Minding Your Ps and Qs

4

Constant of Proportionality in Multiple Representations

MATERIALS

None

Lesson Overview

Students use proportional relationships to create equivalent multiple representations, such as diagrams, equations, tables, and graphs of situations. A proportional relationship may initially be expressed using only words, a table of values, an equation, or a graph. For example, given only the information that "q varies directly with p," students will write an equation, complete a table of values, determine the constant of proportionality, construct a graph from the table of values, and create a scenario to fit the graph.

Grade 7
Proportionality

(4) The student applies mathematical process standards to represent and solve problems involving proportional relationships. The student is expected to:

(A) represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including $d = rt$.

(C) determine the constant of proportionality ($k = \frac{y}{x}$) within mathematical and real-world problems.

ELPS

1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Essential Ideas

- The graph of two variables that are proportional, or that vary directly, is a line that passes through the origin, (0, 0).
- When analyzing the table of two variables that vary directly, the ratios of the y-value to the x-value for any pair are equivalent.
- The equation used to represent a proportional relationship between two values is $y = kx$, where x varies directly with y, and k is the constant of proportionality.
- A table of equivalent ratios, a graph of a straight line through the origin, and an equation of the form $y = kx$ can be created to represent a scenario describing quantities in a proportional relationship.

Lesson Structure and Pacing: 3 Days

Day 1

Engage

Getting Started: Penny's Nickels Are a Quarter of Her Dimes

Students analyze a diagram representing several proportional relationships and use this diagram to solve a problem. They also identify the various constants of proportionality that can be represented in the diagram and write equations to represent each of these perspectives. This activity is designed to engage students in thinking flexibly about representing proportional relationships.

Develop

Activity 4.1: Using a Graph to Write an Equation

Students analyze a graph depicting a proportional relationship. They explain how they know the graph represents a proportional relationship. Next, they determine the constant of proportionality, and describe what it represents in the situation. Students then write an equation for the situation which enables them to solve for an unknown quantity.

Activity 4.2: Using an Equation to Create a Table

Two proportional situations are used in this activity. An equation that represents the situations and partially completed tables of values are given. Students complete the tables using the equation, identify the constant of proportionality, and explain what relevance the constant has to the problem situation.

Day 2

Activity 4.3: Using a Table to Create a Scenario

Students are given a table of values and are asked to create a scenario based on these values. They then analyze their scenario to determine the constant of proportionality and make inferences about the graph representing the scenario.

Activity 4.4: Using a Scenario to Write an Equation

Students are given a scenario and are asked to write an equation to describe the proportional relationship in their scenario. They then analyze the equation and scenario to determine the constant of proportionality and create a graph.

Day 3

Activity 4.5: Multiple Representations of Proportional Relationships

Students complete a table of values and write an equation to represent a proportional relationship. They then summarize how to write the equation to represent the relationship between two variables that are directly proportional if they are given a ratio table or graph.

Demonstrate

Talk the Talk: Every Which Way

Students create a graphic organizer describing four different representations of a proportional relationship scenario, table, graph, and equation. They complete the graphic organizer by representing a proportional relationship in each of the four ways.

Getting Started: Penny's Nickels Are a Quarter of Her Dimes

ENGAGE

Facilitation Notes

In this activity, students use a given diagram to write equations representing proportional relationships. They also identify the constant of proportionality in each relationship.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

- Different methods to write the equations.
- Use of the diagram to determine fractional relationships.
- Reference back to the context.
- Use of values from the problem to write the fractions.

Questions to ask

- How many equal sections are in the model?
- How many of the equal sections represent nickels?
- How many of the equal sections represent dimes?
- How many coins are in each section of the model?
- How much money is 10 nickels?
- How much money is 30 dimes?
- Does $3n = d$ or does $3d = n$?
- Is the relationship three dimes per nickel or three nickels per dime?

Summary

A diagram representing proportional relationships can be used to write equations and identify the constant of proportionality.

Activity 4.1

Using a Graph to Write an Equation

DEVELOP

Facilitation Notes

In this activity, students use the graph of a proportional relationship to determine the constant of proportionality, write an equation for the situation, and solve for an unknown quantity.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Differentiation strategies

To scaffold support,

- Have students draw horizontal and vertical lines from a point on the graph to each axes to determine the values of *y* and *x*.
- Discuss the selection of points that are easy to read from the graph. In some cases, the constant of proportionality is easier to determine from a graph than the unit rate.

