2

Complying with Title IX

Constant of Proportionality

WARM UP

Washington Middle School collects canned food for a local community food bank. Last year, there were 180 students enrolled at the school, and they collected 102 cans of food.

1. Write the ratio representing the number of cans of food contributed to the total number of students in the school.
2. What is the unit rate of cans contributed per student?
3. This year, 210 students are enrolled in the school. Assume the number of cans of food contributed per student for both years is the same. How many cans of food should the school expect to be contributed this year?

LEARNING GOALS

- Determine if there is a constant ratio between two variables.
- Identify the constant of proportionality in proportional relationships.
- Identify the constant of proportionality in equations.
- Represent proportional relationships by equations.

KEY TERM

- constant of proportionality

You know how to recognize proportional relationships from tables and graphs. How do you represent proportional relationships with equations?

Is It Proportional?

Analyze each table to determine if the relationship is proportional. If the table represents a proportional relationship, state the constant ratio that exists between corresponding values of the two quantities.

1. A 30-minute television show has 8 minutes of commercials and 22 minutes of the show. A 120-minute television movie has 32 minutes of commercials and 88 minutes of the movie.

Total Program Length (minutes)	Actual Show Length (minutes)	Commercial Length (minutes)
30	22	8
120	88	32

2. There are 250 boys in 6th grade, and 75 are in the band. There are 200 girls in 6th grade, and 60 are in the band.

6th Grade Class	Total	Band
Boys	250	75
Girls	200	60

3. **Commuters in McKnight and Mitenridge either drive to work or take public transportation.**

Commuters	Drive to Work	Public Transportation to Work
McKnight	175	120
Mitenridge	525	300

4. **Of the 250 middle-school boys who have a subscription to *Boys Noise*, 125 access the magazine through the website. Of the 280 middle-school girls who have a subscription to *Girls Rockstar*, 160 access the magazine through the website.**

ACTIVITY
2.1

Defining the Constant of Proportionality

In a proportional relationship, the ratio of all *y*-values, or outputs, to their corresponding *x*-values, or inputs, is constant. This specific ratio, $\frac{y}{x}$, is called the **constant of proportionality**. Generally, the variable *k* is used to represent the constant of proportionality.

Let's revisit the television show scenario. This situation represents a proportional relationship.

Total Program Length (minutes)	Actual Show Length (minutes)	Commercial Length (minutes)
30	22	8
120	88	32

> The input value is known. The output value is what you are trying to determine.

Suppose you want to determine the actual lengths of your favorite television shows, without commercials, if you know the total program length.

1. Identify the input and output quantities in this scenario.

To determine the length of a program, without commercials, you will need to multiply the total program length by a constant of proportionality.

Analyze the different ideas for determining the constant of proportionality.

Jeremiah

We want to know the actual show length, and we know the total program length, so

$k = \frac{\text{22 minutes of show}}{\text{30 minutes of total length}}$,

or $k = \frac{11}{15}$.

Keisha

To determine if a proportional relationship exists the order of the ratio doesn't matter, so the constant of proportionality can be

$k = \frac{15}{11}$

or $k = \frac{11}{15}$.

Susan

I think the constant of proportionality is

$k = \frac{\text{22 minutes of show}}{\text{8 minutes of commercials}}$,

or $k = \frac{11}{4}$.

JAMie

JEREMIAH'S CORRECT ABOUT WHICH NUMBERS TO USE BUT HE HAS THEM MIXED UP. THE CONSTANT OF PROPORTIONALITY IS

$k = \frac{\text{30 MINUTES OF TOTAL LENGTH}}{\text{22 MINUTES OF SHOW}}$, OR $k = \frac{15}{11}$.

2. Explain why Susan's solution is incorrect.

3. Explain why Jeremiah is correct but Jamie and Keisha are incorrect.

ACTIVITY
2.2 The Meaning of the Constant of Proportionality

"The term *proportional* is used often in the Title IX document. What does this mean for sports at schools?"

