Mirror, Mirror

4

Reflections of Figures on the Coordinate Plane

MATERIALS

Patty paper

Lesson Overview

Students use patty paper to explore reflections of various figures on a coordinate plane. They then generalize about the effects reflecting a figure has on its coordinates. Students verify that two figures are congruent by describing a sequence of translations and reflections that map one figure onto another.

Grade 8

Two-Dimensional Shapes

(10) The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

(A) generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane.

(C) explain the effect of translations, reflections over the *x*- or *y*-axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation.

ELPS

1.A, 1.D, 1.E, 1.G, 2.C, 2.D, 2.G, 2.H, 2.I, 3.A, 3.B, 3.C, 3.D, 3.F, 4.A, 4.B, 4.C, 4.G, 4.K, 5.E

Essential Ideas

- A reflection is a transformation that flips a figure across a reflection line.
- A reflection line is a line that acts as a mirror or perpendicular bisector so that corresponding points are the same distance from the mirror.
- When a geometric figure is reflected across the *y*-axis to form its image, the *x*-values of the ordered pairs of the vertices of the pre-image become opposites and the *y*-values of the ordered pairs of the pre-image remain the same.
- When a geometric figure is reflected across the *x*-axis to form its image, the *y*-values of the ordered pairs of the vertices of the pre-image become opposites and the *x*-values of the ordered pairs of the pre-image remain the same.
- A point with the coordinates (x, y), when reflected across the *x*-axis, has new coordinates $(x, -y)$. A point with the coordinates (x, y), when reflected across the *y*-axis, has new coordinates $(-x, y)$.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Ambulance

Students use informal language about reflections to describe why the word *ambulance* is reversed on the front of ambulances.

Develop

Activity 4.1: Modeling Reflections on the Coordinate Plane

Students reflect figures across the *x*- and *y*-axis using patty paper. Students record the coordinates of the original (pre-image) and reflected (image) figures and explore how the reflection affected the coordinates of the pre-image. The first figure is placed entirely in the first quadrant. The second figure has vertices situated in each quadrant, and students are asked to make conjectures prior to labeling points.

Day 2

Activity 4.2: Reflecting Any Points on the Coordinate Plane

A point with the coordinates (*x*, *y*) is located in the first quadrant. Students reflect the point across the *x*-axis and the *y*-axis and record the coordinates of the images. Next, they are given the coordinates of three vertices of a triangle and graph the triangle. Using reflections across the *x*-axis and *y*-axis, students form two different triangles and record the coordinates of the vertices of the images. Finally, they are given the coordinates of the vertices of a triangle, and without graphing they determine the coordinates of images resulting from different reflections.

Activity 4.3: Verifying Congruence Using Reflections and Translations

Students solve problems to demonstrate that two figures are congruent if the same sequence of reflections and translations maps all the points of one figure onto all the points of the other figure.

Demonstrate

Talk the Talk: Reflecting on Reflections

Students summarize that a reflection of a figure across the *x*-axis multiplies the *y*-coordinate of each point of the figure by −1, while leaving the *x*-coordinates the same. A reflection of a figure across the *y*-axis multiplies the *x*-coordinate of each point of the figure by −1, while leaving the *y*-coordinates the same.

Getting Started: Ambulance

ENGAGE

Facilitation Notes

In this activity, students use informal language about reflections to describe a real-world situation.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- Would you describe the word *ambulance* as written upside down?
- Would you describe the word *ambulance* as written backwards? Or is each letter written backwards?
- Is the order of the lettering reversed?
- Does any letter in the alphabet appear the same when it is written backwards?
- How does viewing a written word in a mirror appear?

Summary

Real-world situations involving reflections may include the use of mirrors, viewing scopes, or camera lenses.

