Slides, Flips, and Spins

2

Introduction to Rigid Motions

MATERIALS

Patty paper
Protractors
Straightedge
Centimeter rulers

Lesson Overview

Students develop a formal understanding of translations, rotations, and reflections in the plane. The terminology of transformations is introduced, including *pre-image, image, translation, reflection, line of reflection, rotation, center of rotation,* and *angle of rotation*. Students use patty paper to investigate each transformation, create images from pre-images, and determine the properties of each transformation. They learn that each rigid motion transformation preserves the size and shape of the original figure, and that translations and rotations also preserve the orientation of the figure. At the end of the lesson, students state the formal name for transformations that carry figures onto congruent figures and reason that an image of an image of a pre-image is congruent to the pre-image.

Grade 8

Two-Dimensional Shapes

(10) The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

(A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane.

(B) differentiate between transformations that preserve congruence and those that do not.

ELPS

1.A, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E

Essential Ideas

- Transformations are mappings of a plane and all the points of a figure in a plane according to a common action or operation.
- Rigid motions are transformations that preserve segment length, angle measure, and parallelism of segments.
- Translations, rotations, and reflections are rigid motions.
- The pre-image of a figure is the original figure. The image is the result of a transformation.
- Translations “slide” a figure in a given direction by a specific distance.
- Reflections “flip” a figure across a line of reflection.
- Rotations “spin” a figure given a center of rotation, by an angle of rotation, in a given direction.

Lesson Structure and Pacing: 3 Days

Day 1

Engage

Getting Started: Design Competition

Students design a logo for a 5K race using their informal knowledge of transformations. They copy a figure onto a sheet of patty paper and by sliding, flipping, and spinning the figure, they create a logo for the race. They recall that these transformations result in congruent figures.

Develop

Activity 2.1: Translations on the Plane

Students explore properties of translations in the plane. They state the relationship between corresponding angles and side lengths of pre-images and images of a translation, including that parallel sides remain parallel after a translation and that the figures are congruent.

Day 2

Activity 2.2: Reflections on the Plane

Students explore properties of reflections in the plane. They state the relationship between corresponding angles and side lengths of pre-images and images of a reflection, including that parallel sides remain parallel after a reflection and that the figures are congruent. They also learn that reflections change a figure's orientation.

Activity 2.3: Rotations on the Plane

Students explore properties of rotations in the plane. They state the relationship between corresponding angles and side lengths of pre-images and images of a rotation, including that parallel sides remain parallel after a rotation and that the figures are congruent.

Day 3

Activity 2.4: Rigid Motions on the Plane

Students use rigid motions to transform non-geometric objects in the plane using either patty paper or the properties of the transformations. They rotate, reflect, and translate different pictures.

Demonstrate

Talk the Talk: Congruence in Motion

Students identify the specific rigid motions that map a figure onto another figure. They formally state that two-dimensional figures are congruent if one can be obtained from the other through a sequence of translations, reflections, and rotations.

ENGAGE

Getting Started: Design Competition

Facilitation Notes

In this activity, students use patty paper to design a logo by first copying the original figure, then they use rigid motion transformations such as sliding, flipping, and spinning. The terms *plane, transformation*, and *rigid motion* are introduced.

Have students work with partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- Is the original running man congruent to each copy of the running man?
- What is the relationship between the corresponding sides of each running man?
- What is the relationship between the corresponding angles of each running man?

Ask a student to read the definitions and information following Question 2 aloud. Discuss as a class.

Differentiation strategies

- Use the idea of a transformer toy to make a connection to transformations. For example, consider a specific transformer that can be a robot or a spaceship. A rigid motion would be if the robot's body position is kept intact, but you slide, flip or spin the robot around. If you change the robot into a spaceship, that is not a rigid motion because the shape and size of the figure are not preserved. Both cases are examples of transformations, but one demonstrates rigid motions and the other does not.
- Have students create a graphic organizer for the terms throughout this lesson. Have students add information as it is introduced. For example,

Transformations

Preserve Size and Shape | Do Not Preserve Size and Shape

Rigid Motions

Translations | Rotations | Reflections
Slide | Spin | Flip
(preserves orientation)

Summary

Sliding, flipping, and spinning preserve the size and shape of geometric figures.

Activity 2.1
Translations on the Plane

DEVELOP

Facilitation Notes

In this activity, students explore properties of translations in the plane. They state the relationship between corresponding angles and side lengths of pre-images and images of a translation, including that parallel sides remain parallel after a translation and that the figures are congruent.

