Proportionality
Topic 3 Overview

How is *Proportionality* organized?

In this topic, students review the meaning of proportionality and linear relationships and differentiate between proportional and non-proportional relationships, including linear relationships that are not proportional. Students begin by sorting and exploring graphs, equations, and scenarios for a variety of relationships. They classify their sorts according to linear or non-linear and proportional or non-proportional. Students recall that in a proportional relationship, the points of the graph form a straight line and pass through the origin and that in a table, all sets of ratios of the corresponding values of the quantities must be constant.

Students then learn about the constant of proportionality, the ratio between the two quantities being compared. They recognize that the constant is determined by the order of the ratio and use proportions to write and analyze direct variation equations, $y = kx$. Students graph proportional relationships and determine the constant of proportionality from the graphs, interpreting this constant as the unit rate in terms of the problem situation. Finally, students use relationships expressed in a variety of ways to practice determining if relationships are proportional, interpreting the meaning of linear proportional relationships, and determining and interpreting the constant of proportionality.

What is the entry point for students?

In grade 6, students developed a strong understanding of ratio and rate reasoning, including reasoning about equivalent ratios from graphs and tables. In the previous topic, students reviewed some of these basic ideas and developed a formal strategy for solving proportions. To begin this topic, students compare ratios presented in table form and decide which ratios are equivalent, and then they draw lines for each ratio to review plotting and comparing ratios using graphs. As students continue comparing relationships, they review relationships that are linear and non-linear and proportional and non-proportional. Once students understand these distinctions, they dig deeper into analyzing graphs and equations of proportional relationships.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic if they can:

- Determine if a proportional relationship exists between two quantities represented in tables, graphs, or equations.
- Represent proportional relationships with equations in the form of $y = kx$.
- Define the constant of proportionality as a rate.
- Identify the constant of proportionality (unit rate) from tables, graphs, equations, and verbal descriptions of proportional relationships.
- Interpret the meaning of the constant of proportionality in real-world situations.
- Interpret the meaning of any point on the graph of a proportional relationship.
- Interpret the meaning of (0,0) and (1, r) on the graph of a proportional relationship, where r is the unit rate.

Why is *Proportionality* important?

Students will continue to apply the constant of proportionality to solve multistep ratio and percent problems in the next topic. They will solve percent problems using the constant of proportionality and equations in the form of $y = kx$. They will relate the constant of proportionality to the scale factor in scale drawings.

The characteristics of proportional relationships, their graphs, and their equations as developed in *Proportionality* provide the underpinnings of algebra and the study of functions. In grade 8, students are expected to graph proportional relationships and interpret the unit rate as the slope of a line that models the relationships. They also represent linear proportional and linear non-proportional relationships with tables, graphs and equations. To understand the connections between proportional relationships, lines, and linear equations, students must first have a thorough understanding of the graphs and equations of proportional relationships, the focus of this topic. Students will connect the constant of proportionality with the slope of a line that passes through the origin. They will then use transformations to translate such lines to create lines in the form $y = mx + b$. However, students should also understand that not all lines are proportional and not all relationships are linear, which is groundwork that begins in this topic.

How do the activities in *Proportionality* promote student expertise in the mathematical process standards?

All Carnegie Learning topics are written with the goal of creating mathematical thinkers who are active participants in class

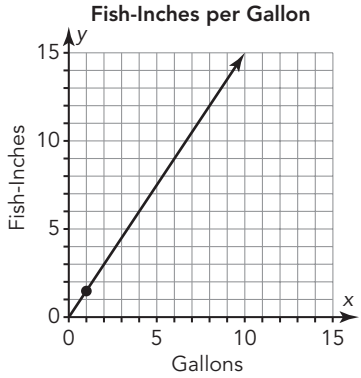
discourse, so elements of the mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work toward solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

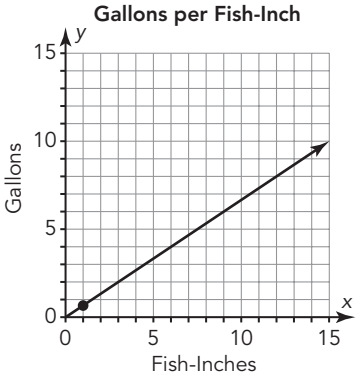
Throughout this topic, students are expected to model real-world scenarios with mathematics and use different mathematical tools and representations to analyze these scenarios. They should notice similarities in the graphs and equations of proportional relationships in order to make generalizations about the characteristics common to all proportional relationships.