Questions to ask

- How can you determine the constant of proportionality using only the graph of attempted free throws?
- How do you determine the number of shots made when the number of attempted free throws is known?
- How do you determine the number of free throws attempted when the number of shots made is known?

Summary

A graph of a proportional relationship can be used to write equations, identify the constant of proportionality and solve for unknown quantities. The point (1, r) represents the unit rate and the ratio $\frac{r}{1}$ represents the constant of proportionality, k.

Activity 4.2

Using an Equation to Create a Table

Facilitation Notes

In the first scenario of this activity, students are given an equation describing a proportional relationship, which is used to complete a table of values and identify the constant of proportionality. In the second scenario, a partially completed table of values is used to write an equation and identify the constant of proportionality.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Differentiation strategies

To scaffold support,

- Help students understand the structure of the problem in order to define the variables. Have students circle *varies directly*. Then, have them underline the phrase in front of it, labeling it *y*. Next, have them underline the phrase behind it, labeling it *x*.
- Discuss that the phrase *varies directly* means that there is a proportional relationship and that it implies what variable is the dependent variable and what variable is the independent variable.

Questions to ask

- How did you determine the number of hours Shaylah worked when she earned $112.85?
- How did you determine Shaylah's earnings when she worked 40 hours?
- How are the hours Shaylah worked determined when you know her earnings?
- How are Shaylah's earnings determined when you know the number of hours she worked?
- How was the table helpful in creating the equation to represent this situation?
- How do you determine Fernando's earnings if he worked 1 hour?
- How are the hours Fernando worked determined when you know his earnings?
- How are Fernando's earnings determined when you know the number of hours he worked?

Differentiation strategy

To extend the activity, ask students to provide examples of pay scales that represent non-proportional relationships. Examples include a base pay with commission and a signing bonus plus an hourly rate. If time permits, have students create a context for a signing bonus plus an hourly rate, create a table, and graph the results. Compare and contrast this non-proportional relationship with the proportional relationships provided.

Summary

Equations representing proportional relationships are written in the form $y = kx$ and can be used to complete tables of values and identify the constant of proportionality.

Activity 4.3

Using a Table to Create a Scenario

Facilitation Notes

In this activity, students are given a table of values describing a proportional relationship, and it is used to create a scenario, define quantities, identify the constant of proportionality, and describe the graph.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- What do you know about this situation?
- What information can you conclude from the table of values?
- How can you tell by looking at the table that the graph of this relationship will pass through the origin?
- Is there enough information to determine the amount of window cleaner needed to clean 1 window?
- How can you determine the amount of window cleaner needed to clean 1 window?
- What is the unit rate in this situation?
- Do the ordered pairs from the table of values form a straight line when graphed?
- Does the graph of the line pass through the origin?
- How is the constant of proportionality represented on the graph?

Summary

Tables of values representing proportional relationships show a constant ratio of $\frac{y}{x}$ and can be used to write an equation and identify the constant of proportionality.

Activity 4.4
Using a Scenario to Write an Equation

Facilitation Notes

In this activity, students are given two scenarios describing proportional relationships, and they are used to define variables, write equations, identify the constant of proportionality and create a graph.

Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

Differentiation strategy

Encourage students to create a table of values before graphing. This will help them think about how to set up the scale and intervals for their graph.

Questions to ask

- What variable represents the number of days the milk is consumed?

- What variable represents the number of liters of milk consumed?
- What is the relevance of 10 in this situation?
- How did you decide how to set up the bounds and intervals for each axis?
- Does the graph of the line pass through the origin?
- Where is the constant of proportionality represented on your graph?
- What variable represents the number of pounds a person weighs on Earth?
- What variable represents the number of pounds a person weighs on the Moon?
- What is the relevance of 6 in this situation?
- Do the ordered pairs from the table of values form a straight line when graphed?
- How did you decide how to set up the bounds and intervals for each axis?
- Does the graph of the line pass through the origin?
- Where is the constant of proportionality represented on your graph?

Summary

Scenarios representing proportional relationships can be used to write an equation of the form $y = kx$, identify the constant of proportionality, and create a graph.

Activity 4.5
Multiple Representations of Proportional Relationships

In this activity, students complete a table of values that describes a proportional relationship. They write an equation to represent the relationship between the two variables using the value of k determined in the table, and then graph the values in the table. Students graph a proportional relationship when given an equation representing the relationship between two quantities that vary directly. They then write an equation representing the relationship between two quantities that vary directly when given a graph.