The Translations Mat located at the end of the lesson provides a place for students to record and organize the findings of their investigations. Provide students with protractors and rulers. Students may wish to use a protractor and a ruler to verify what they notice about the measures of the pre-images and images of each transformation. They will need a ruler for Question 4.

Have students work with partner or in a group to complete Question 1. Share responses as a class.

Questions to ask

- What is the definition of trapezoid?
- Are two sides of the figure parallel? Which sides?
- Are any of the angles congruent? Which angles?
- Why does angle *C* have to be a right angle?

As students work, look for

Confusion between the terms transformation and translation. Using the graphic organizer suggested in the previous activity may help resolve this issue.

Ask a student to read Question 2 and the side note aloud. Discuss the instructions as a class, emphasizing the need to use pencil to transfer the trapezoid onto the Translations Mat. Have students share their translations in their groups and then share responses as a class.

Differentiation strategies

- Assign different groups or different students within a group to use a different line segment on the Translations Mat to get started.
- To extend the activity, have students create a diagonal translation.

Have students work with partner or in a group to complete Question 3. Share responses as a class.

Questions to ask

- Which figure is considered the image? The pre-image?
- Which point of the image maps to point A'?
- Why do you think that corresponding angles are congruent?
- Why do you think that corresponding sides are congruent?
- Why do you think that the sides that were parallel in the pre-image are also parallel in the image?
- Do you think you will get the same results every time you complete a slide of a figure? Why?

Ask a student to read the information and definition following Question 3 aloud. Discuss as a class. Have students work with partner or in a group to complete Questions 4 and 5. Share responses as a class.

Questions to ask

- What do notice about the order in which the vertices are named in the pre-image and in the image?
- Are the corresponding segments congruent?
- Are the corresponding angles congruent?
- Can a figure be translated in any direction?
- What is considered a horizontal translation?
- What is considered a vertical translation?
- Is a translation associated with a slide, a flip, or a spin?
- Do parallel sides remain parallel after a translation?
- Does a translation preserve a figure's size?
- Does a translation preserve a figure's shape?

Summary

A translation is a rigid motion transformation that slides each point of a figure the same distance and direction.

Activity 2.2
Reflections on the Plane

Facilitation Notes

In this activity, students explore properties of reflections in the plane. They state the relationship between corresponding angles and side lengths of pre-images and images of a reflection, including that parallel sides remain parallel after a reflection and that the figures are congruent. They also learn that reflections change a figure's orientation.

The Reflections Mat located at the end of the lesson provides a place for students to record and organize the findings of their investigations. Provide students with at least 3 pieces of patty paper, protractors, and rulers.

Ask a student to read the introduction aloud and complete Question 1 as a class.

Questions to ask

- Do you think this transformation will preserve the size of the trapezoid?
- Do you think this transformation will preserve the shape of the trapezoid?
- Do you think that the sides of the image corresponding to the parallel sides of the pre-image will also be parallel?
- Do you think the orientations of the pre-image and image will be the same?
- What do notice about the order in which the vertices are named in the pre-image and in the image?

Ask another student to read the information following Question 1. Discuss as a class.

Have students work with partner or in a group to complete Questions 2 through 5. Share responses as a class.

As students work, look for

Images traced on the line rather than the same distance from the line as the pre-image.

Differentiation strategies

- Assign different groups or different students within a group to use a different line on the Reflections Mat to get started.
- To extend the activity, have students perform a reflection then translation or translation then reflection and compare images.

Questions to ask

- Which figure is considered the image? The pre-image?
- Does point *A* map onto point *A*′?
- Does point *B* map onto point *B*′?
- Does point *C* map onto point *C*′?
- Does point *D* map onto point *D*′?
- What do you notice about the order in which the vertices are named in the pre-image and in the image?
- Are the corresponding segments congruent?
- Are the corresponding angles congruent?
- Can a figure be reflected across a line of reflection in any direction?
- Is a reflection associated with a slide, a flip, or a spin?
- Do parallel sides remain parallel after a reflection?

Ask a student to read the information and definitions following Question 5. Discuss the instructions and complete Question 6 as a class.

Questions to ask

- Does the location of the figure change as a result of the reflection?
- Does the orientation of the vertices on the figure change as a result of the reflection?
- Do the measurements of the sides change as a result of the reflection?
- Do the measurements of the angles change as a result of the reflection?
- Does a reflection preserve a figure's size? Shape?

