How Does Your Garden Grow?

1

Proportional Relationships

WARM UP

1. A bus travels 18 miles in 15 minutes. At the same rate, what distance will the bus travel in 50 minutes?
2. A copy machine averages 210 copies in 5 minutes. At the same rate, how many copies can the machine make in 12 minutes?

LEARNING GOALS

- Use tables and graphs to explore proportional relationships.
- Decide whether two quantities are in a proportional relationship by testing for equivalent ratios in a table.
- Decide whether two quantities are in a proportional relationship by graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

KEY TERMS

- origin
- proportional relationship
- direct variation

You have learned about the relationship between ratios, a comparison of two quantities, and proportions. How can you determine if a proportional relationship exists between two quantities?

Getting Started

Keep on Mixing!

Amount of Bluish Green Paint	Amount of Yellow Paint	Amount of Blue Paint
3 pt	1 pt	2 pt
5 pt	2 pt	3 pt
6 pt	2 pt	4 pt
12 pt	4 pt	8 pt
15 pt	6 pt	9 pt
20 pt	8 pt	12 pt

The students in Mr. Raith's art class created various quantities of bluish green paint using pints of yellow and blue paint.

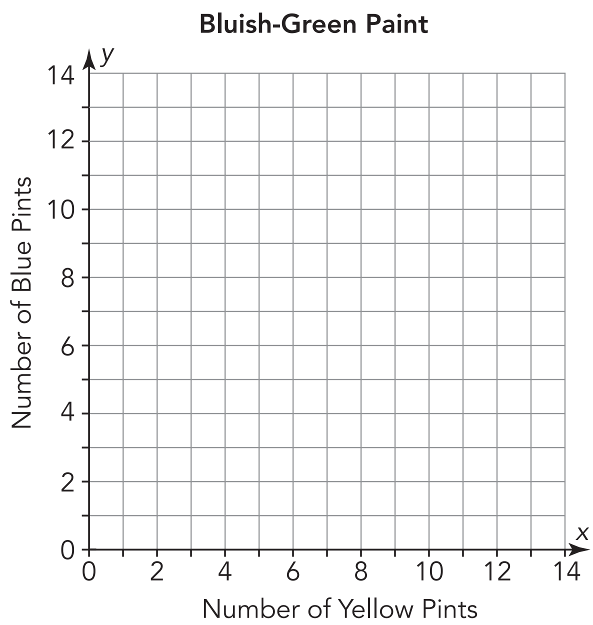
The table shows the different mixtures of paint, in pints, that the students made.

1. How many different shades of paint did the students make? How do you know?

2. Some of the shades of the paint are more yellow than others. Which mixture(s) are the most yellow? Explain your reasoning.

The **origin** is a point on a graph with the ordered pair (0, 0).

3. Plot an ordered pair for each bluish-green paint mixture. Draw a line connecting each point to the origin. What do you notice?



ACTIVITY 1.1

Representations of Varying Quantities

The student government association (SGA) at Radloff Middle School is creating an urban garden at their school for use by their community. They divided up into groups to design different parts of the garden and were asked to (1) describe their project, (2) create an equation to model part of their design or to answer a question about their design, and (3) sketch a graph of their equation.

1. Isaac, the president of the SGA, mixed up the representations of the projects after they were submitted to him. Help Isaac match the scenarios, equations, and graphs.

- **Cut out the scenarios, equations, and graphs at the end of the lesson.**
- **Sort the scenarios, equations, and graphs into corresponding groups.**
- **Tape the representations into the table provided.**

When you connect an equation to a graph, you are establishing a dependency between the quantities. Remember, the independent quantity is always represented on the *x*-axis.

The Urban Garden Project

Scenario			
Equation			
Graph			
Table			

NOTES

ACTIVITY 1.2

Defining Proportional Relationships

When looking over the submissions from the urban garden working groups, Isaac notices that there are two different types of graphical relationships represented: linear and non-linear.

1. Classify each group's graph as representing a linear or a non-linear relationship between quantities.

Isaac notices that the linear graphs are slightly different, but he doesn't know why. He decides to analyze a table of values for each linear graph.

2. Create a table of at least 4 values for each linear relationship in the urban garden project.

Isaac knows that simple equations can represent additive or multiplicative relationships between quantities.

