Fish-Inches

3

Identifying the Constant of Proportionality in Graphs

MATERIALS

None

Lesson Overview

In this lesson, students analyze proportional and non-proportional real-world and mathematical situations that are represented on graphs. When appropriate, they then identify the constant of proportionality. Students write equations to represent the situations from the graphs. Throughout the lesson, students interpret the meaning of points on graphs in terms of proportional relationships, including the meaning of $(1, y)$ and $(0, 0)$.

Grade 7
Proportionality

(4) The student applies mathematical process standards to represent and solve problems involving proportional relationships. The student is expected to:

(A) represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including $d = rt$.

(C) determine the constant of proportionality ($k = \frac{y}{x}$) within mathematical and real-world problems.

(D) solve problems involving ratios, rates, and percents, including multi-step problems involving percent increase and percent decrease and financial literacy problems.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Essential Ideas

- The graph of two variables that are proportional, or that vary directly, is a line that passes through the origin, (0, 0).
- The ratio of the y-coordinate to the *x*-coordinate (their quotient) for any point is equivalent to the constant of proportionality, *k*, when analyzing a graph of two variables that are proportional.
- When analyzing the graph of two variables that are not proportional, the ratios of the *y*-coordinate to the *x*-coordinate for any points are not equivalent.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: The Fish-Inches System of Measurement

Students use a scenario and the rate "3 inches of fish for every 2 gallons of water" to write equations. Students then solve problems using their equations and identify the constant of proportionality shown in each equation, along with its meaning in each equation.

Develop

Activity 3.1: Graphs of Two Constants of Proportionality

Students analyze the two equations they wrote in the Getting Started and graph each equation from a table of values they create. Students then compare the graphs, interpreting the meaning of the origin in each. Students also interpret the meanings of different points on the graphs and identify the unit rate and constant of proportionality using each graph. Finally, students solve problems using each graph, involving points on the graphs and extrapolating to points not shown on the grids.

Day 2

Activity 3.2: Constant of Proportionality from a Graph

The graph given represents the relationship between the time and distance of a runner. Students determine the relationship between the time and distance. They will determine the constant of proportionality, in kilometers per minute, using the *y*-coordinate : *x*-coordinate ratio and explain the constants relevance to the situation. Next, students write an equation to represent the situation and use the equation to solve for unknown quantities. Students then determine the constant of proportionality in terms of kilometers per hour, write a second equation using this constant, and compare both equations.

Activity 3.3: Determining Proportional Relationships from Graphs

Students analyze the four graphs given. They determine if each graph represents two quantities that are proportional to each other. They also determine the constant of proportionality when appropriate.

Demonstrate

Talk the Talk: How Do You Know?

Students complete a writing task, explaining how they know when a graph shows a proportional relationship between quantities, how to determine the unit rate and constant of proportionality from a graph, and the meaning of an arbitrary point on the graph of a relationship.

Getting Started: The Fish-Inches System of Measurement

ENGAGE

Facilitation Notes

In this activity, given a scenario, students define variables, write equations, and use the equations to answer related questions.

Have students work with a partner or in a group to complete Questions 1 through 5. Share responses as a class.

As students work, look for

- Consistent use of labeling y and x in the question.
- Use of the structure $\frac{y}{x}$ and substitution to determine k.
- Reasoning to determine what equation should be used.

Questions to ask

- How is the rule of thumb applied in this situation?
- What is the total length of fish in this situation?
- What variable represents the number of fish-inches?
- What variable represents the gallons of water in the aquarium?
- How did you set up the proportion?
- What is the constant of proportionality?
- What does the constant of proportionality mean in this problem situation?
- How do you determine $\frac{3}{2}$ of 10?
- How do you determine $\frac{2}{3}$ of 18?

Summary

The constant of proportionality is used to define variables, write equations in the form of $y = kx$, and solve real-world problem situations.

Activity 3.1

Graphs of Two Constants of Proportionality

DEVELOP

Facilitation Notes

In this activity, students use the equations written in the previous activity to create tables of values, and plot the ordered pairs to create graphs. The graphs are used to answer questions related to the situation and determine the unit rates.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

As students work look for

- Various methods of completing the table.
- Substitution of *x*-values in an equation and solving for *y*.
- Use of the fact that this is a proportional relationship and creation of equivalent ratios across the table.
- Use of answers from the previous activity.

