Every Which Way

6

Combining Rigid Motions

WARM UP

Determine the distance between each pair of points.

1. (2, 3) and (−5, 3)
2. (−1, −4) and (−1, 8)
3. (6, −2.5) and (6, 5)
4. (−8.2, 5.6) and (−4.3, 5.6)

LEARNING GOALS

- Use coordinates to identify rigid motion transformations.
- Write congruence statements.
- Determine a sequence of rigid motions that maps a figure onto a congruent figure.
- Generalize the effects of rigid motion transformations on the coordinates of two-dimensional figures.

KEY TERMS

- congruent line segments
- congruent angles

You have determined coordinates of images by translating, reflecting, and rotating pre-images. How can you use the coordinates of an image to determine the rigid motion transformations applied to the pre-image?

Getting Started

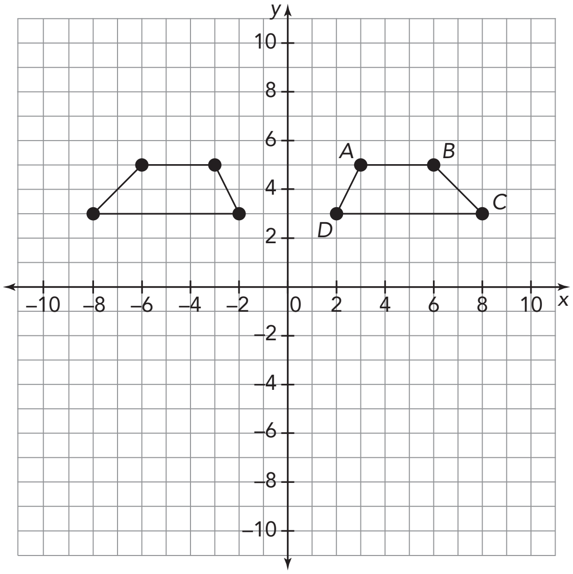
Going Backwards

Use your knowledge of rigid motions and their effects on the coordinates of two-dimensional figures to answer each question.

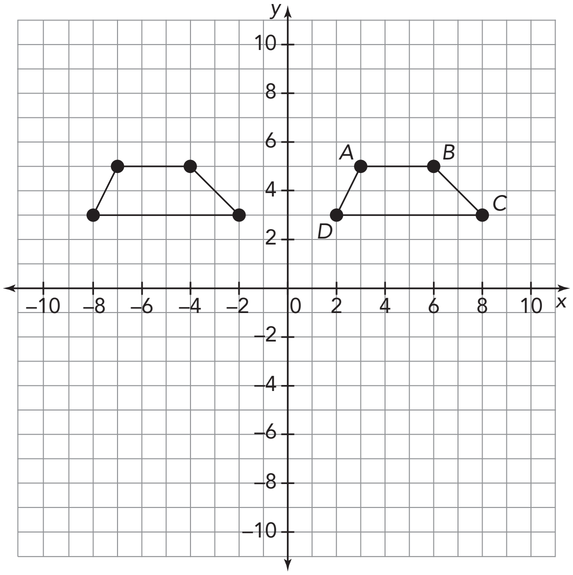
The line of reflection will be an axis, and the center of rotation will be the origin.

1. The pre-image and image of three different single transformations are given. Determine the transformation that maps the pre-image, the labeled figure, to the image. Label the vertices of the image. Explain your reasoning.

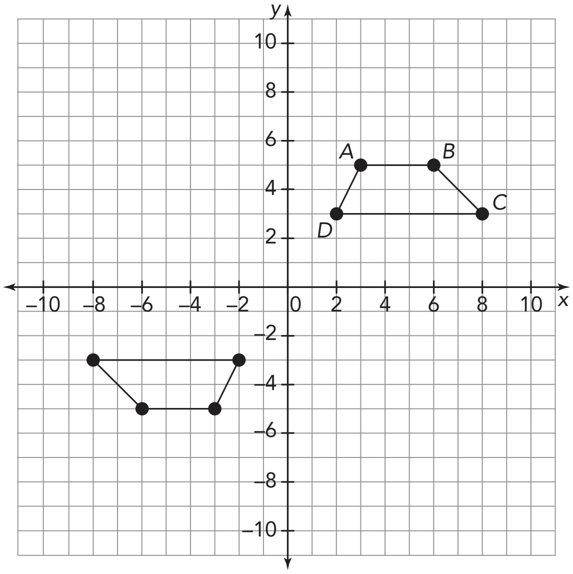
a.



b.