Activity 4.1

Modeling Reflections on the Coordinate Plane

DEVELOP

Facilitation Notes

In this activity, students reflect figures across the *x*- and *y*-axis using patty paper. Students record the coordinates of the original (pre-image) and reflected (image) figures and explore how the reflection affected the coordinates of the pre-image. The first figure is placed entirely in the first quadrant, and the second figure has vertices situated in each quadrant.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Misconception

It is sometimes counter-intuitive to students that reflecting a coordinate point across the *x*-axis changes the sign of the *y*-coordinate and that reflecting a coordinate point across the *y*-axis changes the sign of the *x*-coordinate. Remind students that they can always test a point to check their thinking.

Questions to ask

- When the figure was reflected across the *x*-axis, did the reflection change the *x*-coordinate of each vertex?
- When the figure was reflected across the *x*-axis, did the reflection change the *y*-coordinate of each vertex?
- Which coordinate in every point of the pre-image changed as a result of the reflection across the *x*-axis?
- When the figure was reflected across the *y*-axis, did the reflection change the *x*-coordinate of each vertex?
- When the figure was reflected across the *y*-axis, did the reflection change the *y*-coordinate of each vertex?
- Which coordinate in every point of the pre-image changed as a result of the reflection across the *y*-axis?
- How do you think the coordinates would change if the figure from Quadrant II was reflected into Quadrant III?

Have students work with a partner or in a group to complete Questions 3 through 6. Share responses as a class.

Questions to ask

- When reflecting points across the *x*-axis, what quadrant do points in Quadrant I map onto? Across the *y*-axis?
- When reflecting points across the *x*-axis, what quadrant do points in Quadrant II map onto? Across the *y*-axis?
- When reflecting points across the *x*-axis, what quadrant do points in Quadrant III map onto? Across the *y*-axis?
- When reflecting points across the *x*-axis, what quadrant do points in Quadrant IV map onto? Across the *y*-axis?
- When reflecting points across the *x*-axis, what is a general rule for how the coordinates from the pre-image are affected regardless of the quadrant a point of the pre-image lies in? What evidence do you have to support your response?
- When reflecting points across the *y*-axis, what is a general rule for how the coordinates from the pre-image are affected regardless of the quadrant a point of the pre-image lies in?

Summary

When a figure is reflected across the *x*-axis, the *x*-coordinates of each point remain the same and the *y*-coordinates of the original (pre-image) are written as the opposite value. When a figure is reflected across the *y*-axis, the *y*-coordinates of each point remain the same and the *x*-coordinates of the original (pre-image) are written as the opposite value.

Activity 4.2
Reflecting Any Points on the Coordinate Plane

Facilitation Notes

In this activity, students reflect points and figures across the *x*-axis and the *y*-axis and note how the reflections alter the coordinates. Using this information, they determine the coordinates of images resulting from different reflections without graphing the pre-image or image.

Have students work with a partner or in a group to complete Questions 1 through 3. Share responses as a class.

Questions to ask

- When the figure was reflected across the *x*-axis, did the reflection change the *x*-coordinate of each vertex?
- When the figure was reflected across the *x*-axis, did the reflection change the *y*-coordinate of each vertex?
- Which coordinate in every point of the pre-image changed as a result of the reflection across the *x*-axis?
- When the figure was reflected across the *y*-axis, did the reflection change the *x*-coordinate of each vertex?
- When the figure was reflected across the *y*-axis, did the reflection change the *y*-coordinate of each vertex?
- Which coordinate in every point of the pre-image changed as a result of the reflection across the *y*-axis?

Have students work with a partner or in a group to complete Question 4. Share responses as a class.

Questions to ask

- If the point (x, y) is reflected across the *x*-axis, which coordinate changes to the opposite value?
- If the point (x, y) is reflected across the *y*-axis, which coordinate changes to the opposite value?
- If the reflection of a point (x, y) is the point $(-x, y)$, what do you know about the reflection?
- If the reflection of a point (x, y) is the point $(x, -y)$, what do you know about the reflection?

Summary

A point with the coordinates (x, y) reflected across the *x*-axis becomes the point with the coordinates $(x, -y)$. When a point is reflected across the *y*-axis, it becomes the point with the coordinates $(-x, y)$.