Differentiation strategy

To scaffold support, suggest that students write a vertical version of the table next to each graph.

Questions to ask

- What is the independent variable in this situation?
- What is the dependent variable in this situation?
- How did you determine the values for *x*?
- How did you compute the values for *y*?
- How are the two graphs different?
- Do both graphs give you the same information?
- When does it make sense to use the Fish-Inches per Gallon graph?
- When does it make sense to use the Gallons per Fish-Inch graph?
- What is the number of gallons per fish-inch?
- What is the number of fish-inches per gallon?

Have students work with a partner to complete Questions 5 and 6. Share responses as a class.

Questions to ask

- Which graph did you use to determine the answer?
- Which graph did you use to estimate the answer?

Summary

Equations using the constant of proportionality can be used to create tables of values, create graphs, and determine unit rates.

Activity 3.2

Constant of Proportionality from a Graph

Facilitation Notes

In this activity, students use a graph to define variables, write equations, determine the constant of proportionality, and answer questions related to the situation.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

As students work, look for

- Placement of the coordinate pairs in a table prior to writing an equation.
- Connection of the points, then determining the unit rate directly from the graph.
- Use of a ordered pair on the graph to determine a rate, then converting it to a unit rate.

Questions to ask

- Do the points on the graph appear to have a linear relationship?
- Does the line appear to pass through the origin? What are the implications?
- What do you predict about the relationship of the ratios *y*-coordinate: *x*-coordinate for each point on the graph?
- If the time is known, how do you determine the distance?
- If the distance is known, how do you determine the time?
- Do both equations show that Ella's distance varies directly with time?
- Which constant of proportionality shows how many kilometers Ella runs in a minute?
- Which constant of proportionality shows how many kilometers Ella runs in an hour?

Summary

Graphs can be used to define variables, write equations, determine the constant of proportionality and answer questions related to the situation.

Activity 3.3

Determining Proportional Relationships from Graphs

Facilitation Notes

In this activity, students use graphs to determine proportional relationships. Where possible, the constant of proportionality is identified.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- Do the points on the graph appear to have a linear relationship?
- Does the line appear to pass through the origin?
- How do the *y*-coordinate : *x*-coordinate ratios of the points relate to each other?

Differentiation strategy

To extend the activity, frame questions or have students create *always*, *sometimes*, or *never* statements based on the four graphical representations.

- *Always* statements should include defining factors such as "the graph is a straight line and goes through the origin."
- *Sometimes* statements should include characteristics not related to the definition, such as "the points are connected," or "the points are not connected."
- *Never* statements should include the opposite of the defining factors such as "the points do not line up," or "its *y*-intercept is not the origin."

In addition to discussing the definition of a proportional relationship, extend the discussion so that students understand how *always*, *sometimes,* and *never* statements are developed.

Summary

Graphs of proportional relationships are linear and pass through the origin. Each point on the graph has the same $\frac{y}{x}$ ratio. The point (1, *r*) represents the unit rate and the ratio $\frac{r}{1}$ represents the constant of proportionality, *k*.

DEMONSTRATE

Talk the Talk: How Do You Know?

Facilitation Notes

In this activity, students answer How do you know? questions related to proportional relationships.

Have students work with a partner or in a group to complete Question 1. Share responses as a class.

Misconception

Students may say that the constant of proportionality and unit rate are the same. Clarify that the constant of proportionality is $\frac{y}{x}$. In a unit rate, $\frac{y}{x}$ is written so that $x = 1$.

Questions to ask

- Is the graph of a proportional relationship always linear?
- Does the graph of a proportional relationship always pass through the origin?
- When the graph shows a proportional relationship, what is the quotient of the y-value of any ordered pair divided by the x-value of the ordered pair?
- When the graph shows a proportional relationship, is the unit rate the same as the constant of proportionality?
- How do you determine the unit rate from a graph of a proportional relationship?

Summary

Graphs of proportional relationships are linear and pass through the origin. The unit rate is the same as the constant of proportionality. Given the graph of a proportional relationship the point $(1, r)$ represents the unit rate, and the ratio $\frac{r}{1}$ represents the constant of proportionality, k.

