Proportionality Summary

KEY TERMS

- proportional relationship
- origin
- direct variation
- constant of proportionality

LESSON 1

How Does Your Garden Grow?

One special type of relationship that compares quantities using multiplicative reasoning is a *ratio relationship*. When two equivalent ratios are set equal to each other, they form a *proportion*. The quantities in the proportion are in a **proportional relationship**.

For a table of values to represent a proportional relationship, all the ratios of corresponding *x*- and *y*-values must be constant.

For example, the table displays the growth rate of a certain species of bamboo by comparing the time in days, x, to the height of the bamboo in centimeters, y. Does the table show a proportional relationship?

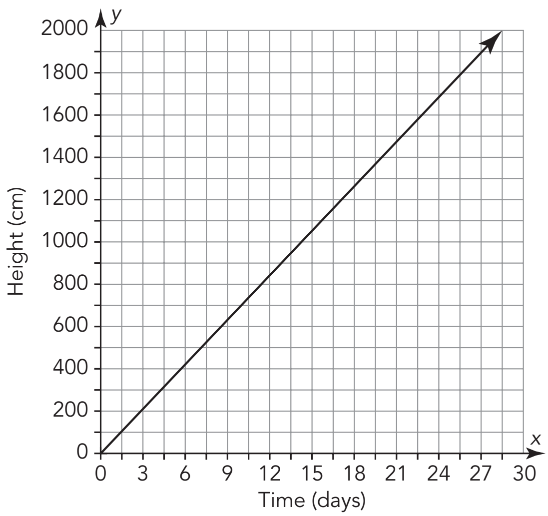
Time (days)	Height of Bamboo (cm)
3	210
10.5	735
18	1260
25.5	1785

$\frac{3}{210} = \frac{1}{70}$ $\quad \frac{10.5}{735} = \frac{1}{70}$ $\quad \frac{18}{1260} = \frac{1}{70}$ $\quad \frac{25.5}{1785} = \frac{1}{70}$

All the values $\frac{x}{y}$ represent the same constant, $\frac{1}{70}$, so the table shows a proportional relationship.

For a graph to represent a proportional relationship, the points of the graph must form a straight line and pass through the origin. The **origin** is the point with the ordered pair (0, 0).

For example, the values from the bamboo situation are graphed here. The graph represents a proportional relationship. It also represents a direct variation. A situation represents a **direct variation** if the ratio between the *y*-value and its corresponding *x*-value is constant for every point. The bamboo grows at a constant rate of 70 centimeters a day. The height varies directly with time.



LESSON 2

Complying with Title IX

If *y* is directly proportional to *x*, the relationship can be represented by the equation $y = kx$, where *k* is the **constant of proportionality.** This means that for any value of *x*, the value of *y* is *x* multiplied by *k*.

For example, the table of values represents a proportional relationship. Determine the constant of proportionality.

Girls	Boys
5	6
10	12
15	18

$y = kx$

$6 = k \cdot 5$

$\frac{6}{5} = k$

You can use the constant of proportionality to determine unknown values in proportions.

For example, using the constant of proportionality represented in the previous table, determine how many girls there are if there are 240 boys.

$$\frac{5}{6} = \frac{g}{240}$$

$$(5)(240) = 6g$$

$$1200 = 6g$$

$$200 = g$$

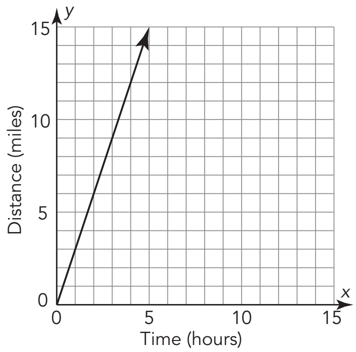
There are 200 girls.

Consider the same ratio of 5 girls to 6 boys. If you know the number of boys and need to determine the number of girls, you can use the equation $y = \frac{5}{6}x$, where x represents the number of boys and y represents the number of girls. In this equation, the $\frac{5}{6}$ represents the constant of proportionality. You can multiply the number of boys by $\frac{5}{6}$ to determine the number of girls. If you know the number of girls and need to determine the number of boys, you can use the equation $y = \frac{6}{5}x$, where x represents the number of girls and y represents the number of boys. In this scenario, the constant of proportionality is $\frac{6}{5}$. You can multiply the number of girls by $\frac{6}{5}$ to determine the number of boys.