3. Analyze the equations.

a. Based on the equations, which graph represents an additive relationship between the variables and which represents a multiplicative relationship?

b. Which variable represents the independent variable (input) and which represents the dependent variable (output)?

One special type of relationship that compares quantities using multiplicative reasoning is a ratio relationship. When two equivalent ratios are set equal to each other, they form a proportion. The quantities in the proportion are in a **proportional relationship**.

You can decide if two quantities are in a proportional relationship by testing that all ratios, $\frac{y}{x}$ or $\frac{x}{y}$, in a table of values are equivalent.

For a relationship to illustrate a proportional relationship, all the ratios $\frac{y}{x}$ or $\frac{x}{y}$, must represent the same constant.

4. Use your tables of values in Question 2 to determine which, if any, of the linear relationships illustrate a proportional relationship. Show the values of the ratios in each relationship.

Isaac says the equation $y = \pi r^2$ represents a proportional relationship between y and r because it includes multiplication between a numerical coefficient and a variable expression.

5. Use a table of values and corresponding ratios in the form of $\frac{y}{x}$ to explain why Isaac is incorrect.

6. Explain to Isaac how the graphs in the urban garden design project are different. Include the terms *linear relationship* and *non-linear relationship, proportional relationship*, and *equivalent ratios.*

ACTIVITY
1.3

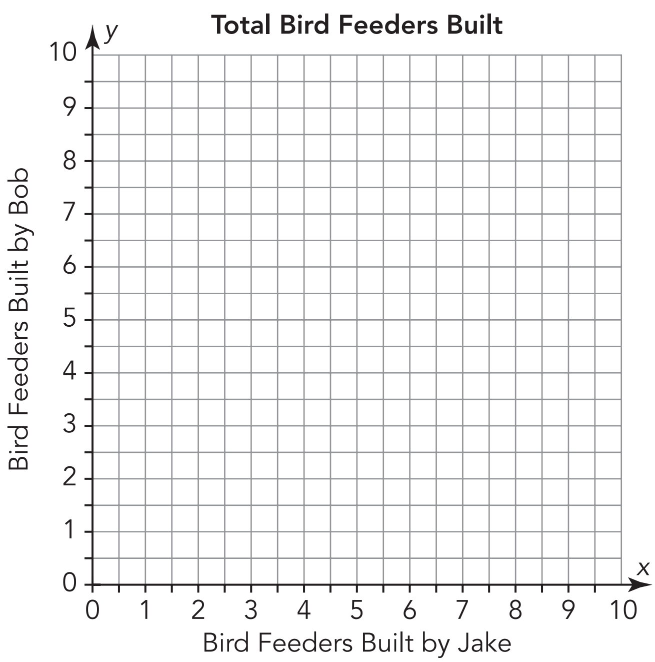
Proportional or Not?

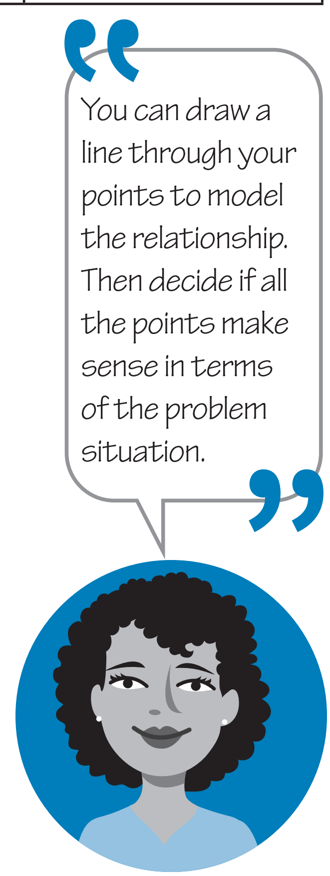
In this activity you will analyze three different problem situations and then determine which represents a proportional relationship.

Bob and his little brother Jake want to build bird feeders to sell at a local farmers market. They have enough money to buy materials to build 10 bird feeders.

1. Complete a table of values by listing possible ways in which they can divide up the work. Assume that each brother only makes whole bird feeders. Then complete the graph.