NOTES

Fish-Inches

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Identifying the Constant of Proportionality in Graphs

WARM UP

Solve each equation for the variable.

1. $\frac{1}{2}a = 5$
2. $\frac{p}{\frac{1}{3}} = 2$
3. $3x = \frac{3}{2}$
4. $\frac{6}{z} = \frac{1}{6}$

LEARNING GOALS

- Determine if relationships represented in words, tables, equations, and graphs are proportional.
- Interpret the meaning of linear proportional relationships represented in words, tables, equations, and graphs.
- Identify and interpret the constant of proportionality for quantities that are proportional and represented in words, tables, equations, and graphs.
- Explain what a point on the graph of a proportional relationship means in terms of the problem situation.
- Explain what the points (0, 0) and (1, *r*) mean on the graph of a proportional relationship, where *r* is the unit rate.

You have determined the constant of proportionality in problem situations and from equations. How can you represent the constant of proportionality in graphs?

Warm Up Answers

1. $a = 10$
2. $p = \frac{2}{3}$
3. $x = \frac{1}{2}$
4. $z = 36$

Answers

1. For 6 fish-inches, 4 gallons of water would be required. Check students' diagrams.
2. Let f equal the number of fish-inches, and let w equal the gallons of water in the aquarium.

3a. $f = \frac{3}{2}w$

3b. $w = \frac{2}{3}f$

Getting Started

The Fish-Inches System of Measurement

You are thinking of purchasing an aquarium. You contact the owner of an aquarium store. You need to know how many fish to purchase for an aquarium, but first you must determine how big the aquarium will be. The owner of the aquarium store tells you his rule of thumb is to purchase "a total length of fish of 3 inches for each 2 gallons of water in the aquarium."

1. How many gallons of water would you need if you had a 4-inch fish and a 2-inch fish? Draw a diagram to explain your reasoning.

2. Define variables for the quantities that are changing in this problem situation.

3. Write an equation for each:

a. fish-inches based on the gallons of water

b. gallons of water based on fish-inches

4. Use one of your equations to solve each problem.

   a. If an aquarium holds 10 gallons of water, how many fish-inches should you purchase?

   b. If you want to purchase a 5-inch fish, two 2-inch fish, and three 3-inch fish, how many gallons of water should the aquarium hold?

5. Determine the constant of proportionality given by each equation and explain what it means in context.

Answers

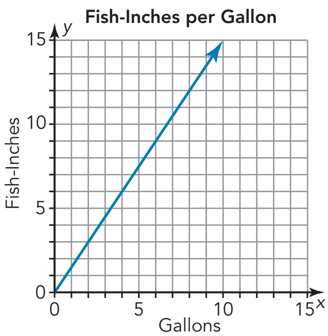
4a. I can buy 15 fish-inches of fish for a 10-gallon aquarium.

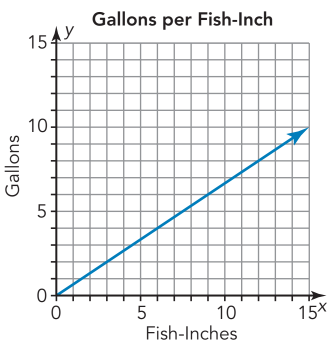
4b. The aquarium should hold 12 gallons.

5. For the equation $f = \frac{3}{2}w$, the constant of proportionality is $\frac{3}{2}$, or $1\frac{1}{2}$. This means that there should be $1\frac{1}{2}$ fish-inches for each gallon of water. For the equation $w = \frac{2}{3}f$, the constant of proportionality is $\frac{2}{3}$. This means that there should be $\frac{2}{3}$ gallon of water for each fish-inch.

Answers

1. See table below. Sample answers.





2. On both graphs, the point (0, 0) means that 0 fish-inches corresponds to 0 gallons of water.

3a. The point (6, 9) on the Fish-Inches per Gallon graph means 6 gallons of water fits 9 inches of fish.

3b. The point (9, 6) on the Gallons per Fish-Inch graph means 9 inches of fish fit in 6 gallons of water.