Activity 4.3

Verifying Congruence Using Reflections and Translations

Facilitation Notes

In this activity, students examine the change in *x*- and *y*-coordinates to determine the congruence of geometric figures. They decide if a sequence of transformations can be used to prove the congruence of figures shown on a graph and then describe that sequence of rigid motions.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Differentiation strategies

Confusion may arise in Question 2 because the vertices to the images are not labeled. To scaffold support for students,

- Suggest they use visual tactics by looking at general side lengths and angle measures to determine corresponding vertices.
- Insert labels for the vertices of the images.

Questions to ask

- How did you determine what vertex in the pre-image corresponded to what vertex in the image?
- Was this figure reflected across the *x*-axis or the *y*-axis? How do you know?
- Was a translation involved in this situation? How do you know?
- What is the *x*-coordinate of each point in the pre-image? What is the *x*-coordinate of each point in the image?
- What is the *y*-coordinate of each point in the pre-image? What is the *y*-coordinate of each point in the image?
- Is there another set of rigid motions that would map the pre-image to the image?
- Does it matter what order you perform the rigid motions? Why do you think that is the case?

Differentiation strategy

To extend the activity, use Question 2 part (a) to make the point that reflections may occur across any line; they are not limited to being across an axis. In Question 2 part (a), a correct answer could be a reflection across the line $y = \frac{1}{2}$. The *Stretch* at the end of this lesson provides two examples of reflections across lines other than the axes. Remind students that they used various lines to reflect on the Reflections Map in Lesson 2, and the same may occur on the coordinate plane.

Summary

Two figures are congruent if the same sequence of reflections and translations moves all the points of one figure onto all the points of the other figure.

Talk the Talk: Reflecting on Reflections

DEMONSTRATE

Facilitation Notes

In this activity, students summarize how a reflection across each of the axes affects the coordinates of points on the graph.

Have students work with a partner or in a group to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- If the point (x, y) is reflected across the x-axis, which coordinate changes to the opposite value?
- If the point (x, y) is reflected across the y-axis, which coordinate changes to the opposite value?
- If the reflection of a point (x, y) is the point $(-x, y)$, what do you know about the reflection?
- If the reflection of a point (x, y) is the point $(x, -y)$, what do you know about the reflection?
- Does the point $(x, -y)$ always mean to make y a negative number? Explain.

Summary

The reflection of a figure across the x-axis multiplies the y-coordinate of each point of the figure by -1, while leaving the x-coordinates the same. A reflection of a figure across the y-axis multiplies the x-coordinate of each point of the figure by -1, while leaving the y-coordinates the same.

NOTES

Mirror, Mirror 4

Reflections of Figures on the Coordinate Plane

WARM UP

Determine each product.

1. -1×6
2. $-\frac{3}{5}(-1)$
3. -1×4.33
4. $4h(-1)$

LEARNING GOALS

- Reflect geometric figures on the coordinate plane.
- Identify and describe the effect of geometric reflections on two-dimensional figures using coordinates.
- Identify congruent figures by obtaining one figure from another using a sequence of translations and reflections.

You have learned to model transformations, such as translations, rotations, and reflections. How can you model and describe these transformations on the coordinate plane?

Warm Up Answers

1. -6
2. $\frac{3}{5}$
3. -4.33
4. $-4h$

ELL Tip

Ask the groups to pass a mirror around and look at themselves in the mirror. Ask the students if they notice anything about their reflection in the mirror, perhaps the writing on their T-shirt. Ask if students know why the writing on a T-shirt would look backward in the mirror. Have students discuss their observations. This will help introduce this lesson and will give English Language Learners another opportunity to explain things in their own words.

Answers

1. Answers will vary.
2. Check students' drawings.

Getting Started

Ambulance

The image shows the front of a typical ambulance.

