Fractional Rates Summary

KEY TERMS

- complex ratio
- proportion
- variable
- means
- extremes
- solve a proportion
- isolate the variable
- inverse operations

LESSON 1 Making Punch

A *rate* is a ratio that compares quantities with different units. A *unit rate* is a rate in which the numerator or denominator (or both) is 1. Two or more ratios or rates can be compared.

For example, two friends make lemon-lime punch using the following recipes.

Jade's Recipe	Kim's Recipe
1 cup lemon-lime concentrate 3 cups club soda	2 cups lemon-lime concentrate 5 cups club soda

Determine which recipe has the stronger taste of lemon-lime.

For Jade's recipe, the ratio of lemon-lime to club soda is 1 cup : 3 cups, so the unit rate is $\frac{1}{3}$ cup lemon-lime per 1 cup club soda. For Kim's recipe, the ratio of lemon-lime to club soda is 2 cups : 5 cups, so the unit rate is $\frac{2}{5}$ cup lemon-lime per 1 cup club soda.

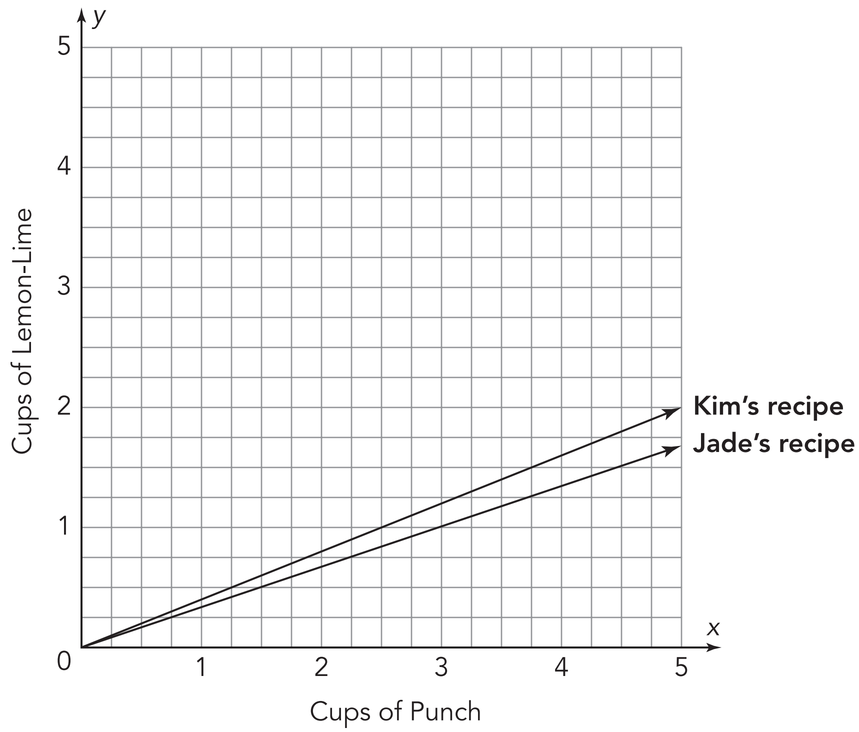
$\frac{1}{3} < \frac{2}{5}$, so Kim's recipe has the stronger taste of lemon-lime.

A unit rate can be represented by the ordered pair (1, *r*) on a coordinate plane.

For example, the graph shows the ratios of lemon-lime to club soda for Jade's and Kim's recipes.

The point $(1, \frac{1}{3})$ represents the unit rate for Jade's recipe, and the point $(1, \frac{2}{5})$ represents the unit rate for Kim's recipe.

The graph of Kim's line is steeper than the graph of Jade's line, which shows that Kim's recipe has a stronger taste of lemon-lime.



LESSON

2

Eggzactly!

A ratio in which one or both of the quantities being compared are written as fractions is called a **complex ratio**.

For example, traveling $\frac{1}{2}$ mile in $\frac{1}{4}$ hour represents a ratio of fractions, or a *complex ratio*. You can write this ratio, which is also a rate, in fractional form: $\frac{\frac{1}{2}}{\frac{1}{4}}$.

To convert a complex rate to a unit rate, you can multiply the numerator and denominator by the reciprocal of the denominator, or you can use the definition of division.

In this example, the unit rate for traveling $\frac{1}{2}$ mile in $\frac{1}{4}$ hour is 2 miles per hour.

$$\frac{\frac{1}{2}}{\frac{1}{4}} \times \frac{\frac{4}{1}}{\frac{4}{1}} = \frac{\frac{4}{2}}{1}$$

$$\frac{2}{1} = 2$$

$$\frac{\frac{1}{2}}{\frac{1}{4}} = \frac{1}{2} \div \frac{1}{4}$$

$$= \frac{1}{2} \cdot 4 = 2$$

To compare unit rates that are given in different units, you can use proportions and conversion rates.

