Fractional Rates
Topic 2 Overview

How is *Fractional Rates* organized?

In this topic, students review terminology about rates, unit rates, proportions, and strategies to determine equivalent ratios. They extend their work with rates to rates with fractional values. To begin the topic, students write, analyze, and use unit rates with whole numbers and fractions to solve problems. Next, students compute and use unit rates associated with ratios of fractions. Finally, students review strategies for solving problems involving equivalent ratios, i.e., proportions. They use unit rates and proportions to convert between measurement systems, i.e., customary system to metric system and vice versa. After reviewing previously developed strategies, students use means and extremes to solve real-world proportion problems.

What is the entry point for students?

In grade 6, students learned about ratios, rates, unit rates, and proportions. They represented ratios and unit rates with tables and graphs. Students used a variety of informal strategies to compare ratios, determine equivalent ratios, and solve simple proportions (e.g., double number lines, scaling up and down by a scale factor, and conversion factors). They also developed fluency with fraction division in grade 6. This topic reviews and builds on these skills by first computing and representing unit rates composed of whole numbers and fractions, then moving to combining unit rate and fraction division to compute unit rates for ratios of fractions.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic if they can:

- Represent unit rates using tables and graphs.
- Compute unit rates associated with ratios of fractions.
- Solve problems with ratios of fractions.
- Use unit rates to convert between measurement systems.
- Use proportions to convert between measurement systems.
- Represent proportional relationships by writing equations.
- Solve proportions using formal strategies.
- Solve real-world problems that involve ratios, rates, unit rates, and proportions.

Why is *Fractional Rates* important?

This topic broadens students' experiences with different types of numbers and strategies for solving ratio and proportion problems, preparing them to dig deeper into representations of proportional relationships in the next topic, and to solve multistep ratio and percent problems in

the next module. The lessons in this topic build on students' informal reasoning and strategies about ratio to develop more formal, abstract strategies for solving proportions with any fractional number.

How do the activities in *Fractional Rates* promote student expertise in the mathematical process standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class discourse, so elements of the mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

In this topic, students reason about ratios and apply previously developed strategies to a larger set of numbers. They notice that the structure and interpretation of ratios does not change because the types of numbers change. Students are also expected to notice that the informal strategies for solving proportions lead to a generalization for a strategy (means and extremes) that can be applied in any proportional situation.

Materials Needed

- None

Strategies to Simplify Complex Rates

- Scale Up or Down

$$\frac{\frac{1}{2}}{\frac{1}{4}} = \frac{\frac{1}{2} \times 4}{\frac{1}{4} \times 4} = \frac{2}{1} = 2$$

Strategies to Solve Proportions

- Scale Up or Down

$$\frac{8}{320} \overset{\times 10}{=} \frac{d}{3200}$$

$$d = 80$$

- Definition of Division

$$\frac{\frac{1}{2}}{\frac{1}{4}} = \frac{1}{2} \div \frac{1}{4}$$

$$= \frac{1}{2} \cdot 4 = 2$$

- Apply the Unit Rate

$$\frac{8 \text{ defective batteries}}{320 \text{ batteries}} \overset{\div 8}{=} \frac{1 \text{ defective battery}}{40 \text{ batteries}} \overset{\times 80}{=} \frac{80 \text{ defective batteries}}{3200 \text{ batteries}}$$

- Product of Means and Extremes

$a : b = c : d$ (extremes: a, d; means: b, c)

$bc = ad$

or

$\frac{a}{b} = \frac{c}{d}$ (means: b, c; extremes: a, d)

$bc = ad$

when $b \neq 0, d \neq 0$

Learning Together

ELPS: 1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Lesson	Lesson Name	TEKS	Days	Highlights
1	Making Punch: Unit Rate Representations	7.4B	1	In this lesson, students recall the concepts of ratio and unit rate and how to represent these mathematical objects using tables and graphs. Students use the unit rate as a measure of a qualitative characteristic: the strength of the lemon-lime taste of a punch recipe. They represent this measure in tables and graphs and with fractions in the numerator.
2	Eggzactly!: Solving Problems with Ratios of Fractions	7.4B 7.4E	2	In this lesson, students determine ratios and write rates, including complex ratios and rates. Students will write proportions and use rates to determine miles per hour. They use common conversions to convert between the customary and metric measurement systems using unit rates and proportions. They will scale up and scale down to determine unknown quantities.
3	Tagging Sharks: Solving Proportions Using Means and Extremes	7.4C 7.4D	2	Students solve several proportions embedded in real-world contexts. The term *variable* is introduced to represent an unknown quantity. Several proportions are solved using one of three methods: the scaling method, the unit rate method, and the means and extremes method. Students learn to isolate a variable in a proportion by using inverse operations.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 7.4B	TEKS: 7.4B, 7.4E	**LESSON 2** continued		TEKS; 7.4C, 7.4D
LESSON 1 **Making Punch** GETTING STARTED ACTIVITY 1 TALK THE TALK	**LESSON 2** **Eggzactly!** GETTING STARTED ACTIVITY 1 ACTIVITY 2	ACTIVITY 3 ACTIVITY 4 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	**LESSON 3** **Tagging Sharks** GETTING STARTED ACTIVITY 1

Day 6	Day 7	Day 8
LESSON 3 continued ACTIVITY 2 ACTIVITY 3 TALK THE TALK	 MATHia® Use LiveLab and Reports to monitor students' progress	**END OF TOPIC ASSESSMENT**

Assessments

There is one assessment aligned to this topic: End of Topic assessment.