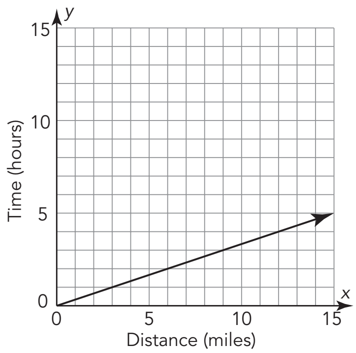
LESSON 3 Fish-Inches

The constant of proportionality of a situation can be graphed in two different ways.

For example, Stephen walks at a constant rate of 3 miles per hour.

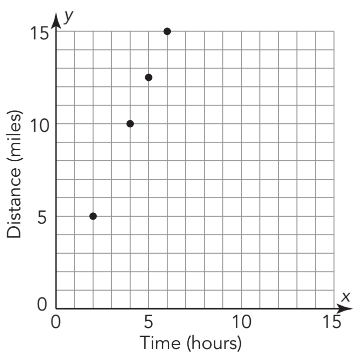


The first graph represents the ratio of distance to time. The point (1, 3) represent the unit rate. The ratio $\frac{3}{1}$ represents the constant of proportionality, $k = 3$.



The second graph represents the ratio of time to distance. The point $(1, \frac{1}{3})$ represents the unit rate. The ratio $\frac{\frac{1}{3}}{1}$ represents the constant of proportionality, $k = \frac{1}{3}$.

You can determine the constant of proportionality from a graph.



For example, the third graph displays the proportional relationship between the time and distance Melanie walks. Determine the constant of proportionality in miles per hour.

Choose an ordered pair from the graph: (4, 10).

$$k = \frac{y}{x}$$
$$k = \frac{10}{4}$$
$$k = 2.5$$

The constant of proportionality is 2.5.

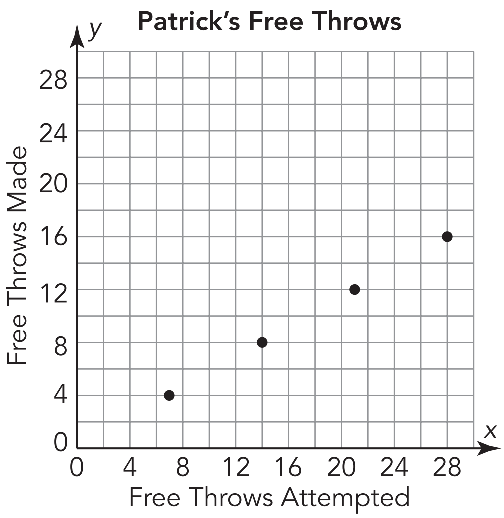
LESSON 4

Minding Your Ps and Qs

Proportional relationships can be represented in many different ways, including diagrams, tables, equations, and graphs. They can also be described in scenarios.

A graph can be used to write an equation.

For example, the graph shows Patrick's total number of free throw attempts and the total number of free throws made. The graph represents a proportional relationship because it is a straight line that goes through the origin. Using the graph, you can determine that the constant of proportionality is $k = \frac{4}{7}$, which means that Patrick made $\frac{4}{7}$ of the free throws he attempted. The equation that represents this is $y = \frac{4}{7}x$.



A scenario can also be used to write an equation.

For example, a blue whale eats 8000 pounds of food a day. Let x represent the independent quantity, the number of days, and let y represent the dependent quantity, the pounds of food eaten. The constant of proportionality is 8000, meaning that for every day that passes, the pounds of food that the blue whale has eaten increases by 8000. The equation that represents this is $y = 8000x$.

A table of values can be created using an equation.

For example, the equation $y = 8000x$ from the blue whale scenario can be used to create the table of values shown.

Number of Days	Pounds of Food
0	0
1	8000
4	32,000
9	72,000