Summary

A reflection is a rigid motion transformation that flips a figure across a line of reflection. A line of reflection is a line that acts as a mirror so that corresponding points are the same distance from the line.

Activity 2.3

Rotations on the Plane

Facilitation Notes

In this activity, students explore properties of rotations in the plane. They state the relationship between corresponding angles and side lengths of pre-images and images of a rotation, including that parallel sides remain parallel after a rotation and that the figures are congruent.

The Rotations Mat located at the end of the lesson provides students with a place to record and organize the findings of their investigations. Provide students with patty paper, protractors, and rulers.

Ask a student to read the introduction aloud and complete Question 1 as a class.

Questions to ask

- Do you think this transformation will preserve the size of the trapezoid?
- Do you think this transformation will preserve the shape of the trapezoid?
- Do you think that the sides of the image corresponding to the parallel sides of the pre-image will also be parallel?
- Do you think the orientations of the pre-image and image will be the same?
- What do you notice about the order in which the vertices are named in the pre-image and in the image?

Have students work with partner or in a group to complete Questions 2 through 8. Share responses as a class.

Differentiation strategies

- Assign different groups or different students within a group to complete a different rotation from the same center of rotation: 90° clockwise, 90° counterclockwise, or 180°.
- To extend the activity, have students perform a rotation then reflection or reflection then rotation and compare images.

Questions to ask

- Which figure is considered the image? The pre-image?
- Does point *A* map onto point *A*′?
- Does point *B* map onto point *B*′?
- Does point *C* map onto point *C*′?
- Does point *D* map onto point *D*′?
- What do you notice about the order in which the vertices are named in the pre-image and in the image?
- Are the corresponding segments congruent?
- Are the corresponding angles congruent?
- Do parallel sides remain parallel after a rotation?
- Is a rotation associated with a slide, a flip, or a spin?
- Can a figure be rotated about any point in any direction?
- Can a figure be rotated about a point on the figure itself?

Ask a student to read the information and definitions following Question 8. Discuss the instructions and complete Question 9 as a class.

Questions to ask

- Does the location of the figure change as a result of the the rotation?
- Does the orientation of the vertices on the figure change as a result of the rotation?
- Do the measurements of the sides change as a result of the rotation?
- Do the measurements of the angles change as a result of the rotation?
- Does a rotation preserve a figure's size? Shape?
- Does a rotation preserve a figure's orientation?

Summary

A rotation is a rigid motion transformation that turns a figure in a plane about a fixed point, called the center of rotation, through a given angle, called the angle of rotation. A center of rotation and an angle of rotation, including a direction, are necessary to perform a rotation.

Activity 2.4

Rigid Motions on the Plane

Facilitation Notes

In this activity, students use rigid motions to transform non-geometric objects in the plane using either patty paper or the properties of the transformations. They rotate, reflect, and translate different pictures.

Provide students with patty paper, straightedges, centimeter rulers, and protractors.

Have students work with a partner or in a group to complete Questions 1 through 4. Share responses as a class.

Differentiation strategies

To scaffold support,

- Have students use patty paper to complete all transformations.
- Have students complete one of the questions according to the directions.

Guidance for completing the transformations

Question 1

Students may choose to use patty paper to aid in their rotation: trace Ali on the patty paper, hold a pencil at the center of rotation, rotate the paper 180°, and then draw Ali. To use the properties of rotations, students should draw segments from a number of points on the pre-image of Ali to the the center of rotation, draw a congruent segment the same length along the same line, and mark the corresponding point of the image of Ali. They can repeat this process with sufficient points to obtain a reasonable copy of Ali.

Question 2

Students should decide how far to the right to translate the googly eyes. They should identify a number of points on the pre-image and create segments of the same length in the same direction (parallel to each other) from those points, creating corresponding image points.

Question 3

Students should choose a center of rotation and label it *O*. They can draw segments from points of the letter E to the center of rotation. They should create right angles on each segment at point *O* such that the direction from the segments drawn from *O* to the pre-image to the segments drawn from *O* to the image is clockwise.

Question 4

Students should reflect the running man across a vertical line. They should show the line of reflection. Students should indicate a number of points on the running man, draw segments perpendicular to the line of reflection through the points. They should then measure the distance from the points on the pre-image to the line of reflection, measure the same distance on the other side of the line of reflection, and place the image of the point. They repeat this until they have sufficient points to draw the running man.