1. Why does the word *ambulance* appear like this on the front?

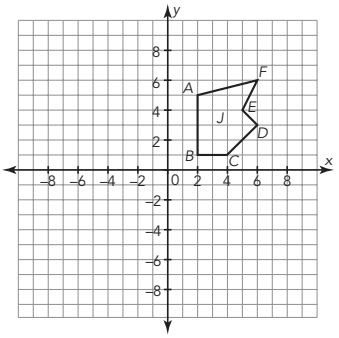
2. Suppose you are going to replace the word *ambulance* with your name. Write your name as it appears on the front of the vehicle. How can you check that it is written correctly?

ACTIVITY
4.1

Modeling Reflections on the Coordinate Plane

In this activity, you will reflect pre-images across the *x*-axis and *y*-axis and explore how the reflection affects the coordinates.

1. Place patty paper on the coordinate plane, trace Figure *J*, and copy the labels for the vertices on the patty paper.



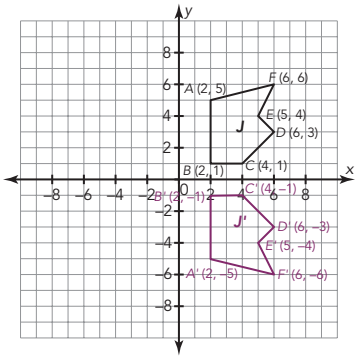
a. Reflect the Figure *J* across the *x*-axis. Then, complete the table with the coordinates of the reflected figure.

Coordinates of *J*	Coordinates of *J'* Reflected Across *x*-Axis
A (2, 5)	
B (2, 1)	
C (4, 1)	
D (6, 3)	
E (5, 4)	
F (6, 6)	

b. Compare the coordinates of Figure *J'* with the coordinates of Figure *J*. How are the values of the coordinates the same? How are they different? Explain your reasoning.

Answers

1a.

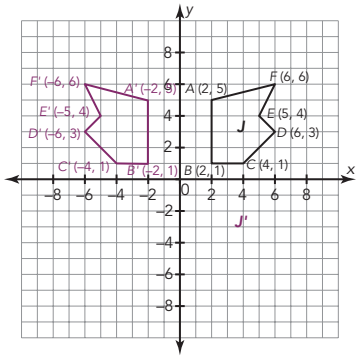


Coordinates of *J*	Coordinates of *J'* Reflected Across *x*-Axis
A (2, 5)	*A'* (2, −5)
B (2, 1)	*B'* (2, −1)
C (4, 1)	*C'* (4, −1)
D (6, 3)	*D'* (6, −3)
E (5, 4)	*E'* (5, −4)
F (6, 6)	*F'* (6, −6)

1b. The *x*-coordinates are all the same. The *y*-coordinates for the image are opposites of the *y*-coordinates for the pre-image.

Answers

2a.



Coordinates of *J*	Coordinates of *J*″ Reflected Across *y*-Axis
A (2, 5)	*A*″ (−2, 5)
B (2, 1)	*B*″ (−2, 1)
C (4, 1)	*C*″ (−4, 1)
D (6, 3)	*D*″ (−6, 3)
E (5, 4)	*E*″ (−5, 4)
F (6, 6)	*F*″ (−6, 6)

2b. The *x*-coordinates for the image are opposites of the *x*-coordinates for the pre-image. The *y*-coordinates are all the same.

NOTES

2. Reflect Figure *J* across the *y*-axis.

a. Complete the table with the coordinates of the reflected figure.

Coordinates of *J*	Coordinates of *J*″ Reflected Across *y*-Axis
A (2, 5)	
B (2, 1)	
C (4, 1)	
D (6, 3)	
E (5, 4)	
F (6, 6)	

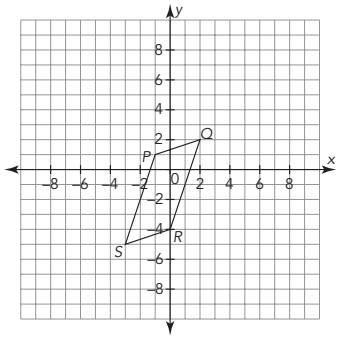
b. Compare the coordinates of Figure *J*″ with the coordinates of Figure *J*. How are the values of the coordinates the same? How are they different? Explain your reasoning.

