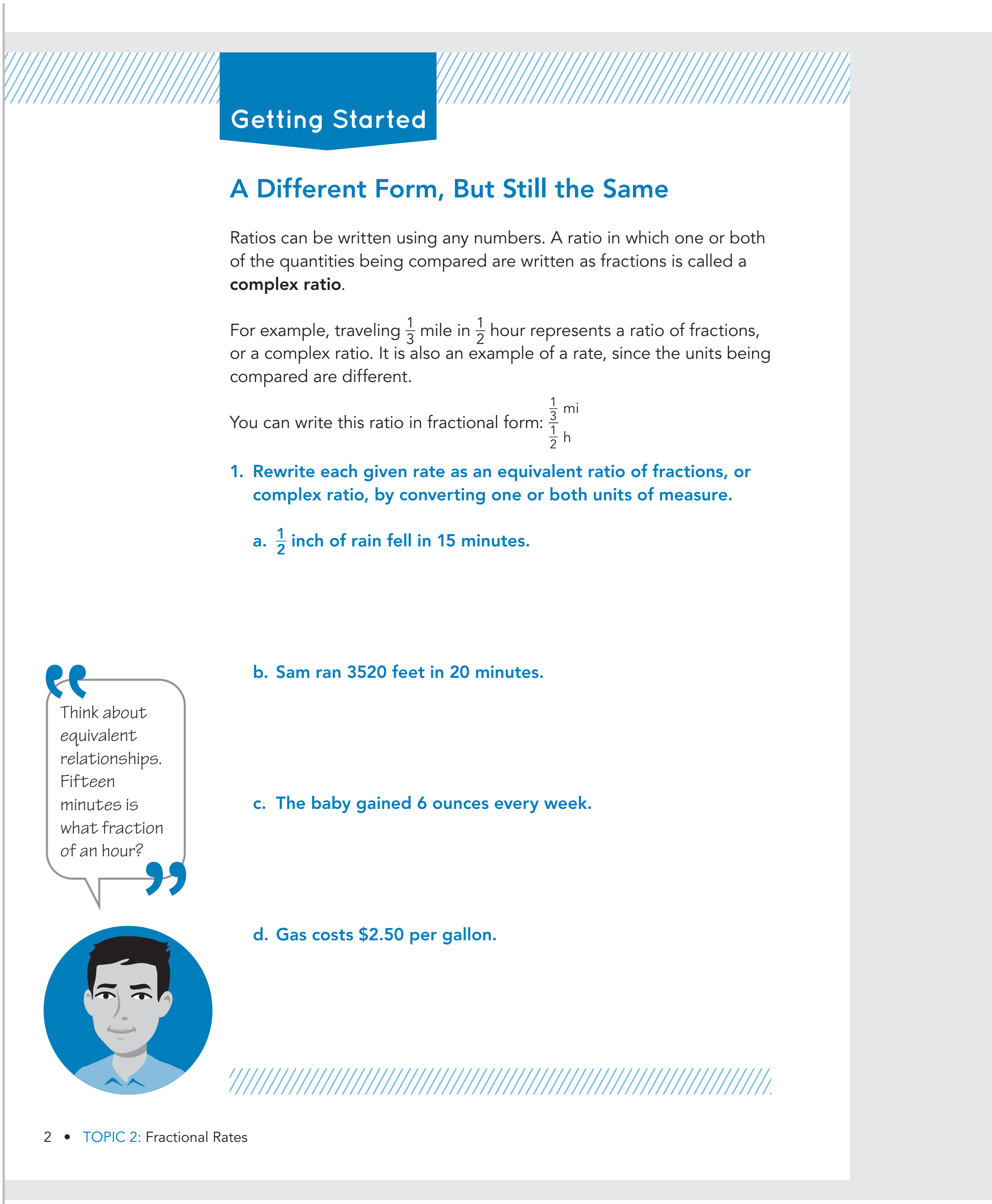
Sample answers.

1a. $\frac{\frac{1}{2}\text{ in.}}{\frac{1}{4}\text{ h}}$

1b. $\frac{\frac{2}{3}\text{ mi}}{\frac{1}{3}\text{ h}}$

1c. $\frac{\frac{3}{8}\text{ lb}}{1\text{ week}}$

1d. $\frac{\frac{5}{2}\text{ dollars}}{1\text{ gallon}}$



Getting Started

A Different Form, But Still the Same

Ratios can be written using any numbers. A ratio in which one or both of the quantities being compared are written as fractions is called a **complex ratio**.

For example, traveling $\frac{1}{3}$ mile in $\frac{1}{2}$ hour represents a ratio of fractions, or a complex ratio. It is also an example of a rate, since the units being compared are different.

You can write this ratio in fractional form: $\frac{\frac{1}{3}\text{ mi}}{\frac{1}{2}\text{ h}}$

1. Rewrite each given rate as an equivalent ratio of fractions, or complex ratio, by converting one or both units of measure.