Bird Feeders Built by Bob	Bird Feeders Built by Jake





2. Describe how the number of bird feeders built by Bob affects the number of bird feeders Jake builds.

3. What is the ratio of bird feeders that Bob builds to the number of bird feeders that Jake builds? Explain your reasoning.

4. Dontrell claims that the number of bird feeders Bob builds is proportional to the number of bird feeders Jake builds. Do you agree with Dontrell's claim? Explain your reasoning.

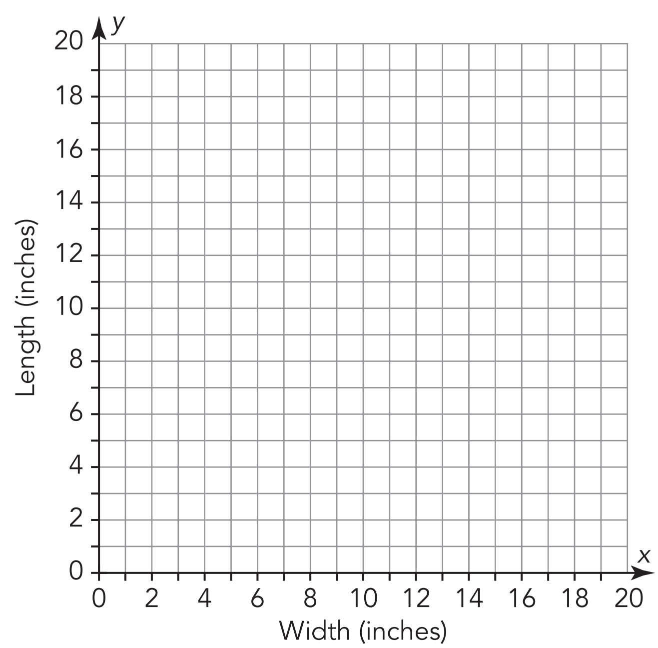
Vanessa was given a math problem to determine how many different rectangles can be constructed with an area of 12 square inches.

5. Vanessa thinks that there are only two: one with a width of 2 inches and a length of 6 inches, and another with a width of 3 inches and a length of 4 inches.

Is she correct? Explain your reasoning.

6. Complete a table of values for the width and length of a rectangle with an area of 12 square inches. Then complete the graph.

Width of Rectangle (in.)	Length of Rectangle (in.)



7. Describe how the width of the rectangle affects the length of the rectangle.

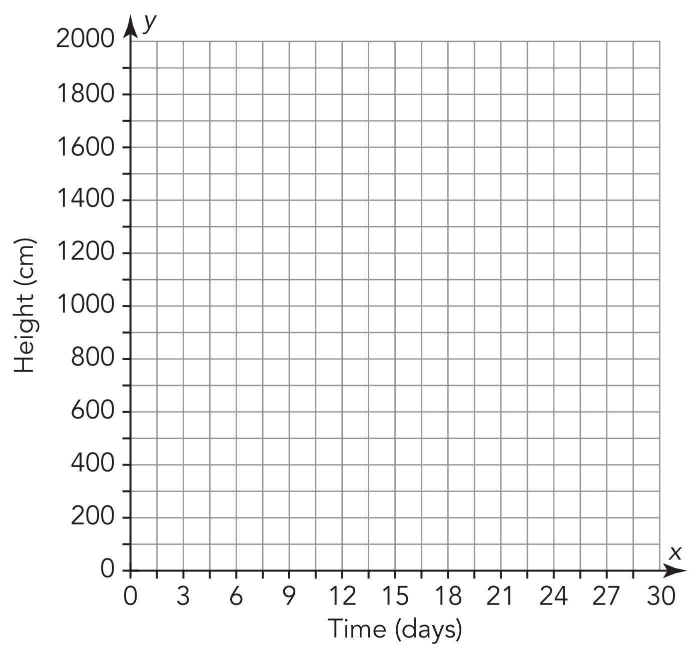
8. Do the width and length of a rectangle with an area of 12 square inches form a proportional relationship? Explain your reasoning.

One species of bamboo can grow at an average rate of 60 centimeters per day.

Why do you think this problem says "average rate" instead of just rate?