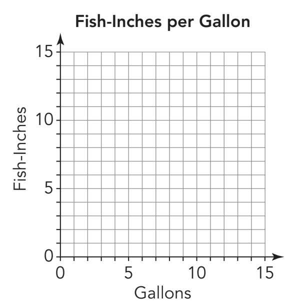
ACTIVITY **3.1**

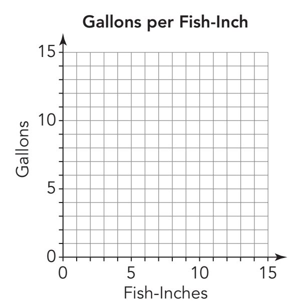
Graphs of Two Constants of Proportionality

Let's graph each equation you wrote in the previous activity.

1. Create a table of ordered pairs. Then plot the ordered pairs to create a graph of each equation.

Fish-Inches							
Gallons							





2. What does the point (0, 0) mean on each graph?

3. Determine the meaning of each point.

a. What does the point (6, 9) on the Fish-Inches per Gallon graph represent?

b. What does the point (9, 6) on the Gallons per Fish-Inch graph represent?

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1.

Fish-Inches	1	$\frac{3}{2}$	3	6	9	12	15
Gallons	$\frac{2}{3}$	1	2	4	6	8	10

c. **What does the point $(1, 1\frac{1}{2})$ on the Fish-Inches per Gallon graph represent?**

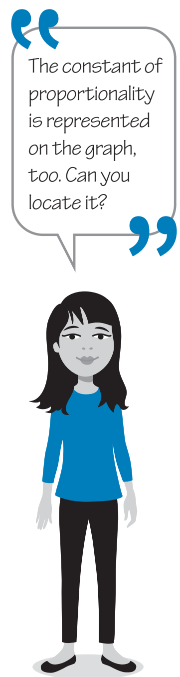
d. **What does the point $(1, \frac{2}{3})$ on the Gallons per Fish-Inch graph represent?**

4. **What is the unit rate for each graph? Explain how you can determine the unit rate using the graph.**

5. **Use one of your graphs to determine each answer.**

   a. **How many fish-inches can fit into 10 gallons of water?**

   b. **How many gallons are needed for $7\frac{1}{2}$ fish-inches?**



6. **Use one of the graphs to estimate each answer. Explain how you used the graph to determine your estimate.**

   a. **How many gallons would be needed for 16 fish-inches?**

   b. **How many fish-inches would fit into 16 gallons?**

Answers

3c. The point $(1, 1\frac{1}{2})$ on the Fish-Inches per Gallon graph represents the unit rate: $1\frac{1}{2}$ fish-inches per gallon.

3d. The point $(1, \frac{2}{3})$ on the Gallons per Fish-Inch graph represents the unit rate: $\frac{2}{3}$ gallon per fish-inch.

4. Determine the y-coordinate of the point with an x-coordinate of 1. The y-value is the unit rate. For the Fish-Inches per Gallon graph, the unit rate is $1\frac{1}{2}$ fish-inches per gallon. For the Gallons per Fish-Inch graph, the unit rate is $\frac{2}{3}$ gallon per fish-inch.

5a. 15 fish-inches can fit into 10 gallons of water.

5b. 5 gallons of water are needed for $7\frac{1}{2}$ fish-inches

6a. Estimates will vary. Estimates should be close to 11 gallons.

6b. Estimates will vary. Estimates should be close to 24 fish-inches.

Answers

1. x = time in minutes
   y = distance in kilometers $y = \frac{1}{6}x$

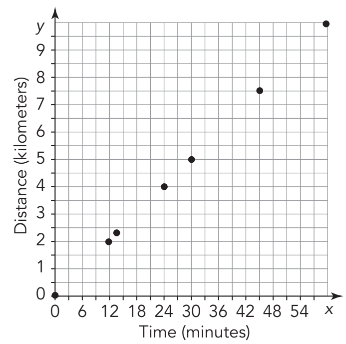
2a. 2.5 kilometers

NOTES

ACTIVITY 3.2

Constant of Proportionality from a Graph

The graph shown displays the relationship between the time and distance Ella runs.