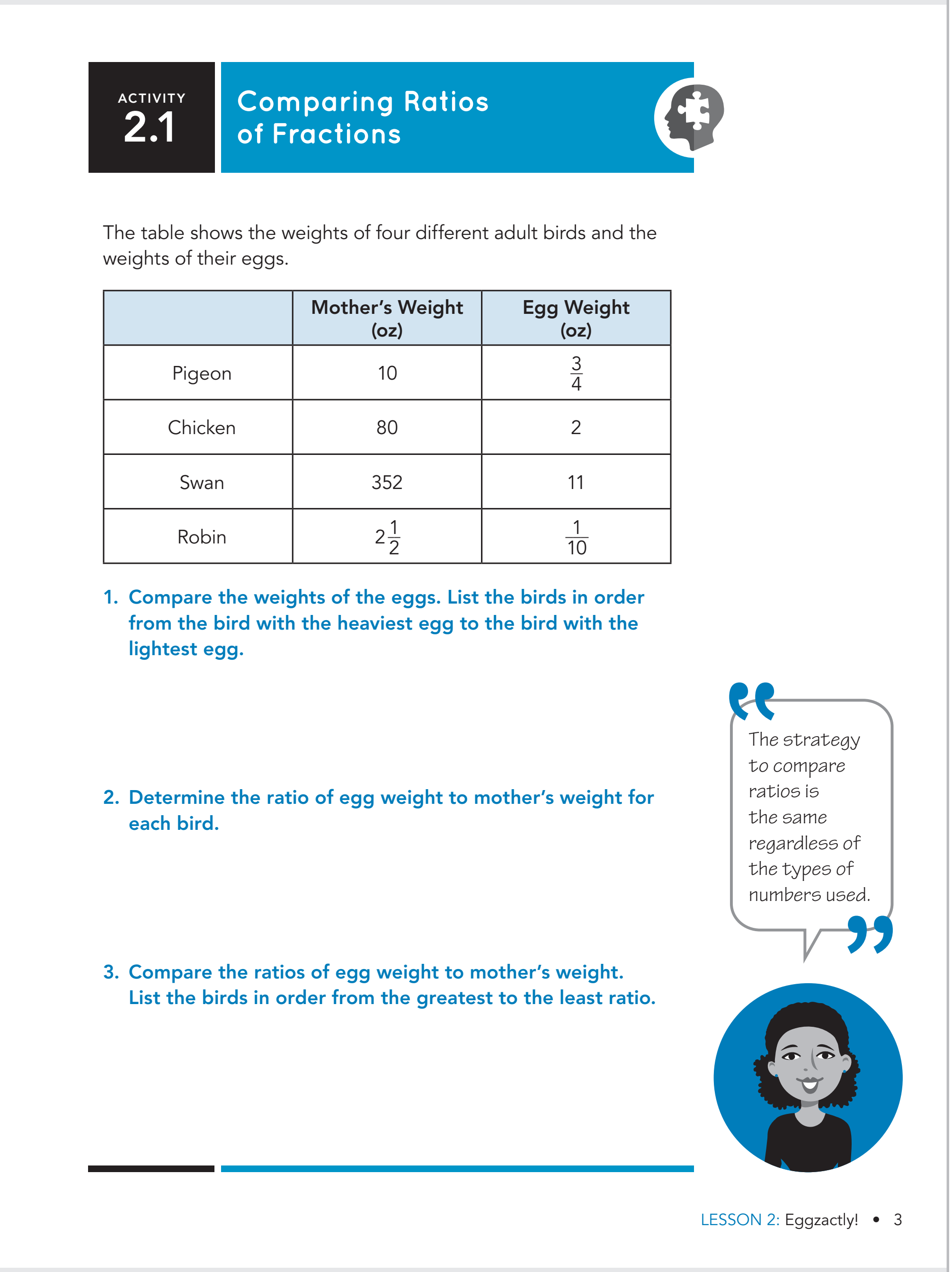
a. $\frac{1}{2}$ inch of rain fell in 15 minutes.

b. Sam ran 3520 feet in 20 minutes.

c. The baby gained 6 ounces every week.

d. Gas costs $2.50 per gallon.

Think about equivalent relationships. Fifteen minutes is what fraction of an hour?



ACTIVITY 2.1

Comparing Ratios of Fractions

The table shows the weights of four different adult birds and the weights of their eggs.

	Mother's Weight (oz)	Egg Weight (oz)
Pigeon	10	$\frac{3}{4}$
Chicken	80	2
Swan	352	11
Robin	$2\frac{1}{2}$	$\frac{1}{10}$

1. Compare the weights of the eggs. List the birds in order from the bird with the heaviest egg to the bird with the lightest egg.

2. Determine the ratio of egg weight to mother's weight for each bird.

3. Compare the ratios of egg weight to mother's weight. List the birds in order from the greatest to the least ratio.

The strategy to compare ratios is the same regardless of the types of numbers used.

Answers

1. swan: 11 oz, chicken: 2 oz, pigeon: $\frac{3}{4}$ oz, robin: $\frac{1}{10}$ oz

2. pigeon: $\frac{\frac{3}{4}}{10}$, chicken: $\frac{1}{40}$, swan: $\frac{11}{352}$, robin: $\frac{\frac{1}{10}}{\frac{5}{2}}$

3. pigeon, robin, swan, chicken

Answers

1a. $\dfrac{22\text{ mi}}{\frac{1}{2}\text{ h}}$

1b. $\dfrac{300\text{ yd}}{\frac{1}{2}\text{ min}}$

1c. $\dfrac{20\text{ yd}}{\frac{5}{2}\text{ s}}$

1d. $\dfrac{200\text{ yd}}{\frac{5}{6}\text{ min}}$

ACTIVITY 2.2

Determining Unit Rates from Ratios of Fractions

Although the ostrich is the largest living bird, it is also the fastest runner. The table shows distances that four birds ran, and the amount of time it took each bird to run that distance.

Bird	Distance Covered	Time
Ostrich	22 miles	$\frac{1}{2}$ hour
Greater Roadrunner	300 yards	$\frac{1}{2}$ minute
Quail	20 yards	$2\frac{1}{2}$ seconds
Pheasant	200 yards	$\frac{5}{6}$ minute

Each row in the table shows a rate. The rate for each bird in this situation is the distance covered per the amount of time.