9. Complete a table of values using the given growth rate of the bamboo plant. Then complete the graph.

Time (days)	Height of Bamboo (cm)



10. **Describe how the time affects the height of the bamboo plant.**

11. **Is the number of days of growth proportional to the height of the bamboo plant? Explain your reasoning.**

ACTIVITY 1.4 Direct Variation

You saw that the height of a bamboo plant varies based on the number of days the bamboo grows. For example, for each increase of one day, the bamboo grows 60 centimeters.

A situation represents a **direct variation** if the ratio between the *y*-value and its corresponding *x*-value is constant for every point. If two quantities vary directly, the points on a graph form a straight line, and the line passes through the origin.

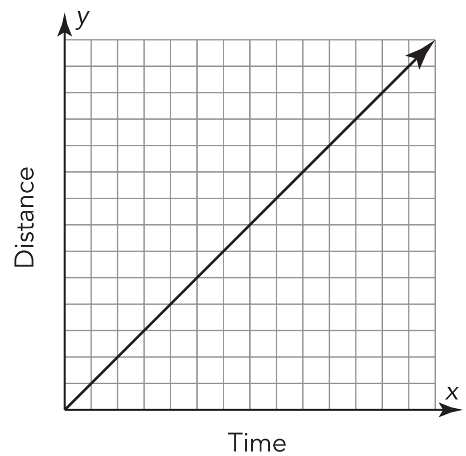
You can describe the quantities of a direct variation relationship as directly proportional.

Examine the Worked Example.

WORKED EXAMPLE

A car driving at a constant rate of 60 miles per hour is an example of direct variation. The distance traveled varies directly with time.

A sketch of a graph that could represent this situation is shown.



When you sketch a graph, be sure to include the labels for each axis. However, you don't *always* have to show values.

1. **Explain how the situation in the Worked Example is an example of a direct variation.**

2. **Explain how you can use each graph in the previous activity, *Proportional or Not?*, to determine which scenarios represent direct variations.**

3. **List another example of quantities that vary directly. Then, sketch a graph that could represent the relationship between the quantities.**

NOTES

TALK the TALK

Determining Proportionality from Tables and Graphs

Go back and examine the graphs in this lesson. Do you see a pattern?

1. **How are all the graphs that display proportional relationships the same?**

2. **Sketch a graph that displays a proportional relationship.**

NOTES

3. **Which tables display linear relationships? Which display proportional relationships? Explain your reasoning.**

a.

x	*y*
1	10
2	11
4	13
5	14

b.

x	*y*
0	0
1	6
3	18
4	24

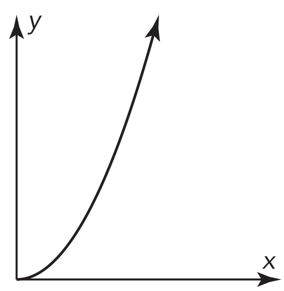
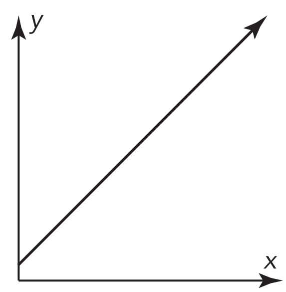
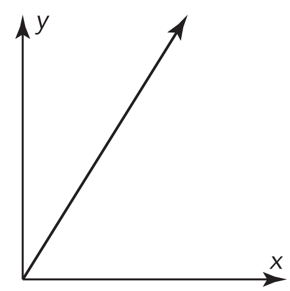
c.

x	*y*
0	4
1	8
2	12
3	16

d.

x	*y*
1	30
2	15
4	10
5	5

Cutouts for The Urban Garden Project

Group A is designing one section of the garden to include fresh herbs grown in circular beds. The group needs to determine the area of each herb bed given the radius of the bed.	Group B is developing a plan to landscape the perimeter of the urban garden. They could only find a meter stick broken at 6 cm, so they reported the dimensions of the garden based on the measurements read off the meter stick. This group needs to determine the actual side lengths of the garden.	Group C is designing the vegetable patches and have decided that each rectangular vegetable patch will be 5 inches wide for every 8 inches long. This group needs to determine the possible dimensions for the lengths of the vegetable patches.
		
$y = 1.6x$	$y = \pi x^2$	$y = x + 6$

Why is this page blank?

So you can cut out the Urban Garden Project materials on the other side.