Facilitation Notes

Students are given a partially completed table of values describing a proportional relationship, which is used to write an equation, identify the constant of proportionality and create a graph. They explain their reasoning.

Have students work with a partner or in a group to complete Questions 1 through 6. Share responses as a class.

Questions to ask

- What is the relationship between p and q?
- What is the product of p and q?
- How do you determine the value of p when the value of q is known?
- How do you determine the value of q when the value of p is known?
- What does the graph of a relationship between two quantities that are proportional look like?
- What information is necessary to write an equation representing a proportional relationship between two quantities?
- What information is necessary to create a graph representing a proportional relationship between two quantities?
- What information is necessary to create a table of values representing a proportional relationship between two quantities?

Summary

Proportional relationships represented in tables show a constant ratio, $\frac{y}{x}$, or the constant of proportionality. The information from a table can be used to write an equation in the form $y = kx$. The information can also be used to graph the proportional relationship as a line passing through the origin and the point (1, r).

DEMONSTRATE

Talk the Talk: Every Which Way

Facilitation Notes

In this activity, students write equations and sketch graphs associated with proportional relationships. They also complete a graphic organizer which shows multiple representations of a proportional relationship.

Have students work with a partner or in a group to complete Question 1 and the graphic organizer. Share responses as a class.

Questions to ask

- What is the relationship between p and q?
- What is the product of p and q?
- How do you determine the value of p when the value of q is known?

- How do you determine the value of *q* when the value of *p* is known?
- What does the table of a relationship between two quantities that are proportional look like?
- What does the equation of a relationship between two quantities that are proportional look like?
- What does the graph of a relationship between two quantities that are proportional look like?
- What information is necessary to write an equation representing a proportional relationship between two quantities?
- What information is necessary to create a table of values representing a proportional relationship between two quantities?
- What information is necessary to create a graph representing a proportional relationship between two quantities?

Summary

Multiple representations of proportional relationships can be in the form of a written scenario, a table of values that show a constant ratio, $\frac{y}{x}$, or the constant of proportionality, an equation in the form $y = kx$, and as a graph that forms a straight line passing through the origin and the point (1, r).

NOTES

Minding Your Ps and Qs

4

Constant of Proportionality in Multiple Representations

WARM UP

Solve each equation.

1. $5p = 2.5$
2. $\frac{1}{3}j = 9$
3. $0.12k = 10.08$
4. $8k = 0$

LEARNING GOALS

- Determine if relationships represented in words, tables, equations, or graphs are proportional.
- Interpret the meaning of linear proportional relationships represented in words, tables, equations, and graphs.
- Determine and interpret the constant of proportionality for quantities that are proportional and represented in words, tables, equations, and graphs.

You have learned how to determine the constant of proportionality. How can you solve problems using this constant in tables, graphs, and diagrams?

Warm Up Answers

1. $p = 0.5$
2. $j = 27$
3. $k = 84$
4. $k = 0$

Answers

1. Extend the lines from the bottom bar to show that 5 equal sections of the model contain coins (4 equal sections of dimes plus the section of nickels). Each of these sections is equal to the number of nickels. So, $5n = 40$. There are 8 nickels and 32 dimes, for a total of $3.60.

2a. $\frac{1}{4}d = n$ or $4n = d$

2b. $d = \frac{4}{5}c$ or $c = \frac{5}{4}d$

2c. $n = \frac{1}{5}c$ or $c = 5n$

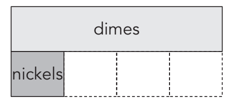
3. Answers may vary depending on the equations written:
   - (a) $\frac{1}{4}$ or 4
   - (b) $\frac{4}{5}$ or $\frac{5}{4}$
   - (c) $\frac{1}{5}$ or 5

Getting Started

Penny's Nickels Are a Quarter of Her Dimes

Penny collects only nickels and dimes. She has one-quarter as many nickels as dimes.

A diagram can represent this problem situation.



All together she has 40 coins. How much money does she have?

1. Explain how you can use the diagram to solve the problem. Determine the solution.

2. Write an equation to represent the proportional relationship between:

a. the number of nickels and the number of dimes.

b. the number of dimes and the total number of coins.

c. the number of nickels and the total number of coins.

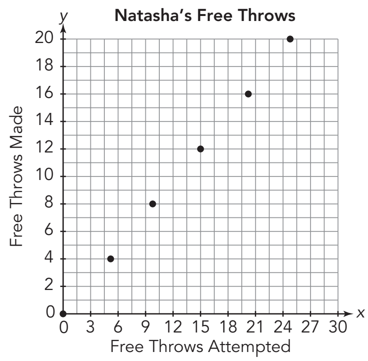
3. Identify the constant of proportionality in each proportional relationship described in Question 2.