In 1972, the federal government passed Title IX, a law requiring educational institutions that receive federal funds to provide equitable athletic opportunities for all students, regardless of their sex. One of the requirements for compliance with Title IX is for institutions to provide opportunities for each subgroup that are *proportional* to their rates of enrollment, or to show that they are working to increase such opportunities. Similarly, athletes of different sexes are to receive athletic scholarship dollars *proportional* to their subgroup's participation.

Let's think about the implications of Title IX at Vista Middle School.

There are 5 girls for every 6 boys enrolled in Vista Middle School.

1. Set up proportions for each question. Then, solve each proportion to determine the unknown value. Use the information from the ratio given.

a. If there are 300 boys enrolled in the school, how many girls are enrolled in the school?

b. If there are 325 girls enrolled in the school, how many boys are enrolled in the school?

2. Define variables for the quantities that are changing in this situation.

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3. **Set up a proportion using your variables for the quantities to the ratio given for the enrollment of girls to boys enrolled in Vista Middle School.**

4. **Use your proportion.**

   a. **Write an equation to determine the number of girls enrolled at Vista Middle School if you know the number of boys enrolled.**

   b. **What is the constant of proportionality in this situation? Where is the constant of proportionality in the equation?**

   c. **What does the constant of proportionality mean in this problem situation?**

5. **Use your proportion.**

   a. **Write an equation to determine the number of boys enrolled at Vista Middle School if you know the number of girls enrolled.**

   b. **What is the constant of proportionality in this situation? Where do you see the constant of proportionality in the equation?**

   c. **What does the constant of proportionality mean in this problem situation?**

6. **What do you notice about the constant of proportionality in each situation?**

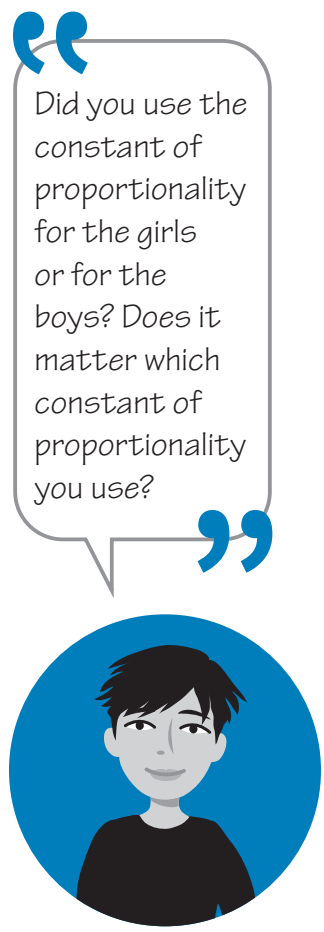
7. **Do you think each constant of proportionality makes sense in terms of the problem situation?**

Sometimes, the constant of proportionality is not a whole number. The constant of proportionality can also be a decimal or a fraction. When the constant of proportionality involves whole items, like people, it may seem strange to think about the constant of proportionality in terms of a fraction. Instead, you can think of the constant of proportionality as a way to predict outcomes of a situation.

8. Use your equations and the information about Title IX to answer each question.

a. If there are opportunities for 79 boys to participate in athletics, how many opportunities must be available for girls?

b. If there are opportunities for 119 girls to participate in athletics, how many opportunities must be available for boys?



ACTIVITY
2.3

Representing Proportional Relationships with Equations

Title IX addresses the number of athletics opportunities provided to different sexes, but the actual participation at schools may differ. Also, the same person may participate in multiple sports.

At Vista Middle School, 5 out of every 7 girls play sports. The guidance counselor, Ms. Shanahan, and the athletics director, Coach Culpepper, are completing reports about the students at Vista Middle School.

Consider the information each person knows and use the constant of proportionality to write equations for each situation.

Guidance Counselor

Ms. Shanahan knows the number of girls in the school on a given day, and she needs to be able to calculate the expected number of girls who play sports.