Summary

Properties of transformations can be used to rotate, reflect, and translate non-geometric objects on a plane.

DEMONSTRATE

Talk the Talk: Congruence in Motion

Facilitation Notes

In this activity, students identify the specific rigid motions that map a figure onto another figure. They formally state that two-dimensional figures are congruent if one can be obtained from the other through a sequence of translations, reflections, and rotations. Students also informally apply the transitive property to congruent figures.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

As students work, look for

- An informal application of the transitive property to congruent figures.
- Use of multiple transformations. The idea that multiple transformations can often be expressed as a single transformation is addressed in Question 3 and will be revisited later in this Topic.

Questions to ask

- Was Figure *A* translated? How can you tell?
- What information do you need to provide for a translation?
- Was Figure *A* reflected? How can you tell?
- What information do you need to provide for a reflection?
- Was Figure *A* rotated? How can you tell?
- What information do you need to provide for a rotation?
- Is it possible that two different transformations could lead to the same image?
- Could you use a combination of transformations to create that image?
- If the orientation of a figure has not changed from the pre-image to the image, does that provide any information regarding what transformation took place?
- Are the figures that result from the same pre-image congruent to each other? Are they congruent to the pre-image?

Summary

Size and shape are preserved when rigid motion transformations such as rotations, reflections, and translations are applied to geometric and non-geometric objects.

Slides, Flips, and Spins 2

Introduction to Rigid Motions

WARM UP

Draw all lines of symmetry for each letter.

1. A
2. B
3. H
4. X

LEARNING GOALS

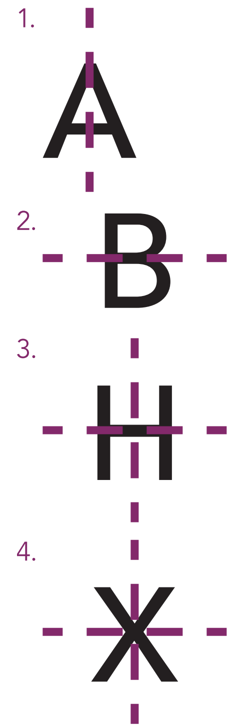
- Model transformations of a plane.
- Translate geometric objects on the plane.
- Reflect geometric objects on the plane.
- Rotate geometric objects on the plane.
- Describe a single rigid motion that maps a figure onto a congruent figure.

KEY TERMS

- plane
- transformation
- rigid motion
- pre-image
- image
- translation
- reflection
- line of reflection
- rotation
- center of rotation
- angle of rotation

When you investigated shapes with patty paper, you used slides, flips, and spins to determine if shapes were congruent. What are the formal names for the actions used to carry a figure onto a congruent figure and what are the properties of those actions?

Warm Up Answers



Answers

1. Answers will vary. Each logo should include an original running man, the running man after a slide in one or a combination of directions, a flip, and a spin.
2. Each copy of the running man has the same size and shape as the original running man. All copies of the running man are congruent.

Getting Started

Design Competition

The Kensington Middle School track club is holding a 5K to raise money for new uniforms. They want to create a logo for the race that includes the running man icon. However, they want the logo to include at least four copies of the running man.

1. Trace the running man onto a sheet of patty paper. Create a logo for the track team on another sheet of patty paper that includes the original running man and three copies, one example each of sliding, flipping, and spinning the picture of the running man.

2. What do you know about the copies of the running man compared with the original picture of the running man?

Each sheet of patty paper represents a model of a geometric *plane*. A **plane** extends infinitely in all directions in two dimensions and has no thickness.

2 • TOPIC 1: Rigid Motion Transformations

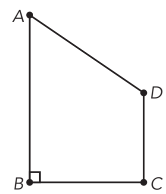
ELL Tip

English Language Learners may need more information to understand what a *rigid motion* is. Add the words *slide, flip,* and *spin* to the board, leaving enough room under each term to add examples. Have one volunteer draw a simple figure under *slide*. Next ask for a volunteer to draw the picture again, after a slide to the right. Continue the activity with *flip* and *spin*.

ACTIVITY
2.1
Translations on the Plane

In this module, you will explore different ways to transform, or change, planes and figures on planes. A **transformation** is the mapping, or movement, of a plane and all the points of a figure on a plane according to a common action or operation. A **rigid motion** is a special type of transformation that preserves the size and shape of the figure. Each of the actions you used to make the running man logo—slide, flip, spin—is a rigid motion transformation.

