How to support your student as they learn about **Thinking Proportionally**

Mathematics is a connected set of ideas, and your student knows a lot. Encourage them to use the mathematics they already know when seeing new concepts in this module.

Module Introduction

In this module your student will develop strategies for solving problems involving ratios and proportional relationships. There are 3 topics in this module: *Circles and Ratio*, *Fractional Rates*, and *Proportionality*. Your student will use what they already know about determining equivalent ratios in this module.

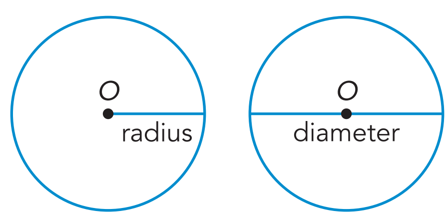
Academic Glossary

Each module will highlight an important term. Knowing and using these terms will help your student think, reason, and communicate their math ideas.

Term	Analyze
Definition	• To study or look closely for patterns. • To break a concept down into smaller parts to gain a better understanding of it.
Questions to Ask Your Student	• Do you see any patterns? • Have you seen something like this before? • What happens if the shape, model, or numbers change?
Related Phrases	• Examine • Evaluate • Determine • Observe • Consider • Investigate • What do you notice?

TABLE OF CONTENTS

Analyze the radius and diameter of Circle O. What do you notice? If the radius measures 3 cm, what will the diameter be? If the diameter measures 20 in., what will the radius be? Do you see a pattern?



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Math Process Standards

Each module will focus on a process (or a pair of processes) that will help your student become a mathematical thinker. The "I can" statements listed below help your student to develop their mathematical learning and understanding.

Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.

I can:

- explain what a problem "means" in my own words.
- create a plan and change it if necessary.
- ask useful questions when trying to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Look for examples of these processes in the Topic Summaries.

The Carnegie Learning Way

Our Institutional Approach

Carnegie Learning's instructional approach is based on how people learn and real-world understandings. It is based on three key components:

ENGAGE	DEVELOP	DEMONSTRATE
Purpose: Provide an introduction that creates curiosity and uses what students already know and have experienced. **Questions to Ask:** How does this problem look like something you did in class?	**Purpose:** Build a deep understanding of mathematics through different activities. **Questions to Ask:** Do you know another way to solve this problem? Does your answer make sense?	**Purpose:** Reflect on and evaluate what was learned. **Questions to Ask:** Is there anything you do not understand?

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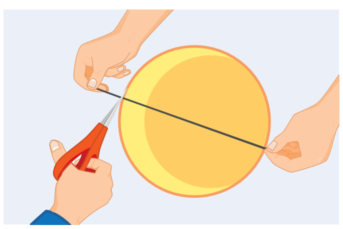
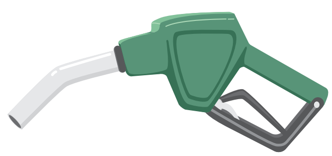
Module Overview

TOPIC 1	TOPIC 2	TOPIC 3
Circles and Ratio	Fractional Rates	Proportionality
9 Days	8 Days	17 Days
Your student will develop formulas for the circumference and area of circles and will develop an understanding of pi (π).	**Your student will write and use unit rates, including those with fractional values.**	**Your student will graph proportional relationships and determine the constant of proportionality.**
Try this at home! Cut a piece of string the length of a circle's diameter and then use that string to measure the **circumference**.  You should see that it takes a little more than 3 times the string's length to measure the circumference.	What in the world? **Unit rates** are used in real life to determine which is the better deal. For example, would you rather pay \$3.05 per gallon of gas or \$2.97 per gallon of gas?  What is the unit rate if you pay \$32 for 10 gallons of gas? [*The unt rate for one gallon of gas is \$3.20.*]	What in the world? If you earn \$15 per hour, then the amount \$15 is the **constant of proportionality**. The amount of money you earn depends on the number of hours you work. money earned = 15 · hours worked  What is the constant of proportionality if you earn \$160 for working an 8 hour day? [*The constant of proportionality is 20.*]