For example, an ostrich can run 22 miles in $\frac{1}{2}$ hour. The Greater Roadrunner can run 300 yards in $\frac{1}{2}$ minute. Which bird runs faster?

$$\frac{\text{distance}}{\text{time}} \rightarrow \frac{22\text{ mi}}{\frac{1}{2}\text{ h}} \underset{\times 2}{\overset{\times 2}{=}} \frac{44\text{ mi}}{1\text{ h}}$$

$$= \frac{44\text{ mi}}{1\text{ h}}$$

The rate of the ostrich is already measured in miles and hours. You can set up a proportion and scale the original rate up to 1 hour.

The ostrich's speed is 44 miles per hour.

The rate of the Greater Roadrunner is written in yards per minute. You can use conversion rates to rewrite the rate in miles per hour.

$$\frac{300\text{ yd}}{\frac{1}{2}\text{ min}} \cdot \frac{60\text{ min}}{1\text{ hr}} \cdot \frac{1\text{ mi}}{1760\text{ yd}}$$

$$\frac{300\ \cancel{\text{yd}}}{\frac{1}{2}\ \cancel{\text{min}}} \cdot \frac{60\ \cancel{\text{min}}}{1\text{ hr}} \cdot \frac{1\text{ mi}}{1760\ \cancel{\text{yd}}}$$

$$\frac{300}{\frac{1}{2}} \cdot \frac{60}{1} \cdot \frac{1\text{ mi}}{1760\text{ hr}}$$

$$600 \cdot \frac{60}{1} \cdot \frac{1\text{ mi}}{1760\text{ hr}} \approx \frac{20.5\text{ mi}}{1\text{ hr}}$$

The Greater Roadrunner's speed is about 20.5 miles per hour.

The ostrich runs faster.

You can use common conversions to convert between measurement systems.

To convert between systems, you can scale up or scale down using ratios.

$$\frac{1\text{ lb}}{0.45\text{ kg}} \underset{\times 2.5}{\overset{\times 2.5}{=}} \frac{2.5\text{ lb}}{1.125\text{ kg}}$$

Write a ratio using the common conversion of 1 lb = 0.45 kg.

Scale up to calculate the number of kilograms in 2.5 pounds.

LESSON 3

Tagging Sharks

A **proportion** is an equation that states that two ratios are equal. A **variable** is a letter or symbol used to represent a number. A proportion of the form $\frac{a}{b} = \frac{c}{d}$ can be written in many different ways. Another example is $\frac{d}{b} = \frac{c}{a}$ or $\frac{c}{a} = \frac{d}{b}$.

In a proportion that is written $a : b = c : d$, the product of the two values in the middle (the **means**) equals the product of the two values on the outside (the **extremes**). Multiplying the means and extremes is like "cross-multiplying."

extremes

$$a : b = c : d$$

means

$$bc = ad$$

or

means $\frac{a}{b} = \frac{c}{d}$ extremes

$$bc = ad$$

when $b \neq 0$, $d \neq 0$

To solve a proportion using this method, first identify the means and extremes. Then, set the product of the means equal to the product of the extremes and solve for the unknown quantity. To **solve a proportion** means to determine all the values of the variables that make the proportion true.

For example, you can rewrite the proportion $\frac{\text{7 books}}{\text{14 days}} = \frac{\text{3 books}}{\text{6 days}}$ as the product of the means equal to the product of the extremes.

7 books : 14 days = 3 books : 6 days

means

extremes

$$(14)(3) = (7)(6)$$

$$42 = 42$$

$$\frac{\text{7 books}}{\text{14 days}} = \frac{\text{3 books}}{\text{6 days}}$$

$$(14)(3) = (7)(6)$$

$$42 = 42$$

When you **isolate the variable** in an equation, you perform an operation, or operations, to get the variable by itself on one side of the equals sign. Multiplication and division are *inverse operations*. **Inverse operations** are operations that "undo" each other.

In the proportion $\frac{a}{b} = \frac{c}{d}$, you can multiply both sides by b to isolate the variable a.

$$b \cdot \frac{a}{b} = \frac{c}{d} \cdot b \longrightarrow a = \frac{cb}{d}$$

Another strategy is to multiply the means and extremes, and then isolate the variable by performing inverse operations.

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

$$\frac{ad}{d} = \frac{bc}{d}$$

$$a = \frac{bc}{d}$$