You are going to start by exploring translations on the plane using the trapezoid shown. Trapezoid *ABCD* has angles *A*, *B*, *C*, and *D*, and sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$.



1. What else do you know about Trapezoid *ABCD*?

2. Use the Translations Mat at the end of the lesson for this exploration.

a. Use a straightedge to trace the trapezoid on the shiny side of a sheet of patty paper.

b. Slide the patty paper containing the trapezoid to align $\overline{AB}$ with one of the segments $\overline{A'B'}$.

c. Record the location of the *image* of Trapezoid *ABCD* on the mat. This image is called Trapezoid *A'B'C'D'*.

Once you have traced the trapezoid on one side, turn the patty paper over and, using a pencil, copy the lines on the back side as well. This will help you to transfer the translated trapezoid back onto the Translations Mat.

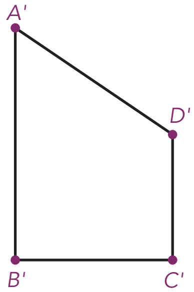
$\overline{AB}$ is read, "line segment *AB*."
A' is read, "*A* prime."

Answers

1. Answers may vary. Trapezoid *ABCD* is a quadrilateral with at least one pair of opposite sides parallel. In this case, $\overline{AB} \parallel \overline{DC}$. In this trapezoid $\angle B$ is a right angle. Students may also reason that $\angle C$ is also a right angle even though it is not marked.
2. Check students' work.

Answers

3a. Students should have a trapezoid labeled as shown on their mats. *A* maps onto *A′*, *B* maps onto *B′*, *C* maps onto *C′*, and *D* maps onto *D′*.



3b. $\overline{AB}$ maps onto $\overline{A'B'}$, $\overline{BC}$ maps onto $\overline{B'C'}$, $\overline{CD}$ maps onto $\overline{C'D'}$, and $\overline{DA}$ maps onto $\overline{D'A'}$.

3c. The corresponding angles have the same measure and are in the same relative order.

3d. The corresponding sides have the same length.

3e. $\overline{A'B'}$ is parallel to $\overline{C'D'}$, just as $\overline{AB}$ is parallel to $\overline{CD}$.

3f. Yes, the image is congruent to the pre-image. All corresponding sides and angles are congruent, so the figures are congruent.

The original trapezoid on the mat is called the **pre-image**.

The traced trapezoid is the **image**. It is the new figure that results from the transformation.

3. Examine your pre-image and image.

a. Which angle in Trapezoid *ABCD* maps to each angle of Trapezoid *A′B′C′D′*? Label the vertices on your drawing of the image of Trapezoid *ABCD*.

b. Which side of Trapezoid *ABCD* maps to each side of Trapezoid *A′B′C′D′*?

c. What do you notice about the measures of the corresponding angles in the pre-image and the image?

d. What do you notice about the lengths of the corresponding sides in the pre-image and the image?

e. What do you notice about the relationship of $\overline{A'B'}$ to $\overline{C'D'}$? How does this relate to the corresponding sides of the pre-image?

f. Is the image congruent to the pre-image? Explain your reasoning.

This type of movement of a plane containing a figure is called a *translation*. A **translation** is a rigid motion transformation that "slides" each point of a figure the same distance and direction. Let's verify this definition.

4. On the mat, draw segments to connect corresponding vertices of the pre-image and image.

a. Use a ruler to measure each segment. What do you notice?

b. Compare your translations and measures with your classmates' translations and measures. What do you notice?

5. Consider the translation you created, as well as your classmates' translations.

a. What changes about a figure after a translation?

b. What stays the same about a figure after a translation?

c. What information do you need to perform a translation?

A figure can be translated in any direction. Two special translations are vertical and horizontal translations. Sliding a figure only left or right is a horizontal translation, and sliding it only up or down is a vertical translation.

Answers

4a. Measures will vary based upon what segment was used on the Translations Mat. All of the segment lengths on a specific Translations Mat should be the same.

4b. Answers will vary. We drew our images in different places on the mat, so the measures of the segments are different; however, on each of our mats, the distance between corresponding vertices is the same.

5a. The figure's location on the plane changes.

5b. All measures and the orientation remain the same. Translations preserve a figure's size, shape, and orientation. Also, parallel lines in the pre-image map to parallel lines in the image.

5c. I would need to know the direction and distance desired for the translation.