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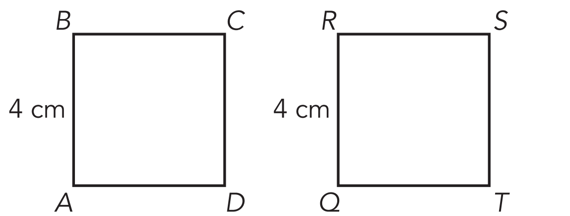
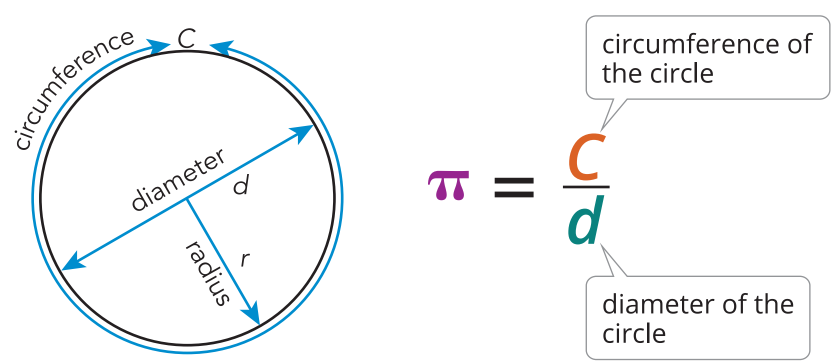
Topic 1: Circles and Ratio

Key Terms	
• congruent • circle • radius • diameter	• circumference • pi • unit rate
Congruent means to have the same size, shape, and measurement. Square *ABCD* is congruent to Square *QRST*. 	A **unit rate** is a comparison of two different measurements in which the numerator or denominator has a value of one unit. The speed 60 miles in 2 hours can be written as a unit rate: $\frac{60\text{ mi}}{2\text{ h}} = \frac{30\text{ mi}}{1\text{ h}}$. The unit rate is 30 miles per hour.
Follow the link to access the Mathematics Glossary: https://www.carnegielearning.com/texas-help/students-caregivers/	

Circles

In this topic, students learn formulas for the **circumference** and area of **circles** and use those formulas to solve mathematical and real-world problems. To fully understand the formulas, students examine the number **pi** (π) as the ratio of a circle's circumference to its **diameter**.



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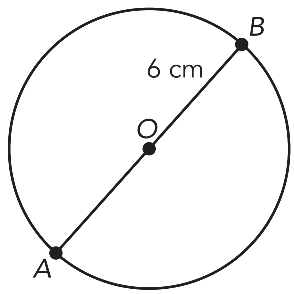
Circumference and Area

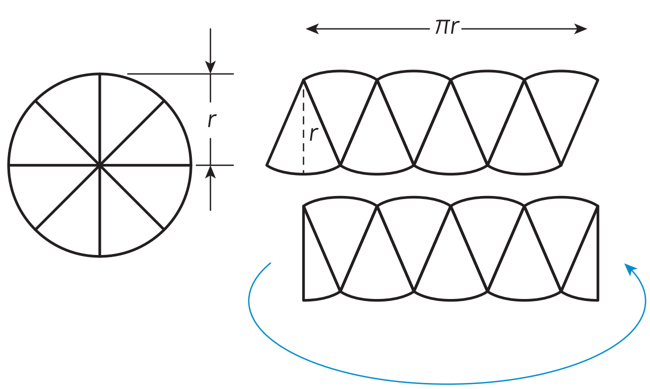
In this topic, students practice using the formulas for the circumference and area of a circle. The distance around a circle is called the circumference of the circle and is calculated using the formula, $C = \pi d$, or $C = 2\pi r$. The formula used to determine the area of a circle is $A = \pi r^2$. Students need to choose the correct formula for a problem based on the information they know and the information they are trying to find.

The diameter of Circle O is 12 centimeters. The circumference of Circle O is 12π centimeters. The area of Circle O is 36π square centimeters.