Athletics Director

Coach Culpepper knows the number of girls participating in sports during a given season, and she needs to be able to calculate the expected number of total girls in the school.

1. **Determine the constant of proportionality for each situation.**

   a. **Guidance Counselor**

   b. **Athletics Director**

2. **Define variables for the quantities that are changing in these situations.**

3. **Use the constants of proportionality to write equations to determine the information needed by each person.**

   a. **Guidance Counselor**

   b. **Athletics Director**

In terms of proportionality, Ms. Shanahan could state that the number of girls who play sports is proportional to the number of total girls in the school at a constant rate equal to the constant of proportionality.

4. **Write Coach Culpepper's situation using the language of proportionality and include the value for the constant of proportionality.**

5. **Consider the given equations, where *y* represents the dependent, or output, quantity and *x* represents the independent, or input, quantity.**

$$\frac{y}{x} = k$$

$$\frac{y}{x} = \frac{k}{1}$$

$$y = kx$$

   a. **Describe how the first equation represents the constant of proportionality.**

   b. **Explain how the second equation represents proportional relationships.**

   c. **Describe how the first equation was rewritten to create the third equation.**

   d. **Explain the meaning of the constant of proportionality, *k*, in the third equation.**

6. **Identify the constant of proportionality in each equation and describe its meaning.**

   a. **$d = 2r$, where d represents the diameter of a circle and r represents the radius of a circle.**

   b. **$P = 3s$, where P represents the perimeter and s represents the sides of an equilateral triangle.**

ACTIVITY
2.4

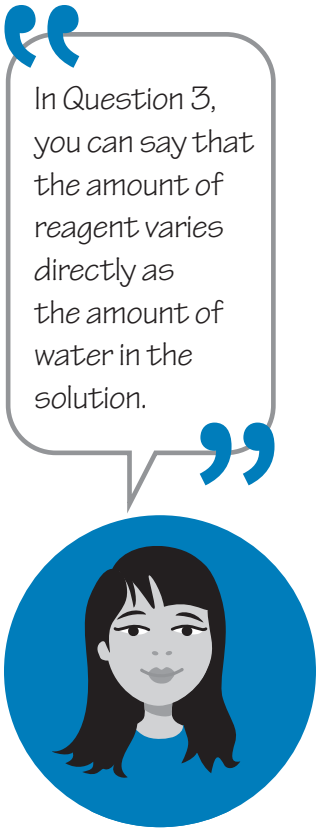
Using the Constant of Proportionality to Solve Problems

A chemist must use a solution that is 30% of reagent and 70% of water for an experiment. A *solution* is a mixture of two or more liquids. A *reagent* is a substance used in a chemical reaction to produce other substances.

1. **Define variables for the quantities that are changing in this problem situation.**

2. **Write an equation for the amount of water needed based on the amount of reagent. What is the constant of proportionality?**

3. Use your equation from Question 2 to write an equation for the amount of reagent needed based on the amount of water. Explain your reasoning.



4. Use your equations to answer each question.

   a. If the chemist uses 6 liters of reagent, how many liters of water will she need to make her 30% solution?

   b. If the chemist uses 77 milliliters of water, how many milliliters of reagent will she need to make her 30% solution?

5. Write an equation to show that y is directly proportional to x using the constant of proportionality given. Then, solve for the unknown value.

   a. $k = 0.7$ and $y = 4$

   b. $k = \frac{3}{11}$ and $x = 9$

   c. $k = 5$ and $x = 1\frac{1}{2}$

   d. $k = \frac{1}{6}$ and $y = 3\frac{1}{3}$

NOTES

TALK the TALK

Turning the Tables

Consider the equation $y = kx$. Use the value of the constant of proportionality assigned to you to answer the questions. You will present your work to your class.

1. **Write a scenario for a proportional relationship that would be represented by the equation. Clearly define your variables and identify the direction of the proportional relationship.**

2. **Interpret the constant of proportionality in the context of your scenario.**

3. **Write and solve at least 2 questions that could be solved using your equation.**