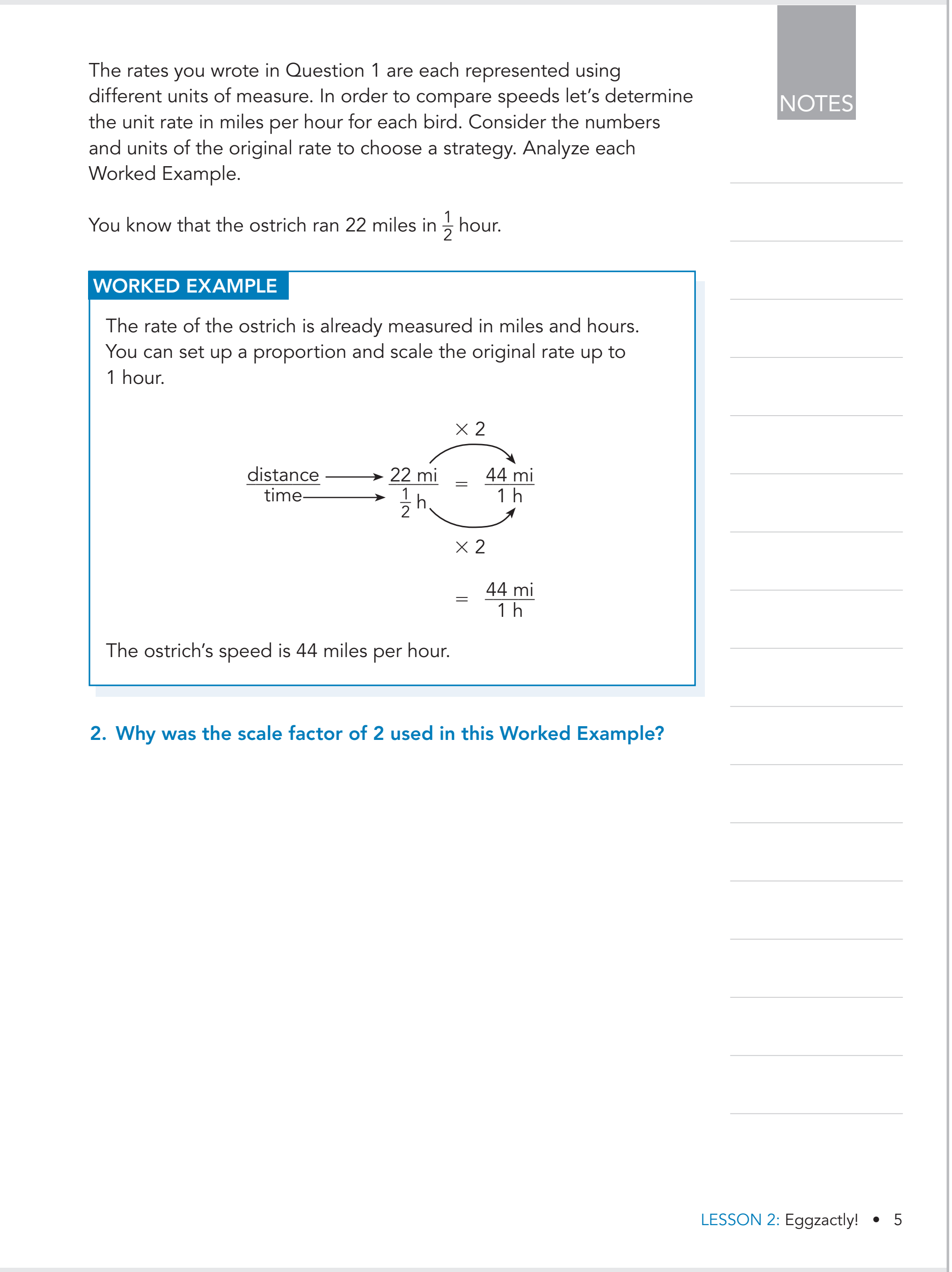
1. Write the rate for each bird as a complex rate.

a. Ostrich

b. Greater Roadrunner

c. Quail

d. Pheasant



NOTES

The rates you wrote in Question 1 are each represented using different units of measure. In order to compare speeds let's determine the unit rate in miles per hour for each bird. Consider the numbers and units of the original rate to choose a strategy. Analyze each Worked Example.

You know that the ostrich ran 22 miles in $\frac{1}{2}$ hour.

WORKED EXAMPLE

The rate of the ostrich is already measured in miles and hours. You can set up a proportion and scale the original rate up to 1 hour.

$$\begin{array}{r}\text{distance} \longrightarrow \\ \text{time} \longrightarrow\end{array} \frac{22\text{ mi}}{\frac{1}{2}\text{ h}} \overset{\times 2}{\underset{\times 2}{=}} \frac{44\text{ mi}}{1\text{ h}}$$

$$= \frac{44\text{ mi}}{1\text{ h}}$$

The ostrich's speed is 44 miles per hour.

2. Why was the scale factor of 2 used in this Worked Example?

Answers

2. To scale $\frac{1}{2}$ to 1, you need to multiply by 2. So, both the numerator and denominator are scaled up by 2.

Answers

3. Answers will vary. The fractional representation is important because it allows you to see the units.

4a. The quail’s speed is 8 yd per second, which is 28,800 yd per hour, or approximately 16.4 mi/h.

4b. The pheasant’s speed is 240 yd per minute, which is 14,400 yd per hour, or approximately 8.2 mi/h.

5. ostrich, Greater Roadrunner, quail, pheasant

There are 1760 yards in 1 mile.

You know that the Greater Roadrunner ran 300 yards in $\frac{1}{2}$ minute.