Modeling the Area of a Circle

Students learn to see how the area of a circle relates to the area of a rectangle. You can divide a circle into a large number of equal-sized pieces. Laying these pieces as shown below, you can see that they almost make the shape of a rectangle. Notice the length of the rectangle and how it relates to what we know about the circle. The area of the rectangle is $l \cdot w = \pi r \cdot r = \pi r^2$. This helps students build the area formula for a circle, πr^2.



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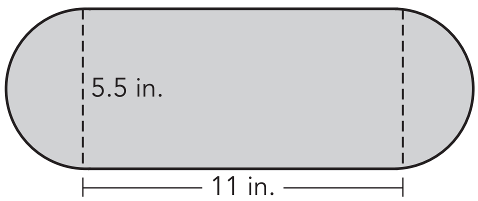
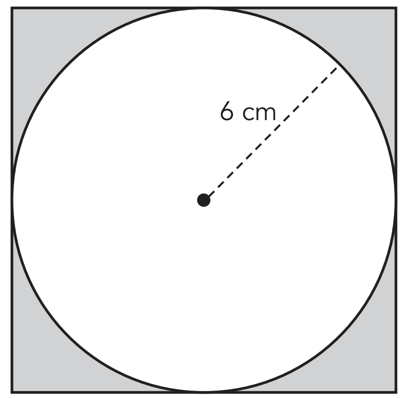
Composite Figures

Finally, students use the formulas to solve different kinds of problems, like calculating the area of composite figures. Students work with composite figures, which are made by putting together different shapes. They add or subtract to find the area of the light or dark part of the image.





Topic 2: Fractional Rates

Key Terms

- complex ratio
- proportion
- variable
- means
- extremes
- solve a proportion
- isolate the variable
- inverse operations

A **variable** is a letter or symbol used to represent a number.

$3x = 81$ $\quad \frac{4}{p} \quad z^2$

variables

In a proportion written $a : b = c : d$, the two values on the outside, a and d, are the **extremes**.

$$\underbrace{7\text{ books} : 14\text{ days} = 3\text{ books} : 6\text{ days}}_{\text{extremes}}$$

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Unit Rates

In this topic, students extend their work with rates to include fractions. To begin the topic, students write, analyze, and use unit rates with whole numbers and fractions to solve problems.

In this example, the unit rate for traveling $\frac{1}{2}$ mile in $\frac{1}{4}$ hour is 2 miles per hour.

$$\frac{\frac{1}{2}}{\frac{1}{4}} \cdot \frac{\frac{4}{1}}{\frac{4}{1}} = \frac{\frac{4}{2}}{1}$$
$$\frac{2}{1} = 2$$

$$\frac{\frac{1}{2}}{\frac{1}{4}} = \frac{1}{2} \div \frac{1}{4}$$
$$= \frac{1}{2} \cdot 4 = 2$$

MATH PROCESS STANDARDS

How do the activities in *Fractional Rates* promote student expertise in the math process standards?

NOTE: This is an example of the math process standard:

Communicate mathematical ideas, reasoning, and their implications using multiple representations including symbols, diagrams, graphs, and language as appropriate.

- I can explain my reasoning and defend my solution.

Refer to page 2 for more "I can" statements.

Two or more ratios or rates can be compared. For example, two friends make lemon-lime punch using the following recipes.

Jade's Recipe	Kim's Recipe
1 cup lemon-lime concentrate 3 cups club soda	2 cups lemon-lime concentrate 5 cups club soda

Which recipe has the stronger taste of lemon-lime? How do you know?

[*Kim's recipe will have a stronger taste of lemon-lime because* $\frac{2}{5} > \frac{1}{3}$.]

Converting Between Systems

Next, students calculate and use unit rates from ratios of fractions. They use unit rates and **proportions** to convert between measurement systems.

To convert between systems, you can scale up or scale down using ratios.

$$\overset{\times 2.5}{\underset{\times 2.5}{\frac{1 \text{ lb}}{0.45 \text{ kg}} = \frac{2.5 \text{ lb}}{1.125 \text{ kg}}}}$$

Write a ratio using the common conversion of 1 lb = 0.45 kg.

Scale up to calculate the number of kilograms in 2.5 pounds.

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Means and Extremes

Finally, students review strategies for solving problems involving equivalent ratios and proportions.