Let's consider a new figure situated differently on the coordinate plane.

3. Reflect Quadrilateral *PQRS* across the *x*-axis.

Make a conjecture about the ordered pairs for the reflection of the quadrilateral across the *x*-axis.

Make a conjecture, investigate, and then use the results to verify or justify your conjecture.



4. Use patty paper to test your conjecture.

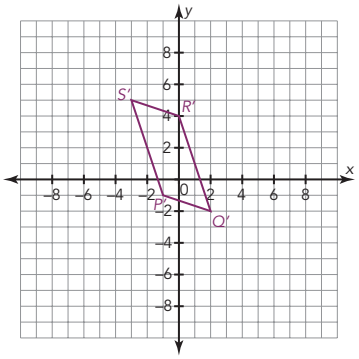
a. Complete the table with the coordinates of the reflection.

Coordinates of Quadrilateral *PQRS*	Coordinates of Quadrilateral *P'Q'R'S'* Reflected Across the *x*-Axis
P (−1, 1)	
Q (2, 2)	
R (0, −4)	
S (−3, −5)	

b. Compare the coordinates of Quadrilateral *P'Q'R'S'* with the coordinates of Quadrilateral *PQRS*. How are the values of the coordinates the same? How are they different? Explain your reasoning.

Answers

3.



Conjectures may vary.

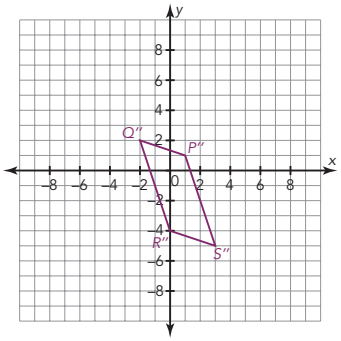
4a.

Coordinates of Quadrilateral *PQRS*	Coordinates of Quadrilateral *P'Q'R'S'* Reflected Across the *x*-Axis
P (−1, 1)	*P'* (−1, −1)
Q (2, 2)	*Q'* (2, −2)
R (0, −4)	*R'* (0, 4)
S (−3, −5)	*S'* (−3, 5)

4b. The *x*-coordinates are all the same. The *y*-coordinates for the image are opposites of the *y*-coordinates for the pre-image.

Answers

5.



5a. Answers will vary.

5b.

Coordinates of Quadrilateral *PQRS*	Coordinates of Quadrilateral *P″Q″R″S″* Reflected Across the *y*-Axis
P (−1, 1)	*P″* (1, 1)
Q (2, 2)	*Q″* (−2, 2)
R (0, −4)	*R″* (0, −4)
S (−3, −5)	*S″* (3, −5)

6. The *x*-coordinates for the image are opposites of the *x*-coordinates for the pre-image. The *y*-coordinates are all the same.

5. Reflect Quadrilateral *PQRS* across the *y*-axis.

a. Make a conjecture about the ordered pairs for the reflection of the quadrilateral across the *y*-axis.

b. Use patty paper to test your conjecture. Complete the table with the coordinates of the reflection.

Coordinates of Quadrilateral *PQRS*	Coordinates of Quadrilateral *P″Q″R″S″* Reflected Across the *y*-Axis
P (−1, 1)	
Q (2, 2)	
R (0, −4)	
S (−3, −5)	

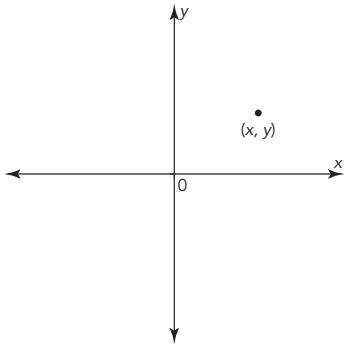
6. Compare the coordinates of Quadrilateral *P″Q″R″S″* with the coordinates of Quadrilateral *PQRS*. How are the values of the coordinates the same? How are they different? Explain your reasoning.