1. Define variables and write an equation to represent the relationship between Ella's distance and time.

2. Use your equation to answer each question.

a. How far can Ella run in 15 minutes?

b. How long does it take Ella to run 7.5 kilometers?

c. How far can Ella run in one hour?

d. Determine the constant of proportionality in kilometers per hour. Then, write another equation that represents how Ella's distance (*d*) varies directly with time (*t*).

e. How is this equation similar to, and different from, the previous equation you wrote?

Answers

2b. 45 minutes

2c. 10 kilometers

2d. $k = \frac{10}{1}$; $d = 10t$

2e. Both equations show that Ella's distance varies directly with time. For $d = \frac{1}{6}t$, the constant of proportionality shows how many kilometers Ella runs in a minute. For $d = 10t$, the constant of proportionality shows how many kilometers Ella runs in an hour.

Answers

1a. The graph does not show a proportional relationship.

1b. The graph does not show a proportional relationship.

ACTIVITY 3.3

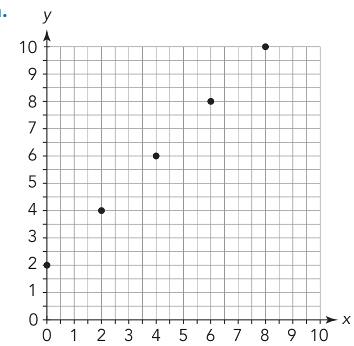
Determining Proportional Relationships from Graphs

A graph establishes dependency. So, if the graph shows a proportional relationship, then *y* varies directly with *x*.

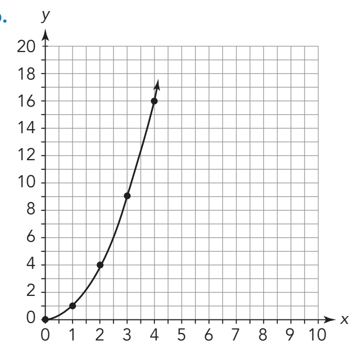
You have seen that proportional relationships can be represented on graphs and that the constant of proportionality can be identified from the graph.

1. Determine if each graph shows a proportional relationship between *x* and *y*. If possible, determine the constant of proportionality. Explain how you determined your answer.

a.



b.



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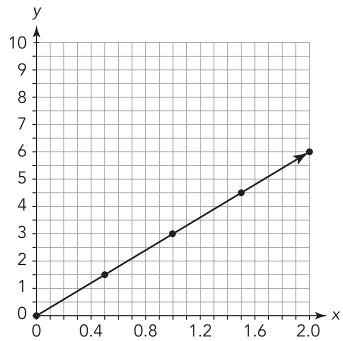
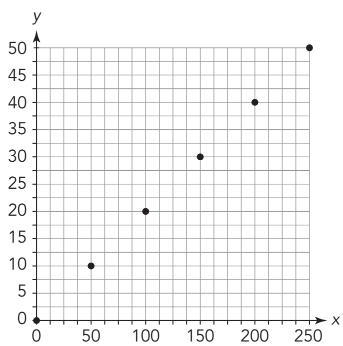
c.



d.



Answers

1c. The graph shows a proportional relationship. $k = 3$

1d. The graph shows a proportional relationship. $k = \frac{1}{5}$

Answers

1a. If the graph is a straight line that passes through the origin, (0, 0), then it is the graph of a proportional relationship.

1b. The constant of proportionality is the quotient of the y-value of any ordered pair divided by the x-value of the ordered pair, when the graph shows a proportional relationship.

1c. When the graph shows a proportional relationship, the unit rate is the same as the constant of proportionality. For the point on the graph whose ordered pair has an x-value of 1, the y-value is the unit rate, $(1, r)$, and constant of proportionality.

1d. For the ordered pair (x, y), x means x units of a quantity and y means y units of a different quantity. The ratio $\frac{y}{x}$ represents the constant of proportionality.

NOTES

TALK the TALK

How Do You Know?

Use examples to explain your answer to each question.

1. **Given a graph of a relationship between two quantities, how do you know:**

   a. **if the graph shows a proportional relationship?**

   b. **what the constant of proportionality is?**

   c. **what the unit rate is?**

   d. **what any ordered pair on the graph represents?**