In the proportion $\frac{a}{b} = \frac{c}{d}$, the terms *b* and *c* are called the ***means***, and the terms *a* and *d* are called the *extremes*.

extremes

$$3 : 4 = 9 : 12$$

means

$$(4)(9) = (3)(12)$$

or

$$\frac{3}{4} = \frac{9}{12}$$

means extremes

$$(4)(9) = (3)(12)$$

You can **solve a proportion** for an unknown variable using this method. First, identify the means and extremes. Then, set the product of the means equal to the product of the extremes. Finally, **isolate the variable** to solve for the unknown quantity.

$$\frac{\text{4 cups of granola}}{\text{1.5 cups of raisins}} = \frac{\text{18 cups of granola}}{x}$$

$$(1.5)(18) = (4)(x)$$

$$27 = 4x$$

$$\frac{27}{4} = \frac{4x}{4}$$

$$6.75 = x$$

There will be 6.75 cups of raisins used in 18 cups of granola.

Topic 3: Proportionality

Key Terms	
• proportional relationship • origin	• direct variation • constant of proportionality
A **proportional relationship** is one in which the ratio of the inputs to the outputs is constant. For a relationship to illustrate a proportional relationship, all the ratios $\frac{y}{x}$ or $\frac{x}{y}$ must represent the same constant.	A situation represents a **direct variation** if the ratio between the *y*-value and its corresponding *x*-value is constant for every point. You can say the quantities vary directly. If Melissa earns \$8.25 per hour, then the amount she earns is in direct variation with the number of hours she works. The amount \$8.25 is the **constant of proportionality**.
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Comparing Two Quantities

In this topic, students learn about the constant of proportionality, which is the ratio between the two quantities being compared. They recognize that the constant is connected to how the numbers are placed in order, and they use proportions to write and analyze direct variation equations.

Time (days)	Height of Bamboo (cm)
3	210
10.5	735
18	1260
25.5	1785

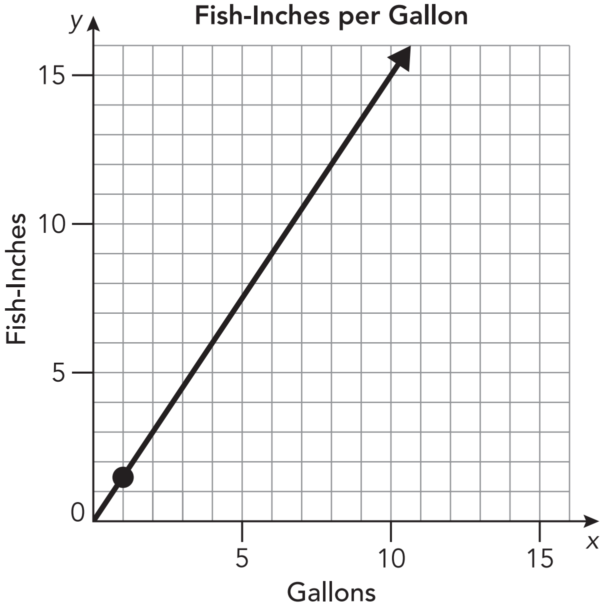
If it took 3 days for the bamboo to grow to 210 centimeters, then the constant of proportionality tells how tall it grew in one day.

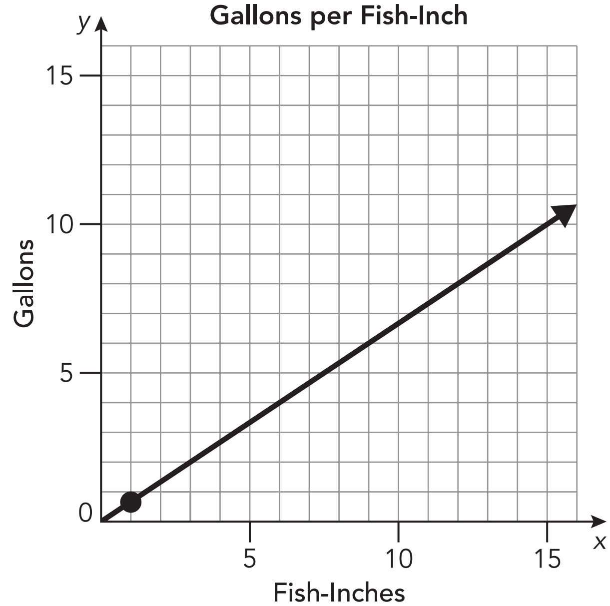
$$\frac{210\text{ cm}}{3\text{ days}} \qquad \frac{210\text{ cm}}{3\text{ days}} \overset{\div 3}{\underset{\div 3}{=}} \frac{7\text{ cm}}{1\text{ day}}$$