Answers

1. Conjectures will vary.
2. Check students' work.

3a. The corresponding angles have the same measure.

3b. The corresponding sides have the same length.

NOTES

ACTIVITY 2.2

Reflections on the Plane

The first transformation you explored was a translation. Now, let's see what happens when you flip, or reflect, the trapezoid. Trace Trapezoid *ABCD* onto a sheet of patty paper. Imagine tracing the trapezoid on one side of the patty paper, folding the patty paper in half, and tracing the trapezoid on the other half of the patty paper.

1. Make a conjecture about how the image and pre-image will be alike and different.

To verify or refine your conjecture, let's explore a reflection using patty paper and the Reflections Mat located at the end of the lesson. Trace the trapezoid from the previous activity on the lower left corner of a new piece of patty paper.

2. Align the trapezoid on the patty paper with the trapezoid on the Reflections Mat. Fold the patty paper along ℓ_1. Trace the trapezoid on the other side of the crease and transfer it onto the Reflections Mat. Label the vertices of the image, Trapezoid *A'B'C'D'*.

3. Compare the pre-image and image that you created.

a. What do you notice about the measures of the corresponding angles in the pre-image and the image?

b. What do you notice about the lengths of the corresponding sides in the pre-image and the image?

c. **What do you notice about the relationship of $\overline{A'B'}$ to $\overline{C'D'}$? How does this relate to the corresponding sides of the pre-image?**

d. **Is the image congruent to the pre-image? Explain your reasoning.**

e. **Draw segments connecting corresponding vertices of the pre-image and image. Measure the lengths of these segments and the distance from each vertex to the fold. What do you notice?**

Notice that the segments you drew are perpendicular to the crease of the patty paper. Why do you think this is true?

4. **Repeat the reflection investigation using Trapezoid *ABCD* and folding along ℓ_2. Record your observations.**

5. **Repeat the reflection investigation using Trapezoid *ABCD* and folding along ℓ_3. Record your observations.**

Answers

3c. $\overline{A'B'}$ is parallel to $\overline{C'D'}$, just as $\overline{AB}$ is parallel to $\overline{CD}$.

3d. Yes, the image is congruent to the pre-image. All corresponding sides and angles are congruent, so the figures are congruent.

3e. Measures will vary by student. The segments connecting corresponding vertices of the pre-image and image will not have the same measure but the distance from a vertex of the pre-image to the fold is the same as the distance from the corresponding vertex of the image to the fold.

4. See Question 3.

5. See Question 3.

Answers

6a. The figure changes orientation of the vertices and location of the figure in the plane.

6b. All measures remain the same. Reflections preserve a figure's size and shape. Also, parallel lines in the pre-image map onto parallel lines in the image.

6c. I would need to know the line of reflection.

Answers

1. Conjectures will vary.

This type of movement of a plane containing a figure is called a *reflection*. A **reflection** is a rigid motion transformation that "flips" a figure across a *line of reflection*. A **line of reflection** is a line that acts as a mirror so that corresponding points are the same distance from the line.

6. Consider the reflections you created.

a. What changes about a figure after a reflection?

b. What stays the same about a figure after a reflection?

c. What information do you need to perform a reflection?

ACTIVITY 2.3 Rotations on the Plane

You have now investigated translating and reflecting a trapezoid on the plane. Let's see what happens when you spin, or rotate, the trapezoid. You are going to use the Rotations Mat found at the end of the lesson for this investigation.

Trace Trapezoid *ABCD* onto the center of a sheet of patty paper. Imagine spinning the patty paper so that the trapezoid is no longer aligned with the trapezoid on the mat.

1. Make a conjecture about how the image and pre-image will be alike and different.

Let's investigate with patty paper to verify or refine your conjecture.

2. Align your trapezoid with the trapezoid on the Rotations Mat.

Put your pencil on point O_1 and spin the patty paper 90° in a clockwise direction.

Then copy the rotated trapezoid onto the Rotations Mat and label the vertices.

3. Compare the pre-image and image created by the rotation.

a. What do you notice about the measures of the corresponding angles in the pre-image and the image?

b. What do you notice about the lengths of the corresponding sides in the pre-image and the image?

c. What do you notice about the relationship of $\overline{A'B'}$ to $\overline{C'D'}$? How does this relate to the corresponding sides of the pre-image?

d. Is the image congruent to the pre-image? Explain your reasoning.