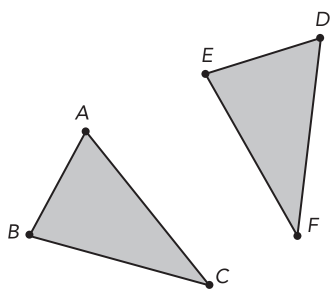


c.



2. Compare the order of the vertices, starting from *A*′, in each image with the order of the vertices, starting from *A*, in the pre-image.

ACTIVITY 6.1 Congruence Statements



You have determined that if a figure is translated, rotated, or reflected, the resulting image is the same size and the same shape as the original figure; therefore, the image and the pre-image are congruent figures.

1. How was Triangle *ABC* transformed to create Triangle *DEF*?

Congruent line segments are line segments that have the same length.

Because Triangle *DEF* was created using a rigid motion transformation of Triangle *ABC*, the triangles are congruent. Therefore, all corresponding sides and all corresponding angles have the same measure. In congruent figures, the corresponding sides are *congruent line segments.*

Think about congruent figures as a mapping of one figure onto the other. When naming congruent segments, write the vertices in a way that shows the mapping.

WORKED EXAMPLE

If the length of line segment *AB* is equal to the length of line segment *DE*, the relationship can be expressed using symbols. These are a few examples.

- $AB = DE$ is read "the distance between *A* and *B* is equal to the distance between *D* and *E*"
- $m\overline{AB} = m\overline{DE}$ is read "the measure of line segment *AB* is equal to the measure of line segment *DE*."

If the sides of two different triangles are equal in length, for example, the length of side *AB* in Triangle *ABC* is equal to the length of side *DE* in Triangle *DEF*, these sides are said to be congruent. This relationship can be expressed using symbols.

- $\overline{AB} \cong \overline{DE}$ is read "line segment *AB* is congruent to line segment *DE*."

2. Write congruence statements for the other two sets of corresponding sides of the triangles.

Likewise, if corresponding angles have the same measure, they are *congruent angles*. **Congruent angles** are angles that are equal in measure.

WORKED EXAMPLE

If the measure of angle A is equal to the measure of angle D, the relationship can be expressed using symbols.

- $m\angle A = m\angle D$ is read "the measure of angle A is equal to the measure of angle D."

If the angles of two different triangles are equal in measure, for example, the measure of angle A in Triangle ABC is equal to the measure of angle D in Triangle DEF, these angles are said to be congruent. This relationship can be expressed using symbols.

- $\angle A \cong \angle D$ is read "angle A is congruent to angle D."

3. Write congruence statements for the other two sets of corresponding angles of the triangles.

You can write a single congruence statement about the triangles that shows the correspondence between the two figures. For the triangles in this activity, $\triangle ABC \cong \triangle DEF$.

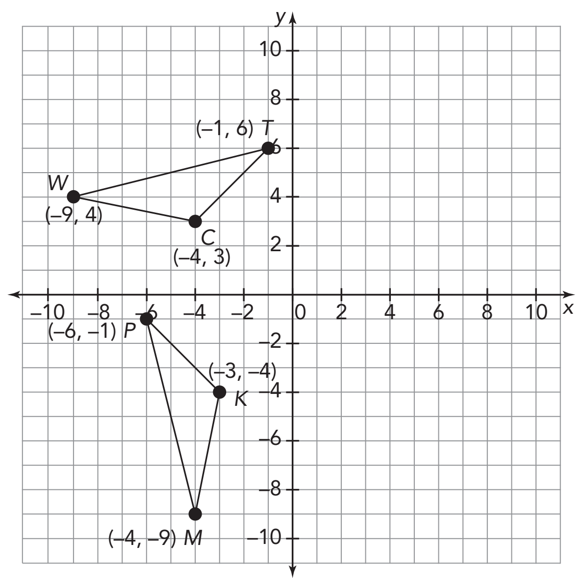
4. Write two additional correct congruence statements for these triangles.