WORKED EXAMPLE

The rate of the Greater Roadrunner is written in yards per minute. You can use conversion rates to rewrite the rate in miles per hour.

$$\frac{300\text{ yd}}{\frac{1}{2}\text{ min}} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{1\text{ mi}}{1760\text{ yd}}$$

$$\frac{300\cancel{\text{ yd}}}{\frac{1}{2}\cancel{\text{ min}}} \cdot \frac{60\cancel{\text{ min}}}{1\text{ hr}} \cdot \frac{1\text{ mi}}{1760\cancel{\text{ yd}}}$$

$$\frac{300}{\frac{1}{2}} \cdot \frac{60}{1} \cdot \frac{1}{1760}\frac{\text{mi}}{\text{hr}}$$

$$600 \cdot 60 \cdot \frac{1}{1760}\frac{\text{mi}}{\text{hr}} \approx \frac{20.5\text{ mi}}{1\text{ hr}}$$

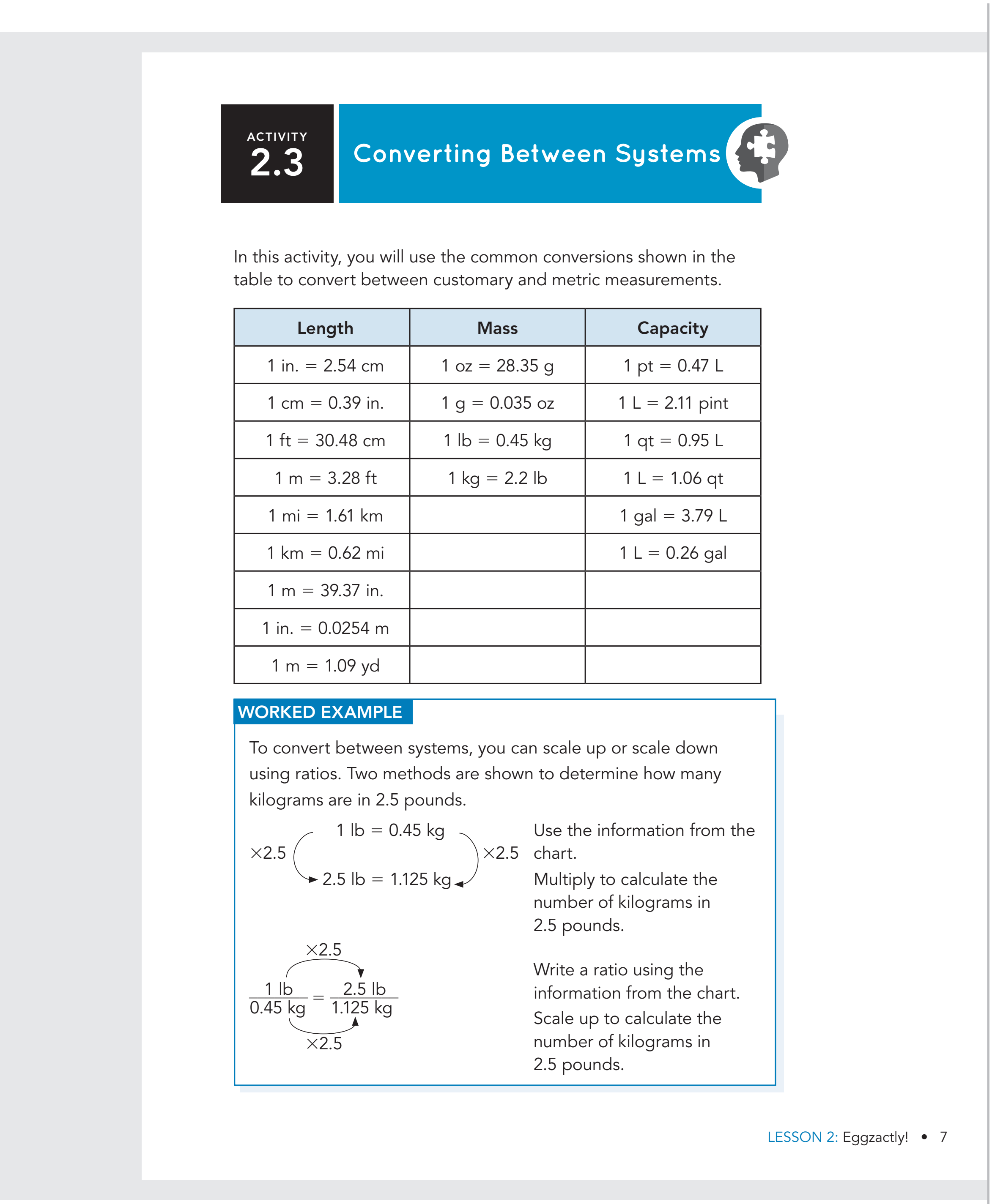
3. Why is the fractional representation of each conversion rate important?

4. Determine the quail’s and pheasant’s speeds in miles per hour.

a. quail’s speed:

b. pheasant’s speed:

5. Write the birds in order from the fastest rate to the slowest rate.



ACTIVITY
2.3

Converting Between Systems

In this activity, you will use the common conversions shown in the table to convert between customary and metric measurements.

Length	Mass	Capacity
1 in. = 2.54 cm	1 oz = 28.35 g	1 pt = 0.47 L
1 cm = 0.39 in.	1 g = 0.035 oz	1 L = 2.11 pint
1 ft = 30.48 cm	1 lb = 0.45 kg	1 qt = 0.95 L
1 m = 3.28 ft	1 kg = 2.2 lb	1 L = 1.06 qt
1 mi = 1.61 km		1 gal = 3.79 L
1 km = 0.62 mi		1 L = 0.26 gal
1 m = 39.37 in.		
1 in. = 0.0254 m		
1 m = 1.09 yd		

WORKED EXAMPLE

To convert between systems, you can scale up or scale down using ratios. Two methods are shown to determine how many kilograms are in 2.5 pounds.

$$\begin{tikzcd} 1\text{ lb} \arrow[d, bend right, "\times2.5"'] & = & 0.45\text{ kg} \arrow[d, bend left, "\times2.5"] \\ 2.5\text{ lb} & = & 1.125\text{ kg} \end{tikzcd}$$

Use the information from the chart.