4. Draw two segments: one to connect point O_1 to A and another to connect point O_1 to A'.

a. Measure the lengths of these segments. What do you notice?

b. Measure the angle formed by the segments. What do you notice?

NOTES

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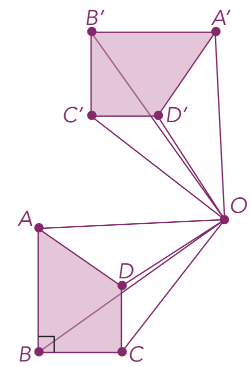
Answers

2. Check students' work.

3a. The corresponding angles have the same measure and are in the same relative order.

3b. The corresponding sides have the same length.



3c. $\overline{A'B'}$ is parallel to $\overline{C'D'}$, just as *AB* is parallel to *CD*.

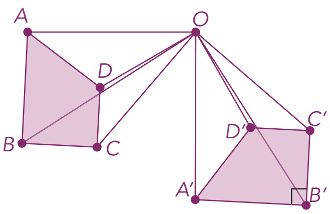
3d. Yes, the image is congruent to the pre-image. All corresponding sides and angles are congruent, so the figures are congruent.

4a. The segments joining corresponding vertices to point *O* have the same measure.

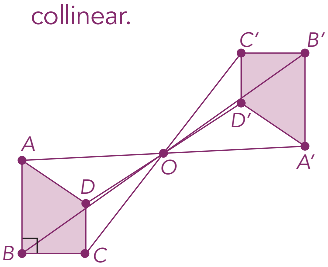
4b. The angle formed with point *O* and corresponding vertices is 90°.

Answers

5. See Question 4a and 4b.
6. See Question 4a.
7. All of the observations will be the same as in Questions 3 and 4.



8. All of the observations will be the same as in Questions 3 and 4a. The only difference is that each angle formed by point *O* and the corresponding vertices is 180°. This means that corresponding vertices and point *O* are collinear.



5. Repeat the process from the previous question with *B* and *B*'. What do you notice about the segment lengths and angle measures?

6. What do you think is true about the segments connecting *C* and *C*' and the segments connecting *D* and *D*'?

7. Repeat the rotations investigation using Trapezoid *ABCD* and spinning the patty paper 90° in a counterclockwise direction around O_1. Record your observations.

8. Repeat the rotations investigation using Trapezoid *ABCD* and spinning the patty paper 180° around O_3. Record your observations.

This type of movement of a plane containing a figure is called a *rotation*. A **rotation** is a rigid motion transformation that turns a figure on a plane about a fixed point, called the **center of rotation**, through a given angle, called the **angle of rotation**. The center of rotation can be a point outside the figure, inside the figure, or on the figure.

Rotations can be clockwise or counterclockwise.

9. Consider the rotations you created.

a. Describe the centers of rotation used for each investigation.

b. How do you identify the angle of rotation, including the direction, in your patty paper rotations?

c. What changes about a figure after a rotation?

d. What stays the same about a figure after a rotation?

e. What information do you need to perform a rotation?

Answers

9a. The center of rotation was point O_1 for the 90° clockwise rotation, O_1 for the 90° counterclockwise rotation, and O_3 for the 180° rotation.

9b. The angle formed with point O and a set of corresponding vertices is the angle of rotation. I can tell if the rotation was clockwise or counterclockwise by determining the direction from the segment containing a vertex of the pre-image to the segment containing a vertex of the image. For example, if I would turn clockwise to move from $\overline{OA}$ to $\overline{OA'}$, then the rotation was clockwise.

9c. The location of a figure on the plane changes after a rotation.

9d. All measures remain the same. Rotations preserve a figure's size and shape. Also, parallel lines in the pre-image map onto parallel lines in the image.

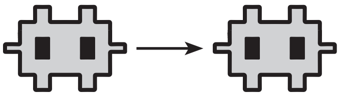
9e. I need a center of rotation and an angle of rotation including a direction.

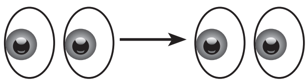
ELL Tip

To add background context for the terms defined, ask students about things that they see on a regular basis that rotate. Encourage students to find things within the classroom that rotate. Draw attention to the clock. The hands on the clock rotate. Ask students if they can determine the center of rotation of the hands on the clock. Ask students if they could determine an angle of rotation on the clock between two numbers.

Answers

1. Answer will vary based on the placement of the center of rotation.



2. 

