Tagging Sharks

3

Solving Proportions Using Means and Extremes

WARM UP

Solve each equation.

1. $w - 5 = 25$
2. $9x = 990$
3. $\frac{c}{12} = 48$
4. $1.15 + m = 10$

LEARNING GOALS

- Rewrite proportions to maintain equality.
- Represent proportional relationships by equations.
- Develop strategies to solve proportions.
- Use proportional relationships to solve multistep problems.

KEY TERMS

- proportion
- variable
- means
- extremes
- solve a proportion
- isolate the variable
- inverse operations

You have learned how to write proportions and calculate unknown values through scaling up and scaling down. Is there a more efficient strategy that works for any unknown in any proportion?

Getting Started

Mix-N-Match

Recall that a **proportion** is an equation that states that two ratios are equal.

A proportion can be written several ways. Each example shows three proportions using the same four quantities.

	Example 1	Example 2
Proportion 1	$\frac{2}{3} = \frac{4}{6}$	$\frac{5}{7} = \frac{15}{21}$
Proportion 2	$\frac{6}{3} = \frac{4}{2}$	$\frac{21}{7} = \frac{15}{5}$
Proportion 3	$\frac{2}{4} = \frac{3}{6}$	$\frac{5}{15} = \frac{7}{21}$

1. In each example, use arrows to show how the numbers were rearranged from the:

a. first proportion to the second proportion.

b. first proportion to the third proportion.

ACTIVITY 3.1

Maintaining Equality with Proportions

Because it is impossible to count each individual animal, marine biologists use a method called the capture-recapture method to estimate the population of certain sea creatures. In certain areas of the world, biologists randomly catch and tag a given number of sharks. After a period of time, such as a month, they recapture a second sample of sharks and count the total number of sharks as well as the number of recaptured tagged sharks. Then, the biologists use proportions to estimate the population of sharks living in a certain area.

Biologists can set up a proportion to estimate the total number of sharks in an area.

$$\frac{\text{Original number of tagged sharks}}{\text{Total number of sharks in an area}} = \frac{\text{Number of recaptured tagged sharks}}{\text{Number of sharks caught in the second sample}}$$

Although capturing the sharks once is necessary for tagging, it is not necessary to recapture the sharks each time. At times, the tags can be observed through binoculars from a boat or at shore.

Biologists originally caught and tagged 24 sharks off the coast of Cape Cod, Massachusetts, and then released them back into the bay. The next month, they caught 80 sharks with 8 of the sharks already tagged. To estimate the shark population off the Cape Cod coast, biologists set up the following proportion:

$$\frac{24 \text{ tagged sharks}}{p \text{ total sharks}} = \frac{8 \text{ recaptured tagged sharks}}{80 \text{ total sharks}}$$

Notice the variable p in the proportion. In this proportion, let p represent the total shark population off the coast of Cape Cod.

A **variable** is a letter or symbol used to represent a number.

1. Write three additional different proportions you could use to determine the total shark population off the coast of Cape Cod.

2. Estimate the total shark population using any of the proportions.

3. Did any of the proportions seem more efficient than the other proportions?

4. Wildlife biologists tag deer in wildlife refuges. They originally tagged 240 deer and released them back into the refuge. The next month, they observed 180 deer of which 30 deer were tagged. Approximately how many deer are in the refuge? Write a proportion and show your work to determine your answer.

A proportion of the form $\frac{a}{b} = \frac{c}{d}$ can be written in many different ways.

Another example is $\frac{d}{b} = \frac{c}{a}$ or $\frac{c}{a} = \frac{d}{b}$.

5. Show how the variables were rearranged from the proportion in the "if" statement to each proportion in the "then" statement to maintain equality.

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{d}{b} = \frac{c}{a}$.

If $\frac{d}{b} = \frac{c}{a}$ then $\frac{c}{a} = \frac{d}{b}$.

6. Write all the different ways you can rewrite the proportion $\frac{a}{b} = \frac{c}{d}$ and maintain equality.

ACTIVITY **3.2**

Solving Proportions with Means and Extremes

The Ready Steady Battery Company tests batteries as they come through the assembly line and then uses a proportion to predict how many batteries in its total production might be defective.

On Friday, the quality controller tested every tenth battery and found that of the 320 batteries tested, 8 were defective. If the company shipped a total of 3200 batteries, how many might be defective?

NOTES

Let's analyze a few methods.

John David

$$\frac{8 \text{ defective batteries}}{320 \text{ batteries}} = \frac{d \text{ defective batteries}}{3200 \text{ batteries}}$$

$$\overset{\times 10}{\frac{8}{320} = \frac{d}{3200}}_{\times 10}$$

$d = 80$

So, 80 batteries might be defective.

Parker

$$\times 10 \left(\begin{array}{c} 8 \text{ defective batteries} : 320 \text{ batteries} \\ \\ d \text{ defective batteries} : 3200 \text{ batteries} \end{array} \right) \times 10$$

$d = 80$

About 80 batteries will probably be defective.

1. How are Parker's and John David's methods similar?

NOTES

Nora

$$\frac{8 \text{ defective batteries}}{320 \text{ batteries}} \underset{\div 8}{\overset{\div 8}{=}} \frac{1 \text{ defective battery}}{40 \text{ batteries}} \underset{\times 80}{\overset{\times 80}{=}} \frac{80 \text{ defective batteries}}{3200 \text{ batteries}}$$

One out of every 40 batteries is defective. So, out of 3200 batteries, 80 batteries could be defective because $3200 \div 40 = 80$.

2. Describe the strategy Nora used.

Natalie

When I write Nora's ratios using colons like Parker, I notice something about proportions . . .