NOTES

ACTIVITY 6.2 Using Rigid Motions to Verify Congruence

You can determine if two figures are congruent by determining if one figure can be mapped onto the other through a sequence of rigid motions. Therefore, if you know that two figures are congruent, you should be able to determine a sequence of rigid motions that maps one figure onto the other.

1. Analyze the two congruent triangles shown.



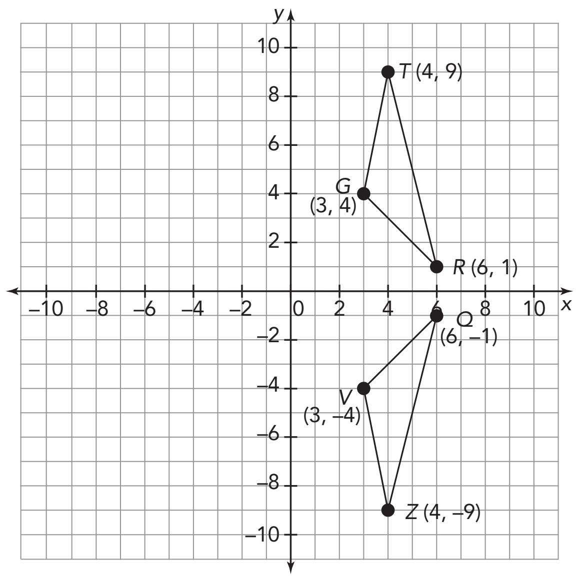
a. Identify the transformation used to create Δ*PMK* from Δ*TWC*.

b. Write a triangle congruence statement.

c. Write congruence statements to identify the congruent angles.

d. Write congruence statements to identify the congruent sides.

2. **Analyze the two congruent triangles shown.**



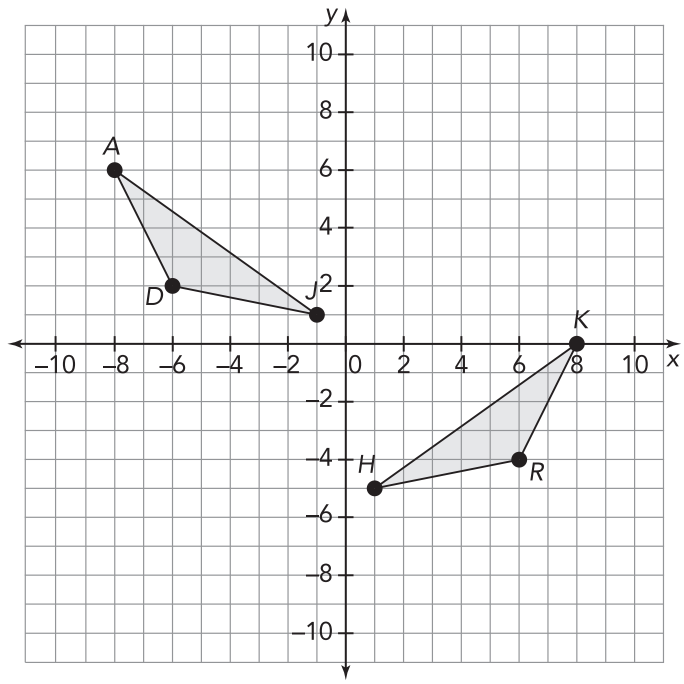
a. Identify the transformation used to create Δ*ZQV* from Δ*TRG*.

b. Write a triangle congruence statement.

c. Write congruence statements to identify the congruent angles.

d. Write congruence statements to identify the congruent sides.