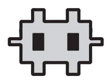
NOTES

ACTIVITY 2.4

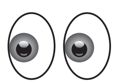
Rigid Motions on the Plane

Use your investigations about the properties of rigid motions to complete each transformation.

1. Rotate Ali the Alien 180°. Be sure to identify your center of rotation.



2. Translate the googly eyes horizontally to the right.

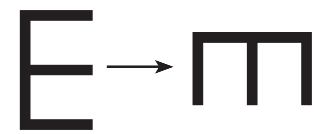


3. **Rotate the letter E 90° clockwise. Be sure to identify your center of rotation.**

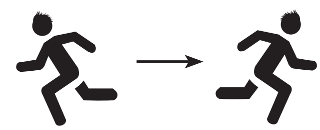
4. **Transform the running man so that he is running in the opposite direction.**

Answers

3. Answer will vary based on the placement of the center of rotation.



4.



Answers

1a. Translate Figure *A* up and to the right to Figure *B*.

1b. Reflect Figure *A* across a vertical line of reflection between Figures *A* and *C*.

1c. Rotate Figure *A* counterclockwise 90 degrees to Figure *E*.

1d. Reflect Figure *C* across a horizontal line of reflection between Figures *C* and *D*.

2. The images that result from the same pre-image are congruent to each other and to the pre-image. When I perform a rigid motion, size and shape are preserved, so the measures all remain the same.

3a. Figure *A* must be congruent to Figure *D*.

3b. Sample answer. I could use the transformations from Figure *A* to Figure *C* and from Figure *C* to Figure *D*. Reflect Figure *A* across a vertical line of reflection between Figures *A* and *C*. Reflect Figure *C* across a horizontal line of reflection between Figures *C* and *D*.

3c. I could rotate Figure *A* 180 degrees to map onto Figure *D*.

NOTES

TALK the TALK

Congruence in Motion

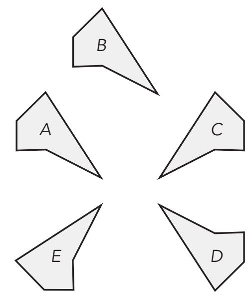
1. Describe a transformation that maps one figure onto the other. Be as specific as possible.

a. Figure *A* onto Figure *B*

b. Figure *A* onto Figure *C*

c. Figure *A* onto Figure *E*

d. Figure *C* onto Figure *D*



2. Explain what you know about the images that result from translating, reflecting, and rotating the same pre-image. How are the images related to each other and to the pre-image?

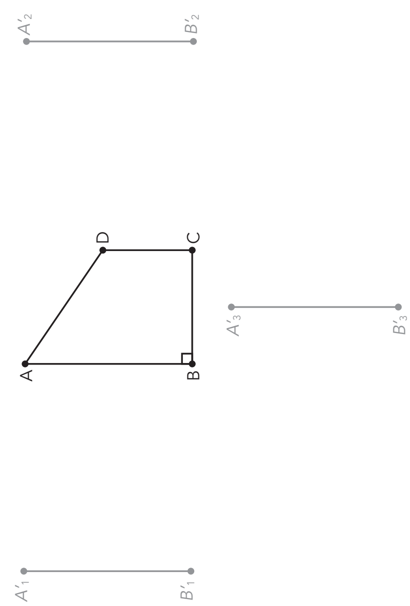
3. If Figure *A* is congruent to Figure *C* and Figure *C* is congruent to Figure *D*, answer each question.

a. What is true about the relationship between Figures *A* and *D*?

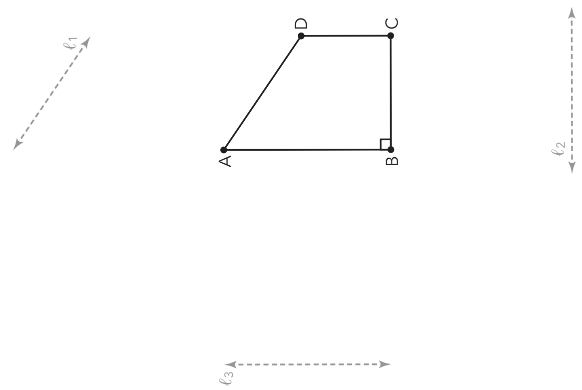
b. How could you use multiple transformations to map Figure *A* onto Figure *D*?

c. How could you use a single transformation to map Figure *A* onto Figure *D*?

Translations Mat



Reflections Mat



Rotations Mat