Multiply to calculate the number of kilograms in 2.5 pounds.

$$\begin{tikzcd} \dfrac{1\text{ lb}}{0.45\text{ kg}} \arrow[r, phantom, "="] \arrow[r, bend left=60, "\times2.5"] \arrow[r, bend right=60, "\times2.5"'] & \dfrac{2.5\text{ lb}}{1.125\text{ kg}} \end{tikzcd}$$

Write a ratio using the information from the chart.

Scale up to calculate the number of kilograms in 2.5 pounds.

Answers

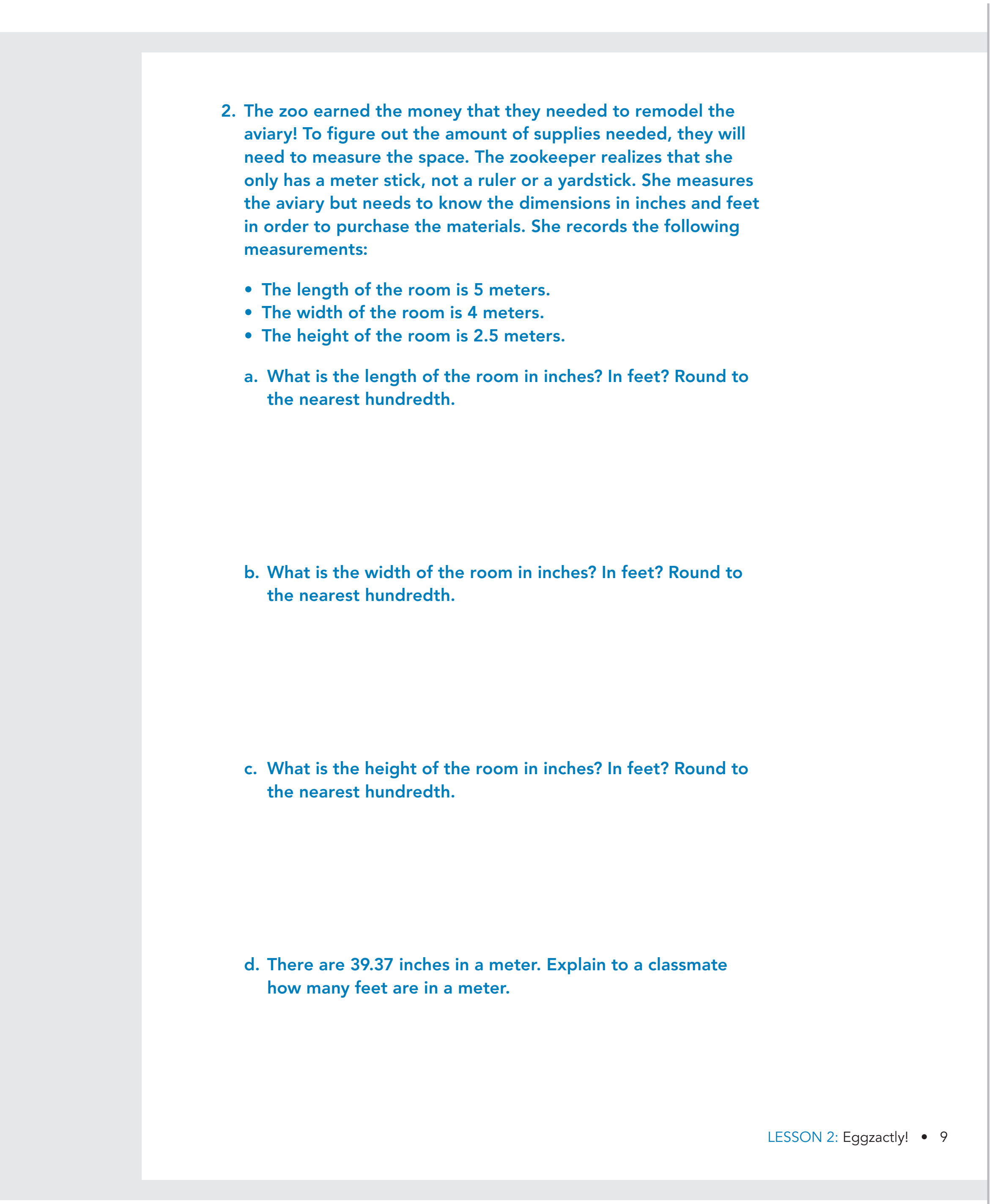
1.

Race	Kilometers	Miles
Short Distance	5	3.1
Medium Distance	10	6.2
Medium Distance	20	12.4
Half Marathon	21.1	13.1
Ultra-marathon	100	62
Ironman Triathlon Swim	3.9	2.4
Ironman Triathlon Bike	180.6	112

The local zoo hosted a marathon to raise money to remodel the aviary. An aviary is a large enclosure for birds, which gives them more living space where they can fly, unlike confining them to birdcages.

1. To train for a marathon, which is 26.2 miles or approximately 42.2 km, runners build up their endurance by running shorter distances. Complete the table shown by writing the unknown measurements. Round to the nearest tenth.