ACTIVITY 4.1

Using a Graph to Write an Equation

NOTES

The graph shows Natasha's total number of free throw attempts and the total number of free throws made.



1. **Explain how you know the graph represents a relationship that is proportional.**

2. **Determine the constant of proportionality and describe what it represents in this problem situation.**

3. **If Natasha attempted 30 free throws, how many would she probably make? First, use your graph to estimate the answer. Then, verify your answer by using an equation.**

Answers

1. The graph passes through (0, 0) and forms a straight line.
2. The constant of proportionality is 0.8. Natasha makes 8 free throws out of 10 free throws attempted, or 80% of her attempted free throws.
3. Natasha would make 24 free throws.

Answers

1. The constant of proportionality represents how much Shaylah earns per hour of work.
2. Sample answer.

Hours Worked	Earnings (dollars)
0	0
12.2	112.85
40	370
1	9.25
10	92.50

NOTES

ACTIVITY 4.2

Using an Equation to Create a Table

Another example of a proportional relationship is the relationship between the number of hours a worker works and their wages earned in dollars.

The amount of money (m) Shaylah earns varies directly with the number of hours (h) she works. The equation describing this relationship is $m = 9.25h$.

1. What does the constant of proportionality represent in this situation?

2. Complete the table based on the equation given. Include the constant of proportionality in the table.

Hours Worked	Earnings (dollars)
0	
	112.85
40	

4 • TOPIC 3: Proportionality

During the summer, Fernando works as a movie attendant. The number of hours he works varies each week.

3. Write an equation to represent this situation. Then complete the table based on your equation and include the constant of proportionality.

Hours Worked	Earnings (dollars)
3	26.88

4. What is the constant of proportionality? What does it mean in this problem situation?

NOTES

Answers

3. $y = 8.96x$
   x = number of hours worked,
   y = earnings (dollars)

   Sample answer.

Hours Worked	Earnings (dollars)
3	26.88
1	8.96
10	89.60
15	134.40
20	179.20

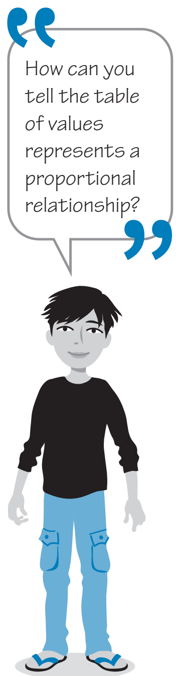
4. The constant of proportionality is 8.96. The constant of proportionality represents how much Fernando earns per hour of work.

Answers

1. Sample answer.
   Any number of windows can be multiplied by a constant, 8, to determine the amount of window cleaner.
2. The constant of proportionality is 8. Each window requires 8 ounces of window cleaner.
3. The ordered pairs from the table would form a straight line that passes through the origin when graphed.

ACTIVITY 4.3

Using a Table to Create a Scenario



Analyze the given table.

Number of Windows	Amount of Window Cleaner (ounces)
0	0
2	16
3	24
4	32
5	40
6	48

1. Describe one possible situation that could be represented by this table of values. Include how the quantities relate to each other.

2. What is the constant of proportionality and what does it represent in your situation?

3. If the table values were used to create a graph, how would the points appear?

ACTIVITY 4.4

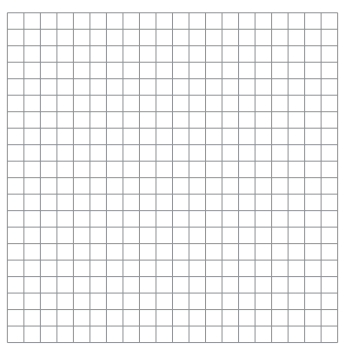
Using a Scenario to Write an Equation

A baby elephant nurses for the first two years of its life. It drinks about 10 liters of milk every day.

1. Define variables and write an equation to represent the relationship between the amounts of milk the baby elephant consumes and the time it spends consuming the milk. Assume the elephant maintains the same rate of consumption.

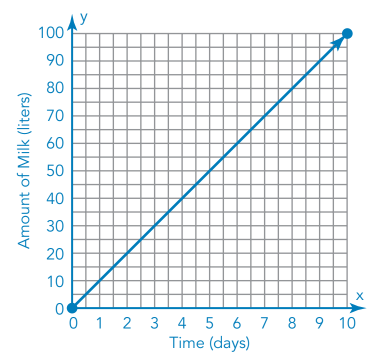
2. Identify the constant of proportionality and describe what it means in this situation.