ACTIVITY
4.2

Reflecting Any Points on the Coordinate Plane

NOTES

Consider the point (x, y) located anywhere in the first quadrant.



1. Use the table to record the coordinates of each point.

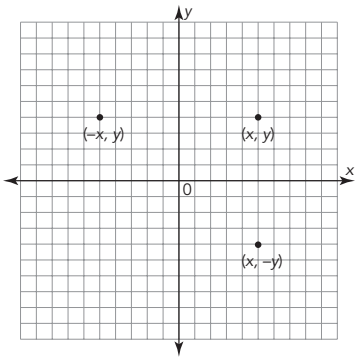
a. Reflect and graph the point (x, y) across the x-axis on the coordinate plane. What are the new coordinates of the reflected point in terms of x and y?

b. Reflect and graph the point (x, y) across the y-axis on the coordinate plane. What are the new coordinates of the reflected point in terms of x and y?

Original Point	Reflection Across the x-Axis	Reflection Across the y-Axis
(x, y)		

Answers

1a.

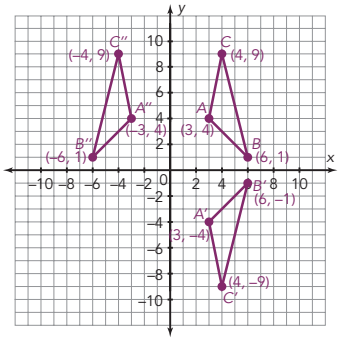


1b.

Reflection Across the x-Axis	Reflection Across the y-Axis
$(x, -y)$	$(-x, y)$

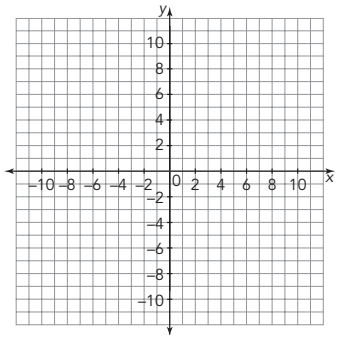
Answers

2.



3a. See table below.

2. Graph Δ*ABC* by plotting the points *A* (3, 4), *B* (6, 1), and *C* (4, 9).



3. Use the table to record the coordinates of the vertices of each triangle.

a. Reflect Δ*ABC* across the *x*-axis to form Δ*A′B′C′*. Graph the triangle and then list the coordinates of the reflected triangle.

b. Reflect Δ*ABC* across the *y*-axis to form Δ*A″B″C″*. Graph the triangle and then list the coordinates of the reflected triangle.

Original Triangle	Triangle Reflected Across the *x*-Axis	Triangle Reflected Across the *y*-Axis
Δ*ABC*	Δ*A′B′C′*	Δ*A″B″C″*
A (3, 4)		
B (6, 1)		
C (4, 9)		

Original Triangle	Triangle Reflected Across the *x*-Axis	Triangle Reflected Across the *y*-Axis
Δ*ABC*	Δ*A′B′C′*	Δ*A″B″C″*
A (3, 4)	*A′* (3, –4)	*A″* (–3, 4)
B (6, 1)	*B′* (6, –1)	*B″* (–6, 1)
C (4, 9)	*C′* (4, –9)	*C″* (–4, 9)

NOTES

Let's consider reflections of a different triangle without graphing.

4. The vertices of Δ*DEF* are *D* (−7, 10), *E* (−5, 5), and *F* (−1, −8).

a. If Δ*DEF* is reflected across the *x*-axis, what are the coordinates of the vertices of the image? Name the triangle.

b. How did you determine the coordinates of the image without graphing the triangle?

c. If Δ*DEF* is reflected across the *y*-axis, what are the coordinates of the vertices of the image? Name the triangle.

d. How did you determine the coordinates of the image without graphing the triangle?