Materials Needed

- Scissors
- Tape or glue sticks

Interpreting the Constant of Proportionality





- The point (1, $1\frac{1}{2}$) on the Fish-Inches per Gallon graph represents the unit rate: $1\frac{1}{2}$ fish-inches per gallon.
- The point (1, $\frac{2}{3}$) on the Gallons per Fish-Inch graph represents the unit rate: $\frac{2}{3}$ gallon per fish-inch.

Learning Together

ELPS: 1.A, 1.C, 1.E, 1.F, 1.G, 2.C, 2.E, 2.I, 3.D, 3.E, 4.B, 4.C, 4.D, 4.J, 5.B, 5.F, 5.G

Lesson	Lesson Name	TEKS	Days	Highlights
1	How Does Your Garden Grow?: Proportional Relationships	7.4A 7.4C	3	Students explore tables and graphs that illustrate proportional relationships. First, students review equivalent ratios and that the graphs of equivalent ratios form straight lines that pass through the origin. They are then given three sets of scenarios, equations, and graphs to match, using any strategy. Each group illustrates a different type of relationship: linear and proportional, linear and non-proportional, or non-linear. Students classify the groups of represetnations as linear and non-linear and use tables of values to classify the linear relationships as proportional or as non-proportional. They summarize the relationships between the terms *linear relationship*, *proportional relationship*, and *equivalent ratios*. Students are then given three new situations to analyze. They create tables of values and graphs and determine if a proportional relationship exists between two quantities. Finally, the term *direct variation* is introduced and explored using multiple representations.

Lesson	Lesson Name	TEKS	Days	Highlights
2	Complying with Title IX: Constant of Proportionality	7.4A 7.4C 7.4D	3	Students learn how to use equations to represent proportional relationships. Students write constants of proportionality based on the direction of the proportional relationship. They then use a scenario to set up a proportion and write two different equations for the scenario, depending on the direction of the proportional relationship. Students identify and interpret the constant of proportionality in the context of a scenario and solve problems using the equations that represent the proportional relationship. Next, students consider an additional situation in which the constant of proportionality and the corresponding equation depend on the question asked. They use the constants of proportionality to write equations, express the equations in terms of proportional relationships, and generalize the equation for proportional relationships. Students then practice using the constant of proportionality to solve for unknown quantities.
3	Fish-Inches: Identifying the Constant of Proportionality in Graphs	7.4A 7.4C 7.4D	2	In this lesson, students analyze real-world and mathematical situations, both proportional and non-proportional, represented on graphs, and then identify the constant of proportionality when appropriate. Students write equations to represent the situations from the graphs. Throughout the lesson, students interpret the meaning of points on graphs in terms of a proportional relationship, including the meaning of (1, *y*) and (0, 0).
4	Minding Your Ps and Qs: Constant of Proportionality in Multiple Representations	7.4A 7.4C	3	Students use proportional relationships to create equivalent multiple representations, such as diagrams, equations, tables, and graphs of the situation. A proportional relationship may initially be expressed using only words, a table of values, an equation, or a graph. For example, given only the information that "*q* varies directly with *p*," students will write an equation, complete a table of values, determine the constant of proportionality, construct a graph from the table of values, and create a scenario to fit the graph.

Suggested Topic Plan

*1 Day Pacing = 45 min. Session

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 7.4A, 7.4Cc **LESSON 1** **How Does Your Garden Grow?** GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 1** continued ACTIVITY 2 ACTIVITY 3	**LESSON 1** continued ACTIVITY 4 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	TEKS: 7.4A, 7.4C, 7.4D **LESSON 2** **Complying with Title IX** GETTING STARTED ACTIVITY 1
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
LESSON 2 continued ACTIVITY 2	**LESSON 2** continued ACTIVITY 3 ACTIVITY 4 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	**MID-TOPIC ASSESSMENT**	TEKS: 7.4A, 7.4C, 7.4D **LESSON 3** **Fish-Inches** GETTING STARTED ACTIVITY 1
Day 11	**Day 12**	**Day 13**	**Day 14**	**Day 15**
LESSON 3 continued ACTIVITY 2 ACTIVITY 3 TALK THE TALK	MATHia® Use LiveLab and Reports to monitor students' progress	TEKS: 7.4A, 7.4C **LESSON 4** **Minding Your Ps and Qs** GETTING STARTED ACTIVITY 1 ACTIVITY 2	**LESSON 4** continued ACTIVITY 3 ACTIVITY 4	**LESSON 4** continued ACTIVITY 5 TALK THE TALK
Day 16	**Day 17**			
MATHia® Use LiveLab and Reports to monitor students' progress	**END OF TOPIC ASSESSMENT**			

Assessments

There are two assessments aligned to this topic: Mid-Topic Assessment and End of Topic Assessment.