3. Create a graph to represent this situation.



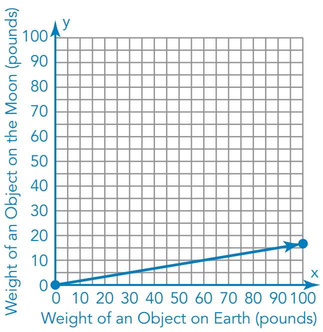
Answers

1. x = number of days the milk is consumed, y = number of liters of milk consumed; $y = 10x$
2. The constant of proportionality is 10. It represents the number of liters of milk a baby elephant consumes per day.
3.



Answers

4. x = weight (pounds) of an object on Earth, y = weight (pounds) of the object on the Moon; $y = \frac{1}{6}x$
5. The constant of proportionality is $\frac{1}{6}$. It means that an object's weight on the Moon is $\frac{1}{6}$ of its weight on Earth.
6.

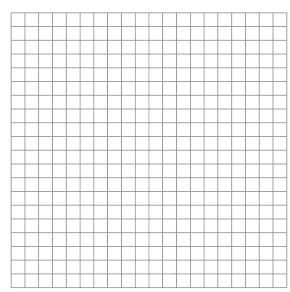


The weight of an object on Earth varies directly with the weight of an object on the Moon. A 150-pound object would weigh approximately 25 pounds on the Moon.

4. Define variables and write an equation to represent the relationship between the weight of an object on Earth and the weight of the object on the Moon.

5. Identify the constant of proportionality and describe what it means in this situation.

6. Create a graph to represent this situation.



ACTIVITY 4.5

Multiple Representations of Proportional Relationships

Suppose q varies directly with p.

1. Complete the table for variables p and q.

p	q
0	
2	6
4	12
0.25	
	3
1.5	4.5

2. Write an equation that represents the relationship between p and q.

3. Summarize how you can write the equation that represents the relationship between two variables that vary directly if you are given a ratio table.

Answers

1.

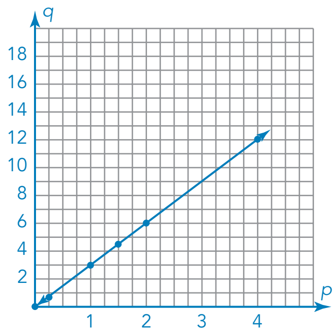
p	q
0	0
2	6
4	12
0.25	0.75
1	3
1.5	4.5

2. $q = 3p$

3. Sample answer.
Choose any one pair of values in the table other than (0, 0). Determine the constant of proportionality, $k = \frac{y}{x}$. Then write the equation $y = kx$, substituting the value of the constant of proportionality for k.

Answers

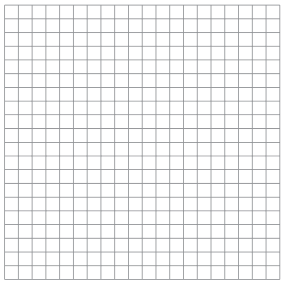
4.



5. Because there is a proportional relationship between p and q, the graph of the values should be a straight line passing through (0, 0).

6. Sample answer. Choose any coordinate pair on the line other than (0, 0). Determine the constant of proportionality, k, and then write the equation $y = kx$, substituting the value of the constant of proportionality for k.

4. Graph your equation. Label your axes.



5. Summarize how to draw a graph from the equation representing the relationship between two quantities that vary directly.

6. Summarize how you can write the equation representing the relationship between two quantities that vary directly if you are given a graph.

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TALK the TALK

Every Which Way

You have seen how to represent proportional relationships in scenarios, on graphs, in tables, and with equations.

1. **Write an equation and sketch a graph to represent each relationship. Label your axes and identify the constant of proportionality on the graph.**

   a. **Suppose the quantity *p* varies directly with the quantity *q*.**

   b. **Suppose the quantity *q* varies directly with the quantity *p*.**

Answers

Sample answers.

1a. $p = kq$; check students' graphs.

1b. $q = kp$; check students' graphs.

Answers

Graphic organizers will vary.

NOTES

2. Write a scenario which describes a proportional relationship between two quantities. Represent this relationship using an equation, a graph, and a table. For each model, identify the constant of proportionality and explain how the model shows that the relationship is proportional.