Race	Kilometers	Miles
Short Distance	5	
Medium Distance	10	
Medium Distance	20	
Half Marathon		13.1
Ultramarathon	100	
Ironman Triathlon Swim		2.4
Ironman Triathlon Bike		112



2. The zoo earned the money that they needed to remodel the aviary! To figure out the amount of supplies needed, they will need to measure the space. The zookeeper realizes that she only has a meter stick, not a ruler or a yardstick. She measures the aviary but needs to know the dimensions in inches and feet in order to purchase the materials. She records the following measurements:
   - The length of the room is 5 meters.
   - The width of the room is 4 meters.
   - The height of the room is 2.5 meters.

   a. What is the length of the room in inches? In feet? Round to the nearest hundredth.

   b. What is the width of the room in inches? In feet? Round to the nearest hundredth.

   c. What is the height of the room in inches? In feet? Round to the nearest hundredth.

   d. There are 39.37 inches in a meter. Explain to a classmate how many feet are in a meter.

LESSON 2: Eggzactly! • 9

Answers

2a. The length of the room is 196.85 inches or 16.40 feet.

2b. The width of the room 157.48 inches or 13.12 feet.

2c. The height of the room is 98.43 inches or 8.20 feet.

2d. There are a little over 3 feet in a meter because a meter is 39.37 inches, and there are 36 inches in 3 feet. Therefore, it is 3 feet with 3.37 inches left over.

Answers

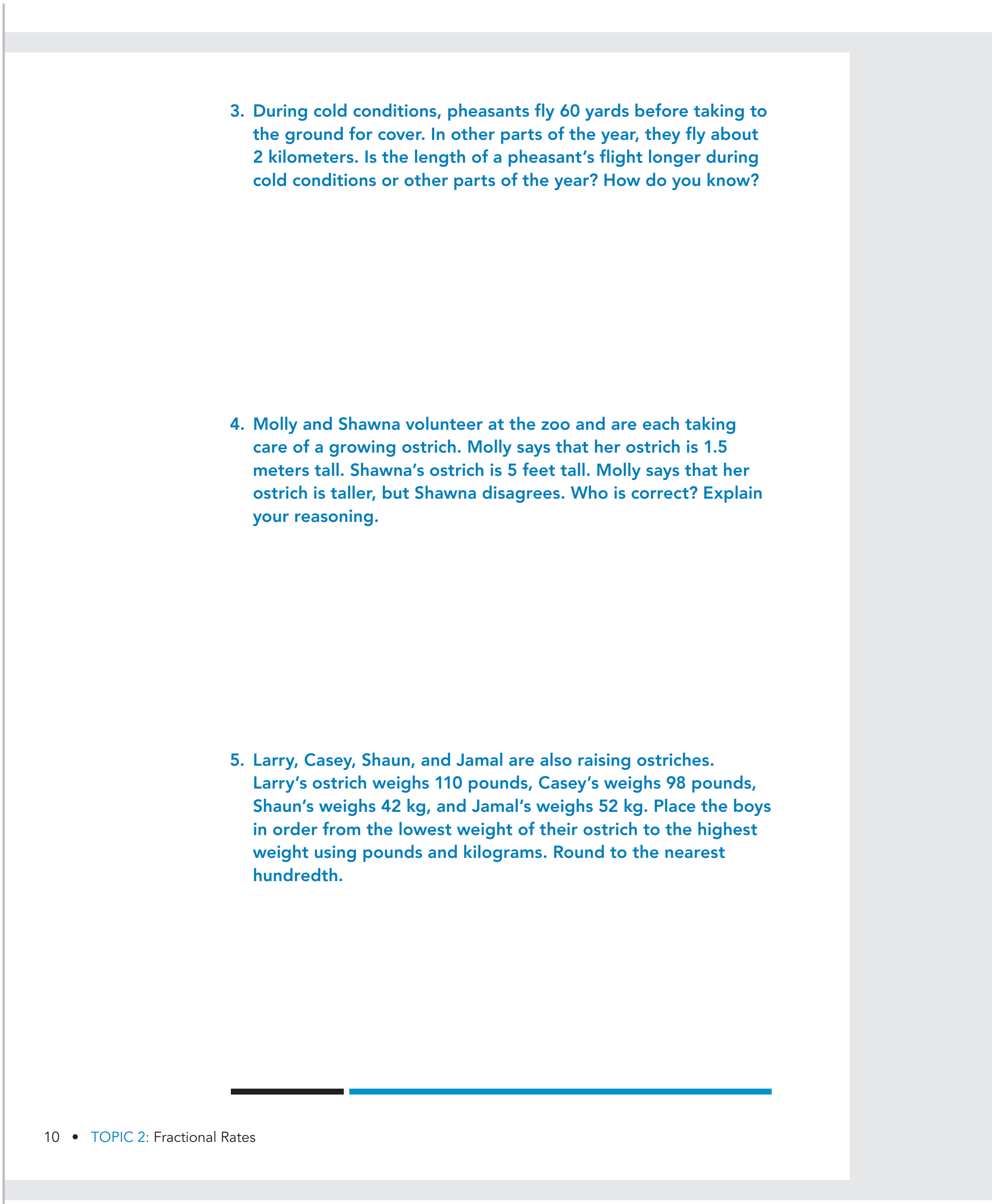
3. Pheasants fly farther in other parts of the year than they do during cold conditions.

   To determine my answer, I have to convert the units of measure to one system. I decided to convert from metric to customary units of measure. First I am converting within the metric system, kilometers to meters. Then I will convert between systems, meters to yards.

$$\frac{1000\text{ m}}{1\text{ km}} = \frac{x\text{ m}}{2\text{ km}}$$
$$x = 2000$$
$$\frac{1.09\text{ yd}}{1\text{ m}} = \frac{x\text{ yd}}{2000\text{ m}}$$
$$x = 2180$$

   Pheasants fly 60 yards during cold conditions before taking to the ground, which is less than the 2180 yards they fly in other parts of the year before taking to the ground.