Answers

4a. The coordinates of the vertices of Triangle *D′E′F′* are *D′* (−7, −10), *E′* (−5, −5), and *F′* (−1, 8).

4b. The coordinates of the vertices of Triangle *D′E′F′* were determined by writing the opposite of each of the *y*-coordinates. The *x*-coordinates stayed the same.

4c. The coordinates of the vertices of Triangle *D″E″F″* are *D″* (7, 10), *E″* (5, 5), and *F″* (1, −8).

4d. The coordinates of the vertices of Triangle *D″E″F″* were determined by writing the opposite of each of the *x*-coordinates. The *y*-coordinates stayed the same.

Answers

1a.

Coordinates of *JKLM*	Coordinates of *J'K'L'M'*
J (2, 2)	J' (−2, 2)
K (4, −1)	K' (−4, −1)
L (2, −2)	L' (−2, −2)
M (1, 0)	M' (−1, 0)

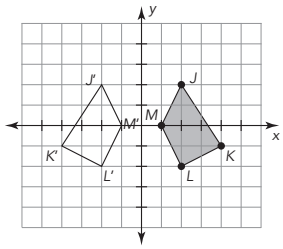
1b. Yes. I can reflect Quadrilateral *JKLM* across the *y*-axis to form Quadrilateral *J'K'L'M'*.

ACTIVITY 4.3

Verifying Congruence Using Reflections and Translations

Just as with translations, one way to verify that two figures are congruent is to show that the same sequence of reflections moves all the points of one figure onto all the points of the other figure.

1. Consider the two figures shown.



a. Complete the table with the corresponding coordinates of each figure.

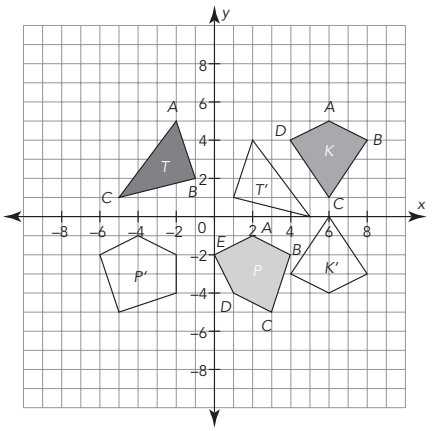
Coordinates of *JKLM*	Coordinates of *J'K'L'M'*

Remember, a rigid motion is a transformation that preserves the size and shape of the figure.

b. Is Quadrilateral *JKLM* congruent to Quadrilateral *J'K'L'M'*? Describe the sequence of rigid motions to verify your conclusion.

NOTES

2. Study the figures shown on the coordinate plane.



Determine whether each pair of figures are congruent. Then describe the sequence of rigid motions to verify your conclusion.

a. Is Figure *K* congruent to Figure *K′*?

b. Is Figure *P* congruent to Figure *P′*?

c. Is Figure *T* congruent to Figure *T′*?

Answers

2a. Sample answer. Yes. Translate *K* down 1 unit and reflect across the *x*-axis.

2b. No.

2c. Sample answer. Yes. Translate *T* down 1 unit and reflect across the *y*-axis.

Answers

1. The ordered pair (x, y) becomes $(x, -y)$.
2. The ordered pair (x, y) becomes $(-x, y)$.

NOTES

TALK the TALK

Reflecting on Reflections

1. **Describe how the ordered pair (x, y) of any figure changes when the figure is reflected across the x-axis.**

2. **Describe how the ordered pair (x, y) of any figure changes when the figure is reflected across the y-axis.**

ELL Tip

Support English Language Learners while they answer the questions in the Talk the Talk. Explain to students that *describe* means that there should be one or two sentences supporting what you are trying to prove. Have students first explain their reasoning verbally before writing it down on paper. Giving their answers verbally may help English Language Learners write their answers with more clarity.