3. **Analyze the two congruent triangles.**



a. **Write a congruence statement for the triangles. How did you determine the corresponding angles?**

b. **Identify a sequence of translations, reflections, and/or rotations that could be used to map one triangle onto the other triangle.**

c. **Reverse the order of the transformations that you used in part (b). Does this order map one figure onto the other?**

d. **Explain why it is not possible to map one figure onto the other using only rotations and translations.**

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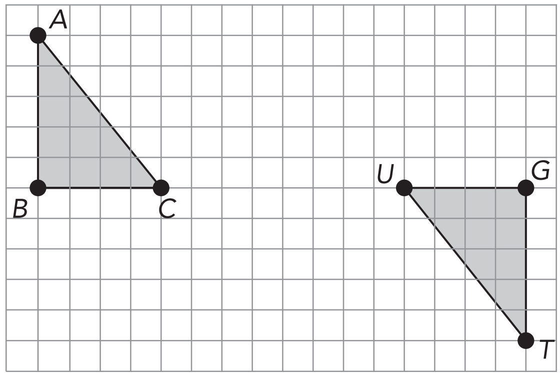
4. Analyze the two congruent triangles.



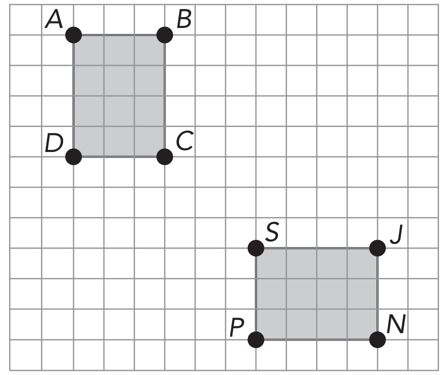
a. Write a congruence statement for the triangles.

b. Identify a sequence of translations, reflections, and/or rotations that could be used to map one triangle onto the other triangle.

c. Reverse the order of the transformations that you used in part (b). Does this order map one figure onto the other?

d. Can you determine a way to map one triangle onto the other in a single transformation? Explain your reasoning.

5. **Analyze the two congruent rectangles.**



   a. **Identify a sequence of translations, reflections, and/or rotations that could be used to verify that the rectangles are congruent.**

   b. **Can you determine a way to map one rectangle onto the other in a single transformation? Explain your reasoning.**

ACTIVITY **6.3**

Transformations with Coordinates

For the triangles in this activity, $\Delta PQR \cong \Delta JME \cong \Delta DLG$.

1. **Suppose the vertices of ΔPQR are P (4, 3), Q (−2, 2), and R (0, 0). Describe the translation used to form each triangle. Explain your reasoning.**

   a. **J (0, 3), M (−6, 2), and E (−4, 0)**

   b. **D (4, 5.5), L (−2, 4.5), and G (0, 2.5)**

2. Suppose the vertices of ΔPQR are P (1, 3), Q (6, 5), and R (8, 1). Describe the rotation used to form each triangle. Explain your reasoning.

   a. J (−3, 1), M (−5, 6), and E (−1, 8)

   b. D (−1, −3), L (−6, −5), and G (−8, −1)

3. Suppose the vertices of ΔPQR are P (12, 4), Q (14, 1), and R (20, 9). Describe the reflection used to form each triangle. Explain your reasoning.

   a. J (−12, 4), M (−14, 1), and E (−20, 9)

   b. D (12, −4), L (14, −1), and G (20, −9)

4. Suppose the vertices of ΔPQR are P (3, 2), Q (7, 3), and R (1, 7).

   a. Describe a sequence of a translation and reflection to form ΔJME with coordinates J (8, −2), M (12, −3), and E (6, −7).

   b. Describe a sequence of a translation and a rotation to form ΔDLG with coordinates D (2, −6), L (3, −10), and G (7, −4).

5. Are the images that result from a translation, rotation, or reflection always, sometimes, or never congruent to the original figure?

NOTES

TALK the TALK

Transformation Match-Up

Suppose a point (x, y) undergoes a rigid motion transformation. The possible new coordinates of the point are shown. Assume c is a positive rational number.

$(y, -x)$	$(x, y - c)$	$(x, -y)$
$(x + c, y)$	$(x - c, y)$	$(-y, x)$
$(-x, -y)$	$(-x, y)$	$(x, y + c)$

1. Record each set of new coordinates in the appropriate section of the table, and then write a verbal description of the transformation. Be as specific as possible.

Translations		Reflections		Rotations	
Coordinates	Description	Coordinates	Description	Coordinates	Description

2. Describe a single transformation that could be created from a sequence of at least two transformations. Use the coordinates to justify your answer.