4. Shawna is correct. 1 m = 3.28 feet, and 0.5 meter = 1.64 feet. Therefore, Molly's ostrich is 3.28 + 1.64 = 4.92 feet tall. Her ostrich is just a little shorter than Shawna's ostrich.
5. Shaun: 92.4 lb, Casey: 98 lb, Larry: 110 lb, and Jamal: 114.4 lb, Shaun: 42 kg, Casey: 44.1 kg, Larry 49.5 kg, and Jamal: 52 kg

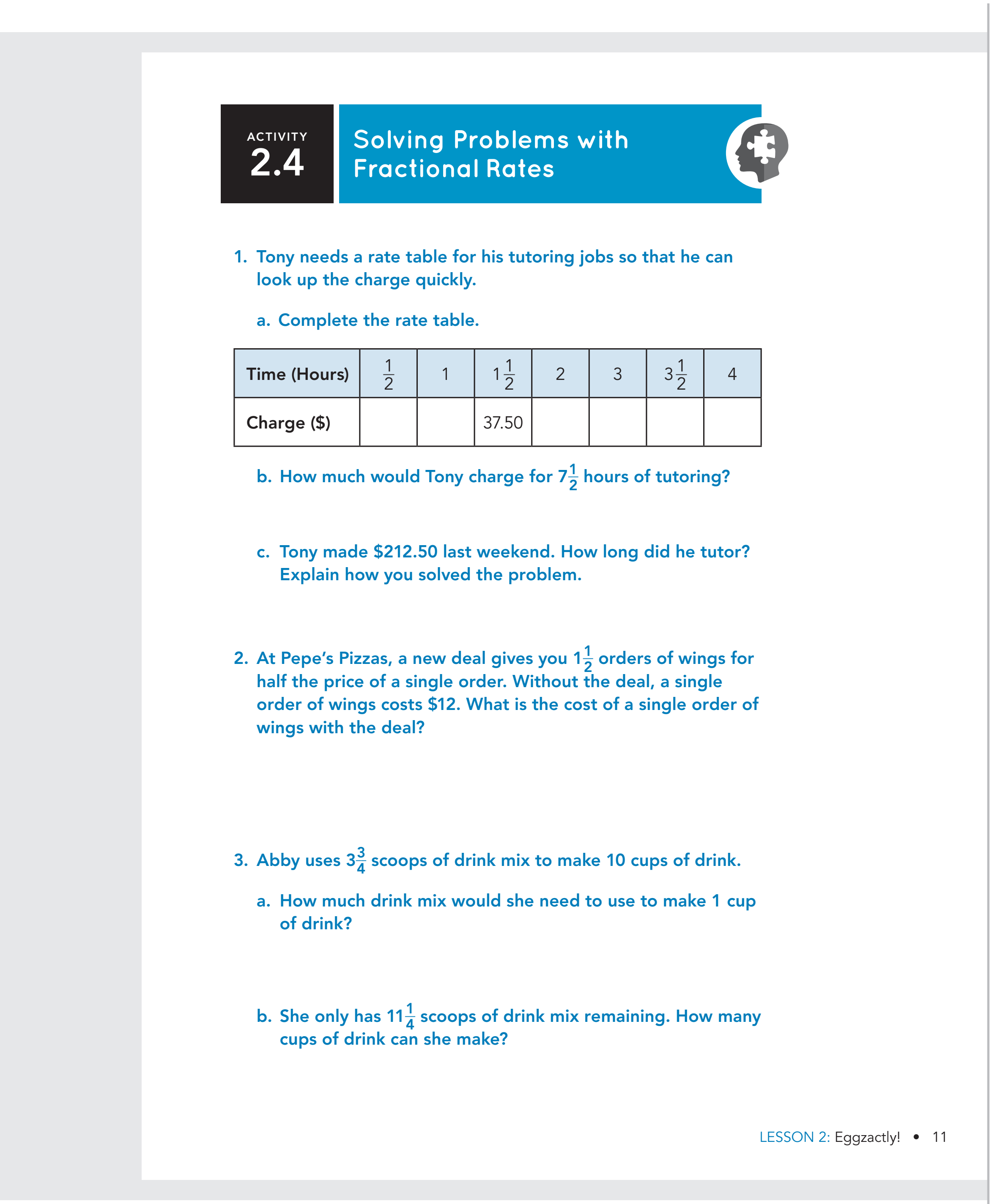


3. During cold conditions, pheasants fly 60 yards before taking to the ground for cover. In other parts of the year, they fly about 2 kilometers. Is the length of a pheasant's flight longer during cold conditions or other parts of the year? How do you know?

4. Molly and Shawna volunteer at the zoo and are each taking care of a growing ostrich. Molly says that her ostrich is 1.5 meters tall. Shawna's ostrich is 5 feet tall. Molly says that her ostrich is taller, but Shawna disagrees. Who is correct? Explain your reasoning.

5. Larry, Casey, Shaun, and Jamal are also raising ostriches. Larry's ostrich weighs 110 pounds, Casey's weighs 98 pounds, Shaun's weighs 42 kg, and Jamal's weighs 52 kg. Place the boys in order from the lowest weight of their ostrich to the highest weight using pounds and kilograms. Round to the nearest hundredth.

10 • TOPIC 2: Fractional Rates



ACTIVITY

2.4 Solving Problems with Fractional Rates

1. Tony needs a rate table for his tutoring jobs so that he can look up the charge quickly.

   a. Complete the rate table.