The bamboo grew 7 centimeters in one day.

Interpreting the Constant of Proportionality

Students graph proportional relationships and determine the constant of proportionality from the graphs in terms of the problem situation.

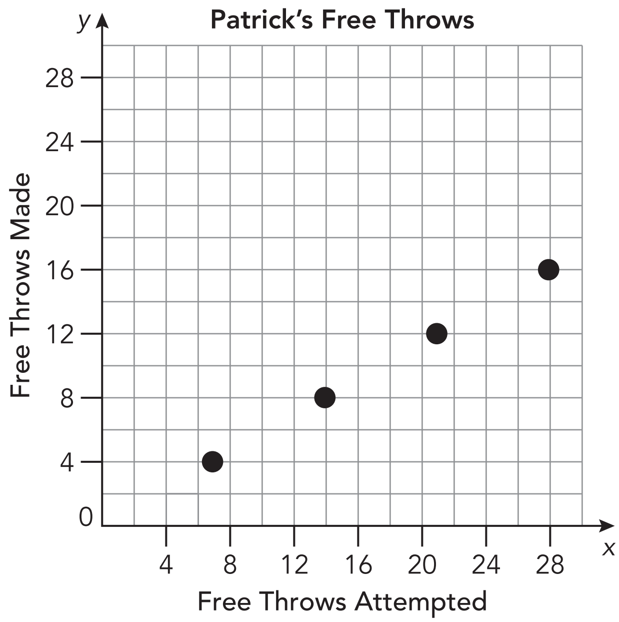




- The point $\left(1, 1\frac{1}{2}\right)$ on the Fish-Inches per Gallon graph represents the unit rate: $1\frac{1}{2}$ fish-inches per gallon.
- The point $\left(1, \frac{2}{3}\right)$ on the Gallons per Fish-Inch graph represents the unit rate: $\frac{2}{3}$ gallon per fish-inch.

Relationships

Students look at relationships in tables, graphs, words, and equations and decide if a relationship is proportional. If a relationship is proportional, students identify and explain the constant of proportionality.



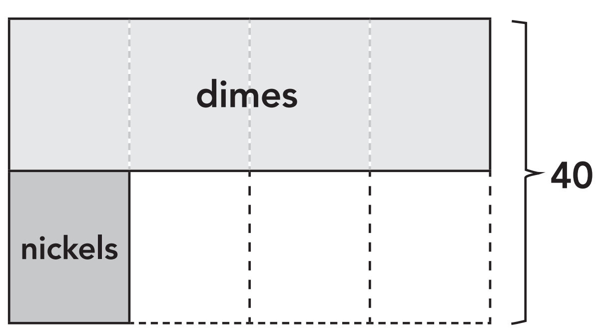
The graph shows Patrick's total number of free throw attempts and the total number of free throws made.

Explain how you know the graph represents a proportional relationship.

Determine the constant of proportionality and describe what it represents in the problem situation.

Using a Diagram to Represent a Proportional Relationship

The diagram shows that there are 4 times as many dimes as nickels. Also, the total number of coins (40) is 8 times the number of nickels and twice the number of dimes.



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Discuss important dates throughout this module such as assessments, assignments, or class events with your student. Use the table to record these dates and reference them as your student progresses through the module.

Important Dates	
Date	Reason

Using the link below, visit the Texas Math Solution Support Center for students and caregivers to access additional resources such as:

- Mathematics Glossaries
- Videos
- Topic Materials
- A Letter to Families and Caregivers

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