$8 : 320 = 1 : 40$

$$\begin{matrix} 8 & : & 1 \\ 320 & : & 40 \end{matrix}$$

$1 : 40 = 80 : 3200$

$$\frac{1}{40} = \frac{80}{3200}$$

. . . the two middle numbers have the same product as the two outside numbers. So, I can solve any proportion by setting these two products equal to each other.

3. Verify that Natalie is correct.

4. **Try the various proportion-solving methods on these proportions and determine the unknown value. Explain which method you used.**

 a. $\dfrac{\text{3 granola bars}}{\text{420 calories}} = \dfrac{g\text{ granola bars}}{\text{140 calories}}$

 b. **8 correct : 15 questions = 24 correct : *q* questions**

 c. $\dfrac{d\text{ dollars}}{\text{5 miles}} = \dfrac{\$9}{\text{7.5 miles}}$

The relationship that Natalie noticed is between the *means* and *extremes*. In a proportion that is written a : b = c : d, the product of the two values in the middle (the **means**) equals the product of the two values on the outside (the **extremes**).

extremes

$a : b = c : d$

means

$bc = ad$

or

$\dfrac{a}{b} = \dfrac{c}{d}$

means extremes

$bc = ad$

when $b \neq 0$, $d \neq 0$

To solve a proportion using this method, first identify the means and extremes. Then, set the product of the means equal to the product of the extremes and solve for the unknown quantity. To **solve a proportion** means to determine all the values of the variables that make the proportion true.

WORKED EXAMPLE

You can rewrite a proportion as the product of the means equal to the product of the extremes.

$$\underbrace{7 \text{ books} : \overbrace{14 \text{ days} = 3 \text{ books}}^{\text{means}} : 6 \text{ days}}_{\text{extremes}}$$

means

extremes

$$(14)(3) = (7)(6)$$

$$42 = 42$$

$$\frac{7 \text{ books}}{14 \text{ days}} = \frac{3 \text{ books}}{6 \text{ days}}$$

$$(14)(3) = (7)(6)$$

$$42 = 42$$

5. You can rewrite the product of the means and extremes from the Worked Example as four different equations. Analyze each equation.

$$3 = \frac{(7)(6)}{14} \qquad 14 = \frac{(7)(6)}{3} \qquad \frac{(3)(14)}{7} = 6 \qquad \frac{(3)(14)}{6} = 7$$

a. Why are these equations all true? Explain your reasoning.

b. Compare these equations to the equation in the Worked Example showing the product of the means equal to the product of the extremes. How was the balance of the equation maintained in each?

6. Why is it important to maintain balance in equations?

NOTES

WORKED EXAMPLE

In the proportion $\frac{a}{b} = \frac{c}{d}$ you can multiply both sides by b to isolate the variable a.

$$b \cdot \frac{a}{b} = b \cdot \frac{c}{d} \longrightarrow a = \frac{bc}{d}$$

When you **isolate the variable** in an equation, you perform an operation, or operations, to get the variable by itself on one side of the equals sign. Multiplication and division are *inverse operations*. **Inverse operations** are operations that "undo" each other.

WORKED EXAMPLE

Another strategy to isolate the variable a is to multiply the means and extremes and then isolate the variable by performing inverse operations.

$$\frac{a}{b} = \frac{c}{d}$$

Step 1: $ad = bc$

Step 2: $\frac{ad}{d} = \frac{bc}{d}$

Step 3: $a = \frac{bc}{d}$

7. Describe each step shown.

8. Rewrite the proportion $\frac{a}{b} = \frac{c}{d}$ to isolate each of the other variables: b, c, and d. Explain the strategies you used to isolate each variable.

ACTIVITY
3.3

Solving Problems with Proportions

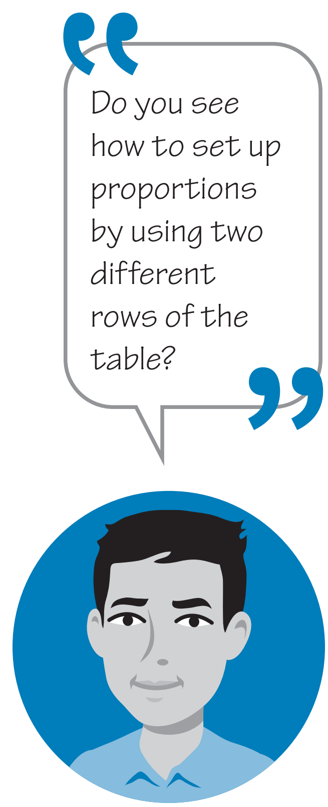
Write and solve proportions to solve each problem.

1. **An astronaut who weighs 85 kilograms on Earth weighs 14.2 kilograms on the Moon. How much would a person weigh on the Moon if they weigh 95 kilograms on Earth? Round your answer to the nearest tenth.**

2. **Water goes over Niagara Falls at a rate of 180 million cubic feet every $\frac{1}{2}$ hour. How much water goes over the falls in 1 minute?**

3. **The value of the U.S. dollar in comparison to the value of foreign currency changes daily. Complete the table shown. Round to the nearest hundredth.**

Euro	U.S. Dollar
1	1.07
	1.00
	6.00
6	
10	



4. **To make 4.5 cups of fruity granola, the recipe calls for 1.5 cups of raisins, 1 cup of granola, and 2 cups of blueberries. If you want to make 18 cups of fruity granola, how much of each of the ingredients do you need?**

NOTES

TALK the TALK

Choose Your Own Proportion Adventure

Write a problem situation for each proportion. Show the solution.

1. $\frac{8}{3} = \frac{2}{n}$

2. $\frac{\frac{1}{2}}{\frac{1}{4}} = \frac{h}{1}$