Time (Hours)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	3	$3\frac{1}{2}$	4
Charge ($)			37.50				

   b. How much would Tony charge for $7\frac{1}{2}$ hours of tutoring?

   c. Tony made $212.50 last weekend. How long did he tutor? Explain how you solved the problem.

2. At Pepe's Pizzas, a new deal gives you $1\frac{1}{2}$ orders of wings for half the price of a single order. Without the deal, a single order of wings costs $12. What is the cost of a single order of wings with the deal?

3. Abby uses $3\frac{3}{4}$ scoops of drink mix to make 10 cups of drink.

   a. How much drink mix would she need to use to make 1 cup of drink?

   b. She only has $11\frac{1}{4}$ scoops of drink mix remaining. How many cups of drink can she make?

Answers

1a. See table below.
1b. $187.50
1c. 8.5 hours

2. $4.00

3a. $\frac{3}{8}$ of a scoop

3b. 30 cups

1a.

Time (hours)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	3	$3\frac{1}{2}$	4
Charge ($)	12.50	25.00	37.50	50.00	75.00	87.50	100.00

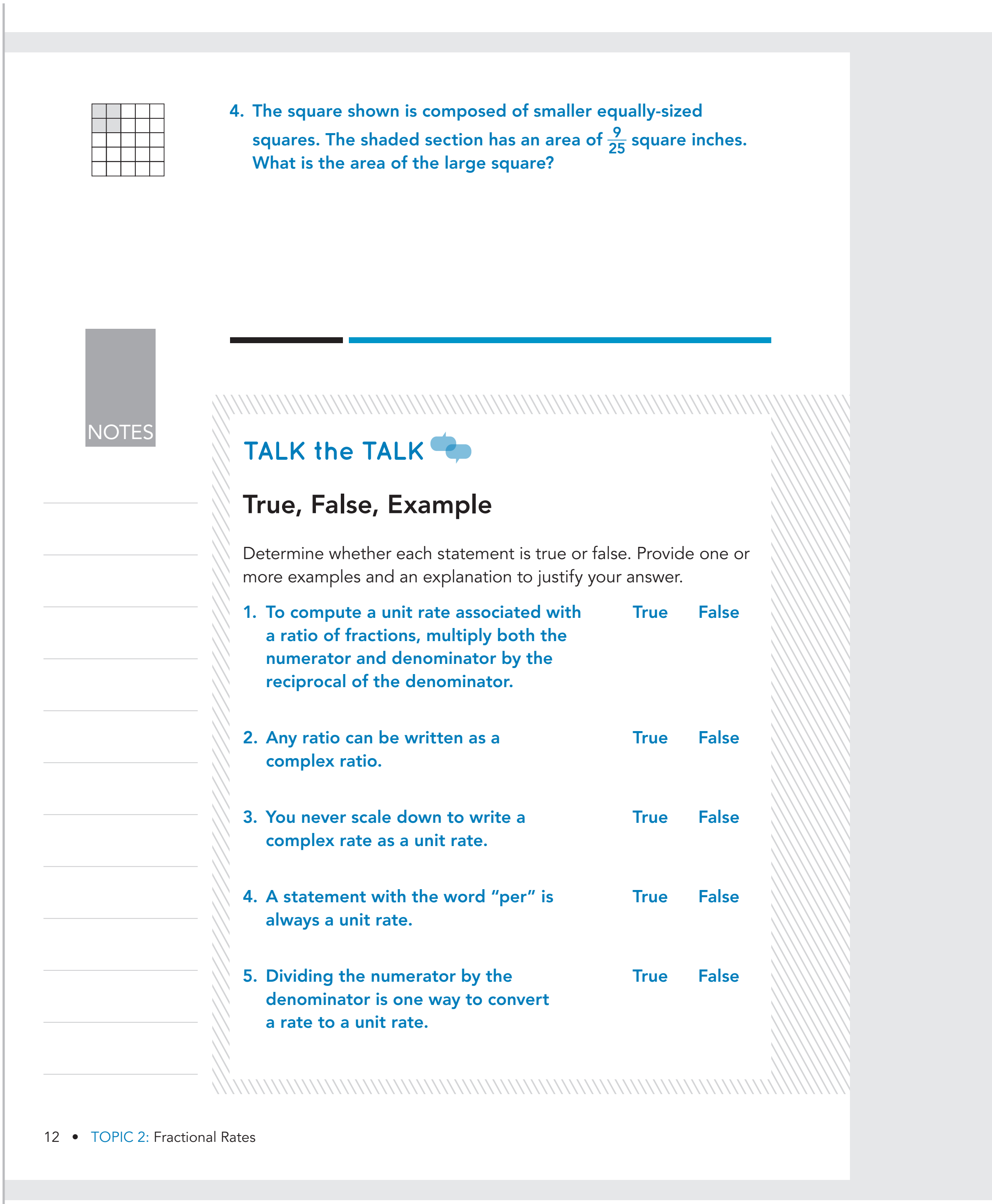
Answers

4. $2\frac{1}{4}$ square inches

Answers

Check students' examples.

1. True
2. True
3. False
4. False
5. True



4. The square shown is composed of smaller equally-sized squares. The shaded section has an area of $\frac{9}{25}$ square inches. What is the area of the large square?

NOTES

TALK the TALK

True, False, Example

Determine whether each statement is true or false. Provide one or more examples and an explanation to justify your answer.

1. To compute a unit rate associated with a ratio of fractions, multiply both the numerator and denominator by the reciprocal of the denominator.	True	False
2. Any ratio can be written as a complex ratio.	True	False
3. You never scale down to write a complex rate as a unit rate.	True	False
4. A statement with the word "per" is always a unit rate.	True	False
5. Dividing the numerator by the denominator is one way to convert a